1) Consider the following Statements.

P: X - Metrizable space. Then, Every bounded sequence has a Convergent Subsequence.

Q: Let $A \subseteq \mathbb{R}$ be Subspace topology of usual topology. Then Every bounded sequence has a Convergent subsequence.

WOTF is False.

a) P only b) Q only c) Both d) Neither

2) Let (X, d) be a metric space and let $N \subseteq X$. For $x \in X$, define,

$$d(x, N) := \inf \{ d(x, n) \mid n \in N \}.$$ If

$d(x, N) = 0 \ \forall x \in X$, then WOTF A true?

a) N - closed b) N - Compact

c) $N = X$ d) N - dense in X.

3) Let $X = [-1, 1] \times \{-1, 1\}$ with dictionary order topology. Consider the following.

P: $\{(0, -1)\}$ is open

Q: $\{(1, 1)\}$ is open.

WOTF is true.

a) P only b) Q only c) Both d) Neither

4) X - Topo. space. Let $A, B \subseteq X$.

P : $A \cup B$ is connected $\Rightarrow$ A or B is connected

Q : $A \cap B$ is connected $\Rightarrow$ $A \cup B$ is connected

WOTF is false.

a) P only b) Q only c) Neither d) Both

5) Let d_1 and d_2 be two distinct metrices & $\alpha, \beta > 0$. Then,

a) $f_1 := \max\{d_1, d_2\}$ is a metric

b) $f_2 := d_1 \cdot d_2$ is never a metric

c) $f_3 := \min\{d_1, d_2\}$ is not a metric

d) $f_4 := \alpha d_1 + \beta d_2$ is a metric

6) Let $X = \mathbb{Z}_-$ (negative integers) with order topology.

P : The set $(-3, -1]$ is not open

Q : The set $(-\infty, -1]$ is closed

WOTF is true.

a) P - true, Q - false b) P - false, Q - true

c) Both are true d) Both are false.

7) ~~WAT~~ ~~W~~ WOTF is / are true?

a) $X = \mathbb{R}$, $\tau_1 := \{ [a, \infty) \mid a \in \mathbb{R} \} \cup \{ \phi, \mathbb{R} \}$. Then

$$a_n = -1/n \to -\frac{1}{e}.$$

b) $X = \mathbb{N}$, $\tau_2 := \{ \{n, n+1, n+2, \dots\} \mid n \in \mathbb{N} \} \cup \{\phi\}$.

Then, $(a_n) = n$ cgs to 15.

c) $X = \mathbb{R}$, $\tau_3 := \{ G \subseteq \mathbb{R} \mid G \subset \mathbb{Q} \text{ or } \mathbb{Q} \subset G \}$

Then, $(a_n) = (1, -1, 1.4, 1, 1.41, -1, 1.414, 1, \dots)$ is convergent.

d) $X = \mathbb{R}$ with $\mathbb{R}_K$ topology, when $K = \{1/n \mid n \in \mathbb{N}\}$.

Then, $(a_n) = 1/n$ converges to 0.

8) P: Intersection of two non comparable topologies can form indiscrete topology.

Q: Intersection of two non comparable topologies forms a topology that is strictly finer than indiscrete topology.

R: Let τ_1, τ_2 be non comparable topologies & let τ_3, τ_4 be some other non comparable topo(s), then $\tau_1 \cap \tau_2$ and $\tau_3 \cap \tau_4$ are non-comparable

WOTF is true.

a) P only b) Q and R c) R only d) P and R

9) Let X be a topo. Space. Consider the following Product topologies.

$\tau_1 := \tau_{usual} \times \mathbb{R}_L$, $\tau_2 := \tau_{usual} \times \tau_f$

$\tau_3 := \tau_{usual} \times \mathbb{R}_L$, $\tau_4 := \tau_{usual} \times \mathbb{R}_U$

Then,

P : $\tau_3 \subseteq \tau_4$

Q : $\tau_1 \supseteq \tau_2$

$\mathbb{R}_L$ - Lower Limit topology

$\mathbb{R}_U$ - Upper Limit topology

a) P and Q are true b) P- true, Q - false

c) P- false, Q - true d) P and Q are false.

10) WOTF are/is false?

For $n > 2$, let

a) $S = \{A \in M_n(\mathbb{R}) \mid A = A^T\}$ with the metric,

$$d(A, B) := \sum_{i,j=1}^{n} |a_{ij} - b_{ij}| \quad \text{where } A = (a_{ij}),\ B = (b_{ij}).$$

Then, S is path connected.

b) For $n > 2$, $GL_n(\mathbb{R})$ with usual metric is closed.

c) $\{(x, y) \in \mathbb{R}^2 \mid x^2 - 3y^2 - 4x - 30y = 80\}$ is Convex.

d) $\mathbb{R}^2 \setminus (\mathbb{Q} \times \mathbb{Q} \cup \mathbb{Q} \times \mathbb{Q}^c)$ is not Connected.

11) Let X - topo. space & A, B be two non empty subsets of X. Then,

a) $A^{\circ} \setminus B^{\circ} \subseteq (A \setminus B)^{\circ}$

b) $\overline{A} \setminus \overline{B} \subseteq \overline{A \setminus B}$

c) $(X \setminus A)^{\circ} \subseteq X \setminus \overline{A}$

d) $(X \setminus A)^{\circ} \supseteq X \setminus \overline{A}$.

12) P : Union of two NON COMPARABLE topologies Can never be a topology.

Q : Union of two non Comparable topologies Can form a topology.

R : Union of two non Comparable topologies forms discrete topology.

WOTF is true.

a) P only b) Q only c) R only d) Q and R.