I- which of the following are true ?

1. If $\{a_n\}, \{b_n\}$ are real seq such that $\{a_n\} \to l$ but $\{b_n\}$ diverges Then

a) $\{a_n \pm b_n\}$ Can be convergent

b) $\{a_n b_n\}$ Can be divergent

c) $\{a_n b_n\}$ can be congt

d) $\{a_n^2 + b_n\}$ may be divergent.

2. Let $\{a_n\}$ be a bdd seq of real numbers Then

a) every subseq of $\{a_n\}$ is convergent

b) There is exactly one subseq of $\{a_n\}$ which is cgt

c) There are infinitly many subseq of $\{a_n\}$ which are convergent

d) There is a subseq of $\{a_n\}$ which is convgt.

3. Let $\{x_n\}$ be a seq of non-nega-tive real numbers. Then which of the following is true?

a) $\lim_{n\to\infty} \inf\{x_n\} = 0 \Rightarrow \lim_{n\to\infty} x_n^2 = 0$

b) $\lim_{n\to\infty} \sup\{x_n\} = 0 \Rightarrow \lim_{n\to\infty} x_n^2 = 0$

c) $\lim_{n\to\infty} \inf\{x_n\} = 0 \Rightarrow \{x_n\}$ is bdd

d) $\lim_{n\to\infty} \inf\{x_n^2\} > 4 \Rightarrow \lim_{n\to\infty} \sup\{x_n\} > 4$

4. Suppose that $\{x_n\}$ is a seq of +ve reals. Let $y_n = \frac{x_n}{1+x_n}$ Then which of the following are true?

a) $\{x_n\}$ is convgnt if $\{y_n\}$ is convergent

b) $\{y_n\}$ is convgnt if $\{x_n\}$ is convergent

c) $\{y_n\}$ is bdd if $\{x_n\}$ is bdd

d) $\{x_n\}$ is bdd if $\{y_n\}$ is bdd.

5. If $\{x_n\}$ is a convgnt seq in $\mathbb{R}$ and $\{y_n\}$ is a bdd seq in $\mathbb{R}$. Then we can conclude that

a) $\{x_n+y_n\}$ is convgnt

b) $\{x_n+y_n\}$ is bdd

c) $\{x_n+y_n\}$ has no convgnt subseq.

d) $\{x_n+y_n\}$ has no bdd subseq.

6. Let $S=\{(x,y) \mid x^2+y^2=\frac{1}{n^2}, n\in\mathbb{N},$ and either $x\in\mathbb{Q}$ or $y\in\mathbb{Q}\}$

a) S is a finite non-empty set

b) S is countable

c) S is uncountable

d) S is empty.

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7. Let $\{x_n\}$ be a seq of real numbers satisfying $a_1 \geq 1$ and $a_{n+2} \geq a_n + 1$ for all $n \geq 1$. Then which of the followings are necessarily true?

a) $\sum_{n=1}^{\infty} \frac{1}{a_n^2}$ diverges

b) $\{a_n\}$ is bdd

c) $\sum_{n=1}^{\infty} \frac{1}{a_n^2}$ converges

d) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ - converges

8. Let $\{x_n\}$ be a seq of a real numbers pick out the case which imply that the seq is cauchy?

a) $|x_n - x_{n+1}| \leq 1/n \ \forall n \in \mathbb{N}$

b) $|x_n - x_{n+1}| \leq 1/n^2, \forall n \in \mathbb{N}$

c) $|x_n - x_{n+1}| \leq 1/2^n, \forall n \in \mathbb{N}$

d) None of the above.

9. If a seq $\{x_n\}$ is monotone and bdd, then

a) $\exists$ a subseq of $\{x_n\}$ that diverges

b) there may exist a subseq of $\{x_n\}$ that not monotone

c) All subseq of $\{x_n\}$ converges to the same limit

d) There exists at least two subseq of $\{x_n\}$ which converges to distinct limits.

10. let $0 < a_1 < b_1$, for $n \geq 1$

$$a_{n+1} = \sqrt{a_n b_n}, \quad b_{n+1} = \frac{a_n + b_n}{2}$$

Which of the followings is not true?

a) Both $\{a_n\}$, $\{b_n\}$ converges but the limits are not equal

b) Both $\{a_n\}$ and $\{b_n\}$ converges and the limits are not equal

c) $\{b_n\}$ is decreasing seq

d) $\{a_n\}$ is an increasing seq.

11. Let $x_n = 2^{2n}\left(1-\cos\left(\frac{1}{2^n}\right)\right), \forall n \in \mathbb{N}$

Then the seq $\{x_n\}$

a) Does not converges

b) converges to zero

c) converges to $1/2$

d) converges to $1/4$

12. Let $s \in (0,1)$ Then decide which of the followings are true?

a) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} \ni s > m/n$

b) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} \ni s < m/n$

c) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} \ni s = m/n$

d) $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} \ni s = m+n.$

13. $\lim_{n\to\infty} \left(1-\frac{1}{n^2}\right)^n$ equal

a) 1 b) $e^{-1/2}$ c) e^{-2} d) e^{-1}

14. Let $p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{100} x^{100}$

Then $\lim_{n\to\infty} \frac{p(n)}{5^n}$ is

a) 5 b) 1 c) 0 d) ∞

15. which of the following is/are correct?

a) $\left(1+\frac{1}{n}\right)^{n+1} \longrightarrow e$ as $n\to\infty$

b) $\left(1+\frac{1}{n+1}\right)^{n+1} \longrightarrow e$ as $n\to\infty$

c) $\left(1+\frac{1}{n}\right)^{n^2} \longrightarrow e$ as $n\to\infty$

d) $\left(1+\frac{1}{n^2}\right)^{n} \to e$ as $n\to\infty$