E

SUBJECT CODE BOOKLET CODE

4 C

2014 (I)
MATHEMATICAL SCIENCES
TEST BOOKLET

Time : 3:00 Hours

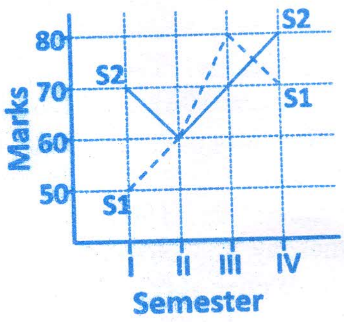
Maximum Marks: 200

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part'A'+40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet of the same code. Likewise, check the answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name and Serial Number of this Test Booklet on the Answer sheet in the space provided. Also put your signatures in the space earmarked.
4. **You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in Part 'A' carries **2** marks, Part 'B' **3** marks and Part 'C' **4.75** marks respectively. There will be negative marking @ 0.5marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have **'ONE'** or **'MORE'** correct options. Credit in a question shall be given only on identification of **'ALL'** the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on answer sheet or sheets for rough work.
9. Use of calculator is not permitted.
10. **After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the invigilator and retain the carbonless copy for your record.**
11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.

PART 'A'

1. Marks obtained by two students S1 and S2 in a four semester course are plotted in the following graph



Which of the following statements is true?

1. S2 got higher marks than S1 in all four semesters. ✗
2. ✓ Over four semesters, S1 improved by a higher percentage compared to S2.
3. Total marks of S1 and S2 are equal. ✗
4. S1 and S2 did not get the same marks in any semester. ✗

2. The following table shows the price of diamond crystals of a particular quality.

Wt. of a diamond crystal (in carat)	Price per carat (in lakh Rs.)
1	4
2	8
3	12
4	16

What will be the price (in lakh Rs.) of a 2.5 carat diamond crystal?

1. ✓ 10   2. 20
3. 25   4. 50

3. How many digits are there in

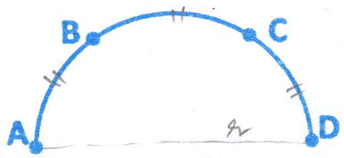
$2^{17} \times 3^2 \times 5^{14} \times 7$?

1. 14   2. 15
3. ✓ 16   4. 17

4. A man on the equator moves along 0° longitude up to 45° N. He then turns east and moves up to 90° E, and returns to the equator along 90°E. The distance covered in multiples of Earth's radius R is

1. $\left(\frac{3}{4}\pi\right)R$ 2. $\left(\frac{\pi}{2}+\frac{\pi}{4\sqrt{2}}\right)R$
3. $\left(\frac{\pi}{2}+\frac{\pi}{2\sqrt{2}}\right)R$ 4. $\left(\frac{\pi}{4}+\frac{\pi}{\sqrt{2}}\right)R$

5.



On a semi-circle of diameter 10m drawn on a horizontal ground are standing 4 boys A, B, C and D with distances AB = BC = CD. The length of line-segment joining A and B is

1. 5 m   2. 6 m
3. 7 m   4. ✓ $\frac{5\pi}{3}$ m

6. What is the next number in the following sequence?
2, 3, 5, 6, 3, 4, 7, 12, 4, 5, 9, . . . ,

1. 10   2. 20
3. 13   4. 6

17 log 2 + 2 log 3 + 14 log 5 + log 7

7. The following sum is

$$1 + 1 - 2 + 3 - 4 + 5 - 6 \ldots - 20 = ?$$

1. 10
2. -10
3. -11
4. -9

8. You are given 100 verbs using which you have to form sentences containing at least one verb, without repeating the verbs, under the condition that the number of verbs (from this set of 100) in any two sentences should not be equal. The maximum number of sentences you can form is

1. 10
2. 13
3. 14
4. 100

9 A 4×4 magic square is given below.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

How many 2×2 squares are there in it whose elements add up to 34?

