Complex Analysis

1. which of the followings are true?

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}} = 0$

c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} = 0$

d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2} = 1$

2) choose correct statements from below

a) $\exists$ non-constant entire fn f ~~$\ni \mathrm{Im}(f) = \mathbb{C}$~~ $\ni \mathrm{Im}(f) = \mathbb{C}\setminus\{i\}$

b) $\exists$ a non-constant entire fn $f \ni \mathrm{Im}(f) = \mathbb{C}\setminus\{1,i\}$

c) let $u(x,y) = x^2$, $\exists$ entire fn $f \ni$ whose $\mathrm{Re}(f) = u$

d) If f is entire then $\overline{\mathrm{Im}(f)} = \mathbb{C}$

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(3) The value of the integral $\int_{|z-1|=1} \frac{e^z}{z^2-2z} dz$ is

a) 0 b) $\pi i e$ c) $(\pi i e) - (\pi i e^{-1})$ d) $e + e^{-1}$

4. Let C be the contour oriented circle of radius $1/2$ centred at $i = \sqrt{-1}$

Then $\int_C \frac{dz}{z^4-1}$ is

a) $-\pi/2$ b) $\pi/2$ c) $-\pi$ d) π

5) Let $\gamma(t) = 3e^{it}$, $0 \le t \le 2\pi$ be the positively oriented circle of radius 3 centred at the origin.

The value of λ for which $\int_{\gamma} \frac{\lambda}{z-2} dz = \int_{\gamma} \frac{1}{z^2-5z+4} dz$

is a) $\lambda = -1/3$ b) $\lambda = 0$ c) $\lambda = 1/3$ d) $\lambda = 1$

6 Let $\gamma_k = \{ke^{ik\theta} \mid 0 \le \theta \le 2\pi\}$ for $k = 1, 2, 3$. Then which of the followings are correct?

a) $\frac{1}{2\pi i} \int_{\gamma_k} \frac{1}{z} dz = 0$ for $k = 1, 2, 3$

b) $\frac{1}{2\pi i} \int_{\gamma_2} \frac{1}{z} dz = 4$ c) $\frac{1}{2\pi i} \int_{\gamma_1} \frac{1}{z} dz = 1$

d) $\frac{1}{2\pi i} \int_{\gamma_3} \frac{1}{z} dz = 3.$

7) Let C be denote the unit ~~disc~~ circle centre at origin then $\frac{1}{2\pi i}\int_{C:|z|=1} |1+z+z^2|^2 dz$,

where the integral is taken anti-clockwise along C equals.

a) 0 b) 1 c) 2 d) 3

8) Let $I_r = \int_{C_r} \frac{dz}{z(z-1)(z-2)}$, where $C_r := \{z \in \mathbb{C} \mid |z| = r\}$

where $r > 0$ then

a) $I_r = 2\pi i$ if $r \in (2,3)$

b) $I_r = \frac{1}{2}$, if $r \in (0,1)$

c) $I_r = -2\pi i$ if $r \in (1,2)$

d) $I_r = 0$ if $r > 3$.

9) $\int_{\gamma: |z|=1} \frac{z^2}{4-z^2} dz$ is a) πi b) $2\pi i$ c) $4\pi i$ d) 0

10. The minimum possible value of $z^2 + (z-3)^2 + (z-6i)^2$ is

a) 15 b) 45 c) 0 d) can't say

11. Given a real number $a > 0$, consider the triangle Δ with vertices $0, a, a+ia$. If Δ is given the counter clockwise orientation, then the contour integral $\int_{\Delta} Re(z) \, dz$ is

a) 0 b) $i\frac{a^2}{2}$ c) ia^2 d) $i\frac{3}{2}a^2$

12) consider the f $f(z) = \frac{\sin\left(\frac{\pi z}{2}\right)}{\sin(\pi z)}$ Then f has poles at

a) all integer b) All even integer c) All odd integer

d) All integer of the form $4K+1$, $K \in \mathbb{Z}$.

13) suppose that $f : \mathbb{C} \to \mathbb{C}$ is analytic fn. Then f is a poly if

a) For any point $\alpha \in \mathbb{C}$ if $f(z) = \sum a_n (z-a)^n$ is a power series expansion at 'a', Then $a_n = 0$ for atleast one n.

b) $\lim_{|z| \to \infty} |f(z)| = \infty$

c) $\lim_{|z| \to \infty} |f(z)| = M$ for some M.

d) $|f(z)| \le M|z|^n$ for $|z|$ sufficiently large and for some n.

