① Let A be a 3×3 real matrix such that A and A^2 have the same char poly Then possible trace of A is?

a) $\{3\}$ b) $\{0,1,3\}$

c) $\{0,1,2,3\}$ d) $\{1,2,3\}$

② Let $\mathbb{P}_n(\mathbb{R})$ be a vector space over $\mathbb{R}$.

Let $T: \mathbb{P}_n(\mathbb{R}) \longrightarrow \mathbb{P}_n(\mathbb{R})$

$$T(f(x)) = x f'(x) - f(x-1)$$

Then $tr(T)$ is?

a) $\frac{(n+1)(n-2)}{2}$ b) $\frac{n(n+1)(n-2)}{2}$

c) $\frac{n(n+1)}{2}$ d) $\frac{n(n-1)}{2}$

(3) Let $V = M_2(\mathbb{R})$ be a v.s over $\mathbb{R}$.

$W_1 = \{ A \in M_2(\mathbb{R}) \mid A\begin{pmatrix}1\\0\end{pmatrix} = \lambda\begin{pmatrix}1\\0\end{pmatrix}$ for some $\lambda \in \mathbb{R} \}$

Then $\dim(W_1)$ is

a) 3 b) 4 c) 1 d) 2.

(4) Let $V = M_3(\mathbb{R})$ be v.s over $\mathbb{R}$.

Let $A \in M_3(\mathbb{R})$ be a fixed matrix such that $\text{rank}(A) = 2$

Let $T : M_3(\mathbb{R}) \to M_3(\mathbb{R})$

$T(B) = AB, \ \forall B \in M_3(\mathbb{R})$

Then which of the followings are ~~not~~ true

a) $\text{rank}(T) = 6$

b) $\text{rank}(T) \geq 3$

c) $\text{rank}(T) \leq 7$

d) $|T| = 0$

~~e) all are false~~

e) all are true

5. Let $V = M_n(\mathbb{R})$ be a v.s over $\mathbb{R}$.

Let $W = \left\{ A = (a_{ij}) \in M_n(\mathbb{R}) \;\middle|\; \sum_{j=1}^{n} a_{ij} = 0, \ \forall i = 1, 2 \dots n \text{ and } \sum_{i=1}^{n} a_{ij} = 0, \ \forall j = 1, 2 \dots n \right\}$

Then $\dim(W)$ is

a) $n^2 - 2n + 1$ b) $\dfrac{n(n+1)(2n+1)}{6}$

c) $\dfrac{n(n+1)}{2}$ d) $\dfrac{n(n-1)(n+1)}{6}$

6. Let $V = M_n(\mathbb{R})$ be a v.s over $\mathbb{R}$, $n \geq 5$.

Let $A \in M_n(\mathbb{R}) \ni AA^T = A^TA = I$.

Then which of the followings are true?

a) A has atleast one real eigenvalue

b) If A has real eigenvalue then it may be ± 1.

~~c) If A has all eigenvalue real then $A = I$~~

d) A is upper triangular matrix over $\mathbb{R}$.

7. Suppose A is an $n \times n$ matrix with real entries (where A is fixed) such that A is commutes with every $B \in M_n(\mathbb{R})$ Then

a) A is diagonalizable over $\mathbb{R}$

b) A is may be nilpotent matrix

c) A is upper triangularizable over $\mathbb{R}$

d) $A^2 + A + I$ is diagonalizable over $\mathbb{R}$.