1. 6
2. 9
3. 4
4. 5

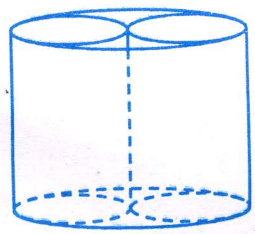
10. To go from the engine to the last coach of his train of length 200 m, a man jumped from his train to another train moving on a parallel track in the opposite direction, waited till the last coach of his original train appeared and then jumped back. In how much time did he reach the last coach if the speed of each train was 60 km/hr?

1. 5 s
2. 6 s
3. 10 s
4. 12 s

11. November 9, 1994 was a Wednesday. Then which of the following is true?

1. November 9, 1965 is a Wednesday and November 9, 1970 is a Wednesday.
2. November 9, 1965 is not a Wednesday and November 9, 1970 is a Wednesday.
3. November 9, 1965 is a Wednesday and November 9, 1970 is not a Wednesday.
4. November 9, 1965 is not a Wednesday and November 9, 1970 is not a Wednesday.

12.



Two identical cylinders of the same height as a bigger hollow cylinder were put vertically into the latter to fit exactly into it. The volume of the bigger cylinder is V. The volume of each of the smaller cylinders is

1. $V/8$
2. $V/4$
3. $V/2$
4. V

13. You get 20% returns on your investment annually, but also pay a 20% tax on the gain. At the end of 5 years, the net gain made by you (as percentage of the capital) is approximately

1. 0
2. 16
3. 80
4. 100

14. Coordinates of a point in x, y, z space is (1, 2, 3). What would be the coordinates of its reflection in a mirror along the x, z plane?

1. (− 1, −2, −3)
2. (1, −2, −3)
3. (1, −2, 3)
4. (− 1, 2, 3)

15. A cubic cavity of edge 20 μm is filled with a fluid with a cubic solid of edge 2 μm. What percent of the cavity volume is occupied by the fluid?

1. 10.0
2. 20.0
3. 90.0
4. 99.9

16. Find the missing number in the sequence

61, 52, 63, 94,__,18, 001, 121.

1. 46
2. 70
3. 66
4. 44

17. What is the length of the longest rod that can be put in a hemispherical bowl of radius 10 cm such that no end of the rod is outside the bowl? (Assume that the rod has negligible thickness.)

1. $10\sqrt{2}$ cm
2. $10\sqrt{3}$ cm
3. $10\sqrt{4}$ cm
4. $10\sqrt{5}$ cm

18. If a 4 digit year (e.g. 1927) is chosen randomly, what is the probability that it is NOT a leap year?

1. $\frac{3}{4}$
2. $\frac{1}{4}$
3. $<\frac{1}{4}$
4. $>\frac{3}{4}$

19. After giving 20% discount on the marked price to a customer, the seller's profit was 20%. Which of the following is true?

1. Sale price = $\frac{\text{Marked price+Cost price}}{2}$,
2. Sale price < $\frac{\text{Marked price+Cost price}}{2}$,
3. $\frac{2}{3}$(Marked price + Cost price) > Sale price > $\frac{\text{Marked price+Cost price}}{2}$,
4. Sale price > $\frac{2}{3}$(Marked price + Cost price).

20. Three years ago, the difference in the ages of two brothers was 2 years. The sum of their present ages will double in 10 years. What is the present age of the elder brother?

1. 6
2. 11
3. 7
4. 9

PART 'B'

Unit - 1

21. Let A be a 5×5 matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in A^3 is

1. 3
2. 15
3. 5
4. 125

22. Given the permutation

$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$, the matrix A is

defined to be the one whose i-th column is the $\sigma(i)$-th column of the identity matrix I. Which of the following is correct?

1. $A = A^{-2}$
2. $A = A^{-4}$
3. $A = A^{-5}$
4. $A = A^{-1}$

23. Let J denote a 101×101 matrix with all the entries equal to 1 and let I denote the identity matrix of order 101. Then the determinant of J − I is

1. 101.
2. 1.
3. 0.
4. 100.

24. Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$ be given by $f(x) = x^2$. Then f is uniformly continuous if

1. A is a bounded subset of $\mathbb{R}$.
2. A is a dense subset of $\mathbb{R}$.
3. A is an unbounded and connected subset of $\mathbb{R}$.
4. A is an unbounded and open subset of $\mathbb{R}$.

