

## KEY

### Section 1: Algebra

**1.1**  $1, 1, \frac{1 \pm i\sqrt{3}}{2}$

**1.2** a,c

**1.3** one

**1.4** a,b

**1.5** 1,2

**1.6**

$$\begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**1.7** Any three linearly independent  $2 \times 2$  matrices with trace zero.

Example:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

**1.8**  $\mathbb{R}$

**1.9** Any  $2 \times 2$  matrix with trace  $-1$  and determinant 1.

**1.10** a,b,c

### Section 2: Analysis

**2.1** a,b,c

**2.2**  $\frac{1}{3} \log 2$

**2.3**  $\mathbb{R} \setminus \{-1\}$

**2.4** a,b,c

**2.5** uniformly on  $[0, 1]^*$

**2.6** a,c

**2.7**  $\frac{2}{3}$

**2.8** a,b

**2.9** a,b

**2.10**  $\pm \frac{\sqrt{3}+i}{\sqrt{2}}$

### Section 3: Topology

**3.1** c

**3.2** a,b,c

**3.3** a,c

**3.4** a,b,c

**3.5** a,c

**3.6** b

**3.7** b

**3.8** a,b

**3.9** a,b,c

**3.10** a

### Section 4: Applied Mathematics

**4.1** 3

**4.2**  $h = 2r$

**4.3**  $\operatorname{div}(\mathbf{u}) = 0$

**4.4**  $Mx'' + cx' + kx = 0; x(0) = x_0; x'(0) = 0,$  where  $k$  and  $c$  are positive constants

**4.5**  $\max z = 17; x = 2; y = 1$

**4.6**  $\min f = u - 2v + 4w$  such that

$u - 2v + w \geq 5; -u - v + 2w \geq 7; u, v, w \geq 0$

**4.7**  $x(t) = c_1 e^{2t} + c_2 e^{-3t}; y(t) = c_1 e^{2t} - 4c_2 e^{-3t}$

**4.8**  $(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

**4.9**  $u(x, y) = y(x^3 - 3x + 1)$

**4.10** 8

### Section 5: Miscellaneous

**5.1** a,b,c

**5.2**  $\frac{1}{4}$

**5.3** a. countable; b. uncountable; c. countable

**5.4**  $\sqrt{14}$

**5.5**  $n + 1$

**5.6** one

**5.7**  $\frac{N}{2}\phi(N)$

**5.8** b,c

**5.9**  $4n2^{n-1} - 2^n + 1$

**5.10**  $1 - \frac{1}{2}5^{\frac{1}{3}}$

#### Note:

Accept any correct equivalent form of the answers.

\* Qn. 2.5: Accept even if the answer is just ‘uniformly’