14) For $z \in \mathbb{C}$, define $f(z) = \frac{e^z}{e^z - 1}$ Then

a) f is entire b) f has infinitly many poles on the imaginary axis

c) The only singularities of f are poles

d) Each pole of f is simple

15) the power series $\sum_{n=0}^{\infty} \frac{1}{3^n}(z-1)^n$ converges if a) $|z| \leq 3$ b) $|z| < \sqrt{3}$

c) $|z-1| < \sqrt{3}$ d) $|z-1| \leq \sqrt{3}$

16) Consider the power series $\sum_{n=1}^{\infty} a_n z^n$, where a_n = number of divisors of n^{50}, then the radius of convergence of $\sum_{n=1}^{\infty} a_n z^n$ is

a) 1 b) 50 c) $\frac{1}{50}$ d) 0

17. let $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ and $q(z) = b_1 z + b_2 z^2 + \cdots + b_n z^n$ be complex polys. If a_0, b_1 are non-zero complex numbers then the residue of $\frac{p(z)}{q(z)}$ at '0' is equal to

a) $\frac{a_0}{b_1}$ b) $\frac{a_1}{b_1}$ c) $\frac{b_1}{a_0}$ d) $\frac{a_0}{a_1}$

18 The fixed points of $f(z) = \frac{2iz+5}{z-2i}$ are

a) $1 \pm i$ b) $1+2i$ c) $2i \pm 1$ d) $i \pm 1$

19. The fn $w(z) = -\left(\frac{1}{z} + bz\right), -1<b<1$ maps $|z|<1$ onto

a) A half-plane b) Exterior of the circle

c) Exterior of an ellipse

d) Interior of an ellipse

20 The fn $f(z) = z^2$ maps the 1^{st} quadrant onto a) itself b) upper half-plane

c) Third quadrant d) Right half-plane

21) The bilinear transformation $w = \frac{2z}{z-2}$ maps $\{z \mid |z-1|<1\}$ onto

a) $\{w \mid Re(w)<0\}$ b) $\{w \mid Im(w)>0\}$

c) $\{w \mid Re(w)>0\}$ d) $\{w \mid |w+2|<1\}$

22) Let $A = \{z \in \mathbb{C} \mid |z| > 1\}$

$B = \{z \in \mathbb{C} \mid z \neq 0\}$

Then

a) $\exists$ is a contg onto fn $f: A \to B$

b) $\exists$ is a contg ~~onto fn~~ 1-1 fn $f: B \to A$

c) $\exists$ is a non-constant analytic fn $f: B \to A$

d) $\exists$ is a non-constant analytic fn $f: A \to B$.

23) Let a, b, c be non-collinear points in the Complex plane and Δ denote the closed triangular region of the plane with vertices a, b, c. For $z \in \Delta$,

Let $h(z) = |z-a||z-b||z-c|$ The maximum value of the fn h is

a) Not attained at any point of Δ

b) attained at an interior point of Δ

c) attained at the centre of gravity of Δ

d) attained at a boundary point of Δ

24) Let $f(z) = 2z^2 - 1$ then the maximum value of $|f(z)|$ on the unit disc $T = \{z \in \mathbb{C} \mid |z| \leq 1\}$ equals

a) 1 b) 2 c) 3 d) 4.

25) The bilinear transformation that maps -1 to 0 and 1 to i, -1 to i respectively. Then $f(1-i)$ equals

a) $-1+2i$ b) $2i$ c) $-2+i$ d) $-1+i$

26) The bilinear transformation as which maps the points $0, 1, \infty$ in the z-plane onto the points $-i, \infty, 1$ in w-plane is

a) $\frac{z-1}{z+i}$ b) $\frac{z-i}{z+1}$ c) $\frac{z+i}{z-1}$ d) $\frac{z+1}{z-i}$

27) $f(z) = \dfrac{z}{3z+1}$ ~~maps~~

a) maps H^+ onto H^+ and H^- onto H^-

b) maps H^+ onto H^- and H^- onto H^+

c) maps H^+ onto L^+ and H^- onto L^-

d) maps H^+ onto L^- and H^- onto L^+

where $H^+ = \{z \in \mathbb{C} \mid Im(z) > 0\}$

$H^- = \{z \in \mathbb{C} \mid Im(z) < 0\}$

$L^- = \{z \in \mathbb{C} \mid Re(x) > 0\}$

$L^+ = \{z \in \mathbb{C} \mid Re(x) < 0\}$

28) $f(z) = \dfrac{2z+1}{5z+3}$

a) maps H^+ onto H^+ and H^- onto H^-

b) maps H^+ onto H^- and H^- onto H^+

c) maps H^+ onto L^+ and H^- onto L^-

d) maps H^+ onto L^- and H^- onto L^+

29) Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$. Then $\exists$ a holomorphic fn $f : D \to \bar{D}$ with $f(0) = 0$ with the property

a) $f'(0) = \frac{1}{2}$ b) $|f(1/3)| = 1/4$

c) $f(1/3) = 1/2$ d) $|f'(0)| = \sec(\pi/6)$

30) Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$ Then

a) $\exists$ a analytic fn $f : D \to D$ with $f(0) = 0$ and $f'(0) = 2$

b) $\exists$ a holomorphic fn $f : D \to D$ with $f(3/4) = 3/4$ and $f'(2/3) = 3/4$

c) $\exists$ a holomorphic fn $f : D \to D$ with $f(3/4) = -3/4$ and $f'(3/4) = -3/4$

d) $\exists$ a holomorphic fn $f : D \to D$ with $f(1/2) = -1/2$ and $f'(\frac{1}{4}) = 1$.