25. Let α, p be real numbers and $\alpha > 1$.

1. If $p > 1$ then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
2. If $p > \frac{1}{\alpha}$ then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
3. If $p < \frac{1}{\alpha}$ then $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx < \infty$.
4. For any $p \in \mathbb{R}$ we have $\int_{-\infty}^{\infty} \frac{1}{|x|^{p\alpha}} dx = \infty$.

26. Let $f: X \to Y$ be a function from a metric space X to another metric space Y. For any Cauchy sequence $\{x_n\}$ in X,

1. if f is continuous then $\{f(x_n)\}$ is a Cauchy sequence in Y.
2. if $\{f(x_n)\}$ is Cauchy then $\{f(x_n)\}$ is always convergent in Y.
3. if $\{f(x_n)\}$ is Cauchy in Y then f is continuous.
4. $\{x_n\}$ is always convergent in X.

27. Let $M_{m \times n}(\mathbb{R})$ be the set of all $m \times n$ matrices with real entries. Which of the following statements is correct?

1. There exists $A \in M_{2\times5}(\mathbb{R})$ such that the dimension of the null space of A is 2.
2. There exists $A \in M_{2\times5}(\mathbb{R})$ such that the dimension of the null space of A is 0.
3. There exist $A \in M_{2\times5}(\mathbb{R})$ and $B \in M_{5\times2}(\mathbb{R})$ such that AB is the 2×2 identity matrix.
4. There exist $A \in M_{2\times5}(\mathbb{R})$ whose null space is $\{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 = x_2, x_3 = x_4 = x_5\}$

28. $\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \cdots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} \right)$ equals

1. $\sqrt{2}$
2. $\frac{1}{\sqrt{2}}$
3. $\sqrt{2}+1$
4. $\frac{1}{\sqrt{2}+1}$

5P3

5x5x5

5x4x3

80

240

125

365

29. Consider the following sets of functions on $\mathbb{R}$.
$W =$ The set of constant functions on $\mathbb{R}$
$X =$ The set of polynomial functions on $\mathbb{R}$
$Y =$ The set of continuous functions on $\mathbb{R}$
$Z =$ The set of all functions on $\mathbb{R}$
Which of these sets has the same cardinality as that of $\mathbb{R}$?
1. Only W. ✗
2. Only W and X. ✗
3. Only W, X and Z.
4. All of W, X, Y and Z. ✓

30. Let $p(x)$ be a polynomial in the real variable x of degree 5. Then $\lim_{n\to\infty} \frac{p(n)}{2^n}$ is

1. 5	2. 1
3. 0	4. ∞

31. For a continuous function $f:\mathbb{R}\to\mathbb{R}$, let $Z(f) = \{x \in \mathbb{R} : f(x) = 0\}$. Then $Z(f)$ is always

1. compact ✗	2. open ✗
3. connected ✗	4. closed ✓

32. For the matrix A as given below, which of them satisfy $A^6 = I$?

1. $A = \begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$

3. $A = \begin{pmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} \end{pmatrix}$

4. $A = \begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Unit - 2

33. If n is a positive integer such that the sum of all positive integers a satisfying $1 \le a \le n$ and $GCD\,(a, n) = 1$ is equal to $240\,n$, then the number of summands, namely, $\varphi(n)$, is

1. 120	2. 124
3. 240	4. 480

34. The total number of non-isomorphic groups of order 122 is

1. 2	2. 1
3. 61	4. 4

35. An ice cream shop sells ice creams in five possible flavours: Vanilla, Chocolate, Strawberry, Mango and Pineapple. How many combinations of three scoop cones are possible? [Note: The repetition of flavours is allowed but the order in which the flavours are chosen does not matter.]

1. 10 ✗	2. 20 ✗
3. 35 ✓	4. 243

36. Let $A \subseteq \mathbb{R}^2$ and $X = \mathbb{R}^2 \setminus A$ be subsets with subspace topology inherited from the usual topology on $\mathbb{R}^2$. Then
1. A is countable dense implies that X is totally disconnected.
2. A is unbounded implies that X is compact. ✗
3. A is open implies that X is compact. ✗
4. A is countable implies that X is path-connected.

37. Let f, g be meromorphic functions on $\mathbb{C}$. If f has a zero of order k at $z = a$ and g has a pole of order m at $z = 0$, then $g(f(z))$ has
1. a zero of order km at $z = a$
2. a pole of order km at $z = a$
3. a zero of order $|k - m|$ at $z = a$
4. a pole of order $|k - m|$ at $z = a$

$X = \mathbb{R}^2 \setminus A$.

MPS

38. Let $p(x)$ be a polynomial of the real variable x of degree $k \geq 1$. Consider the power series $f(z) = \sum_{n=0}^{\infty} p(n) z^n$ where z is a complex variable. Then the radius of convergence of $f(z)$ is

1. 0
2. 1
3. k
4. ∞

39. Let G denote the group of all the automorphisms of the field $F_{3^{100}}$ that consists of 3^{100} elements. Then the number of distinct subgroups of G is equal to

1. 4
2. 3
3. 100
4. 9

40. Let p, q be distinct primes. Then

1. $\mathbb{Z}/_{p^2 q\mathbb{Z}}$ has exactly 3 distinct ideals.
2. $\mathbb{Z}/_{p^2 q\mathbb{Z}}$ has exactly 3 distinct prime ideals.
3. $\mathbb{Z}/_{p^2 q\mathbb{Z}}$ has exactly 2 distinct prime ideals.
4. $\mathbb{Z}/_{p^2 q\mathbb{Z}}$ has a unique maximal ideal.

Unit - 3

41. The homogeneous integral equation

$$\varphi(x) - \lambda \int_0^1 (3x-2)t\, \varphi(t) dt = 0,$$

has

1. One characteristic number
2. Three characteristic numbers
3. Two characteristic numbers
4. No characteristic number

42. The initial value problem $\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = x, 0 \leq x \leq 1,\ t > 0$ and $u(x,0) = 2x$ has

1. a unique solution $u(x,t)$ which $\to \infty$ as $t \to \infty$
2. more than one solution
3. a solution which remains bounded as $t \to \infty$
4. no solution

43. Let $Y_1(x)$ and $Y_2(x)$ defined on $[0,1]$ be twice continuously differentiable functions satisfying $Y''(x) + Y'(x) + Y(x) = 0$. Let $W(x)$ be the Wronskian of Y_1 and Y_2 and satisfy $W\left(\frac{1}{2}\right) = 0$. Then

1. $W(x) = 0\ for\ x \in [0,1]$
2. $W(x) \neq 0\ for\ x \in [0, 1/2) \cup (\frac{1}{2}, 1]$
3. $W(x) > 0\ for\ x \in (1/2, 1]$
4. $W(x) < 0\ for\ x \in [0, 1/2)$

44. Consider two waves of same angular frequency ω, same angular wave number k, same amplitude a traveling in the positive direction of $x-$ axis with the same speed, and with phase difference ϕ. Then the superposition principle yields a resultant wave with

1. Amplitude $2a$ and phase ϕ.
2. Amplitude $2a$ and phase $(\phi/2)$.
3. Amplitude $2a\cos(\phi/2)$ and phase$(\phi/2)$.
4. Amplitude $2a\cos(\phi/2)$ and phase ϕ.

45. Let $x = x(s),\ y = y(s), u = u(s),\ s \in \mathbb{R}$, be the characteristic curve of the PDE

$$\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) - u = 0,$$

passing through a given curve

$$x = 0, y = \tau, u = \tau^2, \tau \in \mathbb{R}.$$

Then the characteristics are given by

1. $x = 3\tau(e^s - 1),\ y = \frac{\tau}{2}(e^{-s} + 1);$ $u = \tau^2 e^{-2s}$
2. $x = 2\tau(e^{-s} - 1),\ y = \tau(2e^{2s} - 1),$ $u = \frac{\tau^2}{2}(1 + e^{-2s})$

$e^{-s}=1$

$2e^{-2s}=1$

3. $x = 2\tau(e^s - 1),\ y = \frac{\tau}{2}(e^s + 1),$
   $u = \tau^2 e^{2s}$
4. $x = \tau(e^{-s} - 1),\ y = -2\tau\left(e^{-s} - \frac{3}{2}\right),$
   $u = \tau^2(2e^{-2s} - 1)$

46. Let $f(x) = ax + b$ for $a, b \in \mathbb{R}$. Then the iteration $x_{n+1} = f(x_n)$ starting from any given x_0 for $n \geq 0$ converges
1. for all $a \in \mathbb{R}$
2. for no $a \in \mathbb{R}$.
3. for $a \in [0,1)$
4. only for $a = 0$.

