# LINEAR ALGEBRA.

1. let $J$ denote a $101 \times 101$ matrix with all the entries equal to $1$ and let $I$ denote the identity matrix of order $101$. Then the determinant of $J-I$ is

(a) 101

(b) 1

(c) 0

(d) 100.

2. let $A$ and $B$ be $n \times n$ matrices over $\mathbb{C}$. Then

(a) $AB$ and $BA$ always have the same set of eigenvalues.

(b) If $AB$ and $BA$ have the same set of eigen values then $AB = BA$.

(c) If $A^{-1}$ exists then $AB$ and $BA$ are similar

(d) The rank of $AB$ is always the same as the rank of $BA$.

3. let $A$ be an invertible $4 \times 4$ real matrix.

Which of the following are not true?

(a) Rank $A = 4$

(b) For every vector $b \in \mathbb{R}^4$, $Ax = b$ has exactly one solution.

(c) dim (nullspace $A$) $\geq 1$

(d) $0$ is an eigen value of $A$.

4. let $A$ be an $n \times n$ complex matrix. Assume that $A$ is self-adjoint and let $B$ denote the inverse of $A + iI_n$. Then all eigen values of $(A - iI_n)B$ are

(a) purely Imaginary

(b) of modulus one

(c) Real

(d) of modulus less than one.

5. let $V$ denote the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to $n$. which of the following defines a norm on $V$?

(a) $\|p\|^2 = |p(1)|^2 + \dots + |p(n+1)|^2$, $p \in V$

(b) $\|p\| = \sup_{t \in [0,1]} |p(t)|$, $p \in V$

(c) $\|p\| = \int_0^1 |p(t)|\,dt$, $p \in V$.

(d) $\|p\| = \sup_{t\in[0,1]} |p'(t)|$, $p \in V$.

6. Consider the vector space $V$ of real polynomial of degree less than or equal to $n$. Fix distinct real numbers $a_0, a_1, \ldots, a_k$. For $p \in V$ $\max\{|p(a_j)| : 0 \le j \le k\}$ defines a norm on $V$

(a) only if $k < n$

(b) only if $k \ge n$

(c) if $k+1 \le n$

(d) if $k \ge n+1$.

7. Suppose $A, B$ are $n\times n$ positive definite matrices and $I$ be the $n\times n$ Identity matrix Then which of the following are positive definite.

(a) $A+B$

(b) $ABA^*$

(c) $A^2 + I$

(d) $AB$.

8. Which of the following are positive definite?

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

9. If A is $(5\times5)$ matrix and the dimension of the solution space of $Ax=0$ is atleast two then

(a) Rank $(A^2) \leq 3$ (b) Rank $(A^2) \geq 3$

(c) Rank $(A^2) = 3$ (d) Det $(A^2) = 0$.

10. let A be a $4\times7$ real matrix and B be a $7\times4$ real matrix such that $AB = I_4$. Which of the following is/are always true?

a) Rank $(A) = 4$

b) Rank $(B) = 7$

c) Nullity $(B) = 0$

d) $BA = I_7$ where $I_7$ is the $7\times7$ identity matrix.

11) let T be the linear transformation from the real vector space $\mathbb{R}[x]$ to itself. given by $T(f) = f'$; where $f'$ is the derivative of f. Consier the following statements about T:

P: T is nilpotent

Q: The only eigen value of T is 0.

(a) only P is correct (b) only Q is correct

(c) Both P and Q are correct

(d) Both P and Q are not correct.

12. let a $2\times 2$ matrix $A(\neq I)$ with real entries such that $A^3 = I$ then

(a) $tr(A) = 2$ (b) $\det(A) = I$

(c) $tr(A) = -1$ (d) matrix A not exist.

13. let x and y in $\mathbb{R}^n$ be non-zero column vectors. From the matrix $A = xy'$, where $y'$ is transpose of y. Then the rank of A is

(a) 2 (b) 0 (C) atleast $n/2$

(d) none of the above

14. let $A \in M_n(\mathbb{R}) \ni$ Rank $(A) = 1$. , $n \geq 2$

a) If $\exists$ a $v \neq 0 \ni A^n v \neq 0 \; \forall n$ then $A$ is not diagonaizable.

b) The degree of minimal polynomial of $A$ is 2.

c) The dimension of null space of corresponding Jacobian matrix is $n-1$.

d) The degree of minimal polynomial of $A$ is $\left[\frac{n}{2}\right]$.

15. let $V$ be an $n$-dimensional vector space and let $T: V \rightarrow V$ be such that

Rank $T \leq$ Rank $T^3$, Then which of the following statements are necessarily true?

(a) Null space $(T) =$ Range $(T)$

(b) Null space $(T) \cap$ Range $(T) = \{0\}$

(c) If $T^2 = 0$, then $T$ is zero transformation

(d) Rank $(T^2) =$ Rank $(T)$.

16. let $A \in M_3(\mathbb{R})$ and let $X = \{C \in GL_3(\mathbb{R}) \mid CAC^{-1}$ is triangular$\}$, Then

(a) $X \neq \phi$

(b) If $X = \phi$, then $A$ is diagonalizable over $\mathbb{C}$ and $\mathbb{R}$.

(c) If $X = \phi$ then $A$ is diagonalizable over $\mathbb{C}$

(d) If $X = \phi$, then $A$ has no real eigenvalue.

17. let $A$ be a real matrix with characterstic polynomial $(x-1)^3$. pick the statements from below.

(a) $A$ is necessarily diagonalizable

(b) If the degree of minimal polynomial is strictly less than the G.M of 1 then $A$ is not diagonalizable.

(c) Characterstic polynomial of $A^n$ is $(x-1)^n$

(d) If $A$ has exactly two Jordan blocks, then $(A-I)^2$ is diagonalizable.

18. A linear operator T on a complex vector space V has characteristic polynomial $x^3(x-5)^2$ and minimal polynomial $x^2(x-5)$. Choose the correct options.

(a) The Jordan form of T is uniquely determined by the given information.

(b) There are exactly 2 Jordan blocks in the Jordan decomposition of T.

(c) The operator induced by T on the quotient space $V/\ker(T)$ is a scalar multiple of the identity operator.

(d) The operator induced by T on the quotient space $V/\ker(T-5I)$ is nilpotent.

19. Let $\mathbb{R}^n, n \geq 2$ be equipped with standard inner product. Let $\{v_1, v_2, \cdots v_n\}$ be n column vectors forming an orthonormal basis of $\mathbb{R}^n$. Let A be the $n \times n$ matrix formed by the column vectors then

a) $A = A^{-1}$ b) $A = A^T$ c) $A^{-1} = A^T$ d) $\mathrm{Det}(A) = \mathbb{I}$

20. let $A$ be an $n \times n$ matrix with real entries. Define $\langle x, y \rangle_A := \langle Ax, Ay \rangle$, $x, y \in \mathbb{R}^n$. Then $\langle x, y \rangle_A$ defines an inner product if and only if

(a) $\operatorname{Ker} A = \{0\}$ (b) $\operatorname{rank} A = n$

(c) All eigen values of $A$ are positive

(d) All eigen values of $A$ are non-negative.