47. Consider the initial value problem in $\mathbb{R}^2$
$Y'(t) = AY + BY;\quad Y(0) = Y_0$, where

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Then $Y(t)$ is given by
1. $e^{tA}e^{tB}Y_0$
2. $e^{tB}e^{tA}Y_0$
3. $e^{t(A+B)}Y_0$
4. $e^{-t(A+B)}Y_0$

48. The curve extremizing the functional

$$I(y) = \int_1^2 \frac{\sqrt{1 + (y'(x))^2}}{x}\,dx,$$

$y(1) = 0, y(2) = 1$ is

1. an ellipse
2. a parabola
3. a circle
4. a straight line

Unit - 4

49. Let Y_1, Y_2, Y_3 be uncorrelated random variables with common unknown variance σ^2 and expectations given by
$E(Y_1) = \beta_0 + \beta_1,$
$E(Y_2) = \beta_0 + \beta_2,$
$E(Y_3) = \beta_0 + \beta_3,$
where $\beta_0, \beta_1, \beta_2, \beta_3$ are unknown parameters. Which of the following statements is true?

1. The degrees of freedom associated with the error sum of squares is 1.
2. An unbiased estimator of σ^2 is $\frac{1}{6}[(Y_1 - Y_2)^2 + (Y_1 - Y_3)^2 + (Y_2 - Y_3)^2]$.
3. $\beta_0, \beta_1, \beta_2,$ and β_3 are each individually estimable.
4. $\beta_1 - 2\beta_2 + \beta_3$ is estimable.

50. Let X be a $p \times 1$ random vector such that $X \sim N_p(\mathbf{0}, \Sigma)$ where rank$(\Sigma) = p$. Which of the following is true?
1. $E(X'\Sigma^{-1}X) = 2p,\ V(X'\Sigma^{-1}X) = 2p.$
2. $E(X'\Sigma^{-1}X) = 2p,\ V(X'\Sigma^{-1}X) = p.$
3. $E(X'\Sigma^{-1}X) = p,\ V(X'\Sigma^{-1}X) = p.$
4. $E(X'\Sigma^{-1}X) = p,\ V(X'\Sigma^{-1}X) = 2p.$

51. A finite population has 8 units, labeled $u_1, u_2, \ldots, u_8$ and the value of a study variable for the unit u_i is Y_i $(i = 1, 2, \ldots, 8)$. Let $\bar{Y} = (1/8)\sum_{i=1}^{8} Y_i$. A sample of size 4 units is drawn from this population in the following manner: a simple random sample (SRS) of size 2 is drawn from the units $u_2, u_3, \ldots, u_7$ and the sample so selected is augmented by the units u_1 and u_8 to get a sample of size 4. Let $\bar{y}$ be the sample mean based on the SRS of size two and let $T = (Y_1 + 6\bar{y} + Y_8)/8$. Which of the following statements is true?
1. T is a biased estimator of $\bar{Y}$.
2. T is unbiased for $\frac{1}{6}\sum_{i=2}^{7} Y_i$.
3. T is unbiased for $\bar{Y}$ and $V(T) = 3V(\bar{y})/4$.
4. T is unbiased for $\bar{Y}$ and $V(T) = 9V(\bar{y})/16$.

$x_{n+1} = ax_n + b.$

$|x_{n+1} - x_n|$

52. At a doctor's clinic patients arrive at an average rate of 10 per hour. The consultancy time per patient is exponentially distributed with an average of 6 minutes per patient. The doctor does not admit any patient if at any time 10 patients are waiting. Then at the steady state of this M/M/1/R queue the expected number of patients waiting is

1. 0.
2. 5.
3. 9.
4. 10.

53. Let T be a statistic whose distribution under the null hypothesis H_0 is uniform(0,1). Let the distribution of T under an alternative hypothesis H_1 be triangular distribution with density

$$g(x) = \begin{cases} x, & : \ 0 \le x \le 1, \\ 2 - x, & : \ 0 \le x \le 2. \end{cases}$$

Then the power β of the most powerful test for testing H_0 against the alternative H_1 based on the statistic T with size 0.1 satisfies

1. $0 < \beta \le 0.5$.
2. $0.5 < \beta \le 0.55$.
3. $0.55 < \beta \le 0.7$.
4. $0.7 < \beta \le 1$.

54. Let X be a random variable following a Poisson distribution with parameter $\lambda > 0$. To estimate λ^5, consider an estimator

$T = X(X-1)(X-2)(X-3)(X-4)$.

Which of the following statements is true?

1. T is not unbiased.
2. T is unbiased but not UMVUE.
3. T is UMVUE.
4. UMVUE for λ^5 does not exist.

55. Let $(X_n)_{n\geq 0}$ be a Markov chain on the state space $S = \{0,1\}$. Then

1. The chain has a unique stationary distribution.
2. $\mathbb{P}(X_n = 0 | X_0 = 0)$ converges as $n \to \infty$.
3. The chain may have one recurrent and one transient state. ✓
4. The chain is always irreducible.

56. Suppose X, Y and Z are three independent random variables each with finite variance. Let $U = X + Z$ and $V = Y + Z$. Suppose U and V have the same distribution. Then

1. X and Y have the same distribution.
2. It is possible to have $Corr\ (U,V) < 0$.
3. $U + V$ and $U - V$ are always independent.
4. We must have $Corr\ (U,V) < 1$.

57. Suppose you have a coin with probability $\frac{3}{4}$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H,H),(H,T),(T,H),(T,T)\}$ be the sample space. Then it is possible to have an event $E \subseteq \Omega$ such that

1. $\mathbb{P}(E) = \frac{1}{3}$.
2. $\mathbb{P}(E) = \frac{1}{9}$.
3. $\mathbb{P}(E) = \frac{1}{4}$.
4. $\mathbb{P}(E) = \frac{7}{8}$.

58. Suppose $X_1, X_2, \cdots, X_n$ are independent random variables each having a $Bin\left(8, \frac{1}{2}\right)$ distribution. Then $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} (-1)^k X_k$ converges in distribution to

1. $N(0, 1)$.
2. $N(0, 2)$.
3. $N(4,2)$.
4. $N(4, 1)$.

59. Let $X_1, X_2, \cdots, X_n$ be iid with common density $f_\theta(x) = \begin{cases} \theta e^{-\theta}, & x > 0, \\ 0, & x \le 0, \end{cases}$ where $\theta > 0$. For testing H_0: $\theta = 1$ versus H_1: $\theta = 2$, let r_n be the power of the most powerful test of size $\alpha = 0.05$ wih sample size n. Then

1. r_n increases to $1 - \alpha$.
2. r_n may not converge.

3. r_n increases to 1.
4. r_n may not be an increasing sequence.

60. Consider the following three sets of sample observations.
Sample 1: $x_1, x_2, \dots, x_n$.
Sample 2: $y_1, y_2, \dots, y_m$.
Sample 3: $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_m$.
Let $\bar{m}_i, \tilde{m}_i, \widehat{m}_i$ and σ_i^2 denote mean, median, mode and variance respectively of the i^{th} sample for $i = 1, 2, 3$. Assume $\bar{m}_1 = \bar{m}_2$. Which of the following is NOT always true?
1. $\bar{m}_3 = \bar{m}_1$.
2. $min(\tilde{m}_1, \tilde{m}_2) \le \tilde{m}_3 \le max(\tilde{m}_1, \tilde{m}_2)$.
3. $min(\widehat{m}_1, \widehat{m}_2) \le \widehat{m}_3 \le max(\widehat{m}_1, \widehat{m}_2)$.
4. $min(\sigma_1^2, \sigma_2^2) \le \sigma_3^2 \le max(\sigma_1^2, \sigma_2^2)$.

PART 'C'

Unit - 1

61. Let V denote a vector space over a field F and with a basis $B = \{e_1, e_2, \dots, e_n\}$. Let $x_1, x_2, \dots, x_n \in F$. Let $C = \{x_1 e_1, x_1 e_1 + x_2 e_2, \dots, x_1 e_1 + x_2 e_2 + \dots + x_n e_n\}$. Then
1. C is a linearly independent set implies that $x_i \neq 0$ for every $i = 1,2,\dots,n$.
2. $x_i \neq 0$ for every $i = 1,2,\dots,n$ implies that C is a linearly independent set.
3. The linear span of C is V implies that $x_i \neq 0$ for every $i = 1,2,\dots,n$.
4. $x_i \neq 0$ for every $i = 1,2,\dots,n$ implies that the linear span of C is V.

62. Let V denote the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to n. Which of the following defines a norm on V?
1. $\| p \|^2 = |p(1)|^2 + \dots + |p(n+1)|^2$, $p \in V$
2. $\| p \| = \sup_{t \in [0,1]} |p(t)|$, $p \in V$
3. $\| p \| = \int_0^1 |p(t)|\, dt$, $p \in V$
4. $\| p \| = \sup_{t \in [0,1]} |p'(t)|$, $p \in V$.

63. Let u, v, w be vectors in an inner-product space V, satisfying $\| u \| = \| v \| = \| w \| = 2$ and $< u, v > = 0, < u, w > = 1, < v, w > = -1$. Then which of the following are true?
1. $\| w + v - u \| = 2\sqrt{2}$
2. $\{\frac{1}{2}u, \frac{1}{2}v\}$ forms an orthonormal basis of a two dimensional subspace of V.
3. w and $4u - w$ are orthogonal to each other.
4. u, v, w are necessarily linearly independent.

64. Let A be a 4×4 matrix over $\mathbb{C}$ such that $rank(A) = 2$ and $A^3 = A^2 \neq 0$. Suppose that A is not diagonalizable. Then
1. One of the Jordan blocks of the Jordan canonical form of A is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
2. $A^2 = A \neq 0$.
3. There exists a vector v such that $Av \neq 0$ but $A^2 v = 0$.
4. The characteristic polynomial of A is $x^4 - x^3$.

65. Let $l^2 = \{\mathbf{x} = (x_1, x_2, \dots) : x_n \in \mathbb{C}\ \forall\, n \ge 1 \text{ and } \sum_{n=1}^{\infty} |x_n|^2 < \infty\}$ and $e_n \in l^2$ be the sequence whose n-th element is 1 and all other elements are zero. Equip the space l^2 with the norm $\|x\| = \sqrt{\sum_{n=1}^{\infty} |x_n|^2}$. Then the set $S = \{e_n : n \ge 1\}$
1. is closed.
2. is bounded.
3. is compact.
4. contains a convergent subsequence.

66. Let $\varphi: \mathbb{R}^2 \longrightarrow \mathbb{C}$ be the map $\varphi(x, y) = z$, where $z = x + iy$. Let $f: \mathbb{C} \longrightarrow \mathbb{C}$ be the function $f(z) = z^2$ and $F = \varphi^{-1} f \varphi$. Which of the following are correct?

1. The linear transformation $T(x, y) = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ represents the derivative of F at (x, y).
2. The linear transformation $T(x, y) = 2\begin{pmatrix} x & y \\ y & x \end{pmatrix}$ represents the derivative of F at (x, y).
3. The linear transformation $T(z) = 2z$ represents the derivative of f at $z \in \mathbb{C}$.
4. The linear transformation $T(z) = 2z$ represents the derivative of f only at 0.

67. Let $X = \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 < 5\}$ and $K = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 2 \text{ or } 3 \leq x^2 + y^2 \leq 4\}$. Then,

1. $X \setminus K$ has three connected components.
2. $X \setminus K$ has no relatively compact connected component in X.
3. $X \setminus K$ has two relatively compact connected components in X.
4. All connected components of $X \setminus K$ are relatively compact in X.

68. For two subsets X and Y of $\mathbb{R}$, let $X + Y = \{x + y : x \in X, y \in Y\}$.

1. If X and Y are open sets then $X + Y$ is open.
2. If X and Y are closed sets then $X + Y$ is closed.
3. If X and Y are compact sets then $X + Y$ is compact.
4. If X is closed and Y is compact then $X + Y$ is closed.

69. Let $\{f_n\}$ be a sequence of continuous functions on $\mathbb{R}$.

1. If $\{f_n\}$ converges to f pointwise on $\mathbb{R}$, then $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx = \int_{-\infty}^{\infty} f(x)dx$.
2. If $\{f_n\}$ converges to f uniformly on $\mathbb{R}$ then $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx = \int_{-\infty}^{\infty} f(x)dx$.
3. If $\{f_n\}$ converges to f uniformly on $\mathbb{R}$, then f is continuous on $\mathbb{R}$.
4. There exists a sequence of continuous functions $\{f_n\}$ on $\mathbb{R}$, such that $\{f_n\}$ converges to f uniformly on $\mathbb{R}$, but $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx \neq \int_{-\infty}^{\infty} f(x)dx$.

70. Let V be the vector space of polynomials over $\mathbb{R}$ of degree less than or equal to n. For $p(x) = a_0 + a_1x + \cdots + a_nx^n$ in V, define a linear transformation $T: V \to V$ by $(Tp)(x) = a_0 - a_1x + a_2x^2 - \cdots + (-1)^n a_nx^n$. Then which of the following are correct?

1. T is one-to-one.
2. T is onto.
3. T is invertible.
4. det T = 0.

71. Let $\{a_n\}$, $\{b_n\}$ be given bounded sequences of positive real numbers. Then (Here $a_n \uparrow a$ means a_n increase to a as n goes to ∞, similarly, $b_n \downarrow b$ means b_n decreases to b as n goes to ∞)

1. if $a_n \uparrow a$, then $\sup_{n\geq 1}(a_nb_n) = a\,(\sup_{n\geq 1} b_n)$.
2. if $a_n \uparrow a$, then $\sup_{n\geq 1}(a_nb_n) < a\,(\sup_{n\geq 1} b_n)$.
3. if $b_n \downarrow b$, then $\inf_{n\geq 1}(a_n b_n) = (\inf_{n\geq 1} a_n)b$.
4. if $b_n \downarrow b$, then $\inf_{n\geq 1}(a_n b_n) > (\inf_{n\geq 1} a_n)b$.

72. Let $S \subset \mathbb{R}^2$ be defined by $S = \{ \left(m + \frac{1}{2^{|p|}}, n + \frac{1}{2^{|q|}}\right) : m, n, p, q \in \mathbb{Z} \}$. Then,

1. S is discrete in $\mathbb{R}^2$.
2. the set of limit points of S is the set $\{(m,n) : m,n \in \mathbb{Z}\}$.
3. $\mathbb{R}^2 \setminus S$ is connected but not path connected.
4. $\mathbb{R}^2 \setminus S$ is path connected.

73. Consider a homogeneous system of linear equation $Ax = 0$, where A is an $m \times n$ real matrix and $n > m$. Then which of the following statements are always true?
1. $Ax = 0$ has a solution.
2. $Ax = 0$ has no nonzero solution.
3. $Ax = 0$ has a nonzero solution.
4. Dimension of the space of all solutions is at least $n - m$.

74. Let a, b, c be positive real numbers, $D = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : {x_1}^2 + {x_2}^2 + {x_3}^2 \le 1\}$,

$E = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : \frac{{x_1}^2}{a^2} + \frac{{x_2}^2}{b^2} + \frac{{x_3}^2}{c^2} \le 1\}$

and $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, $\det A > 1$. Then, for a compactly supported continuous function f on $\mathbb{R}^3$, which of the following are correct?
1. $\int_D f(Ax)dx = \int_E f(x)dx$
2. $\int_D f(Ax)dx = \frac{1}{abc}\int_D f(x)dx$
3. $\int_D f(Ax)dx = \frac{1}{abc}\int_E f(x)dx$
4. $\int_{\mathbb{R}^3} f(Ax)dx = \frac{1}{abc}\int_{\mathbb{R}^3} f(x)dx$

75. Let $f : (0,1) \to \mathbb{R}$ be continuous. Suppose that $|f(x) - f(y)| \le |\sin x - \sin y|$ for all x, y $\in (0, 1)$. Then
1. f is discontinuous at least at one point in $(0, 1)$.
2. f is continuous everywhere on $(0, 1)$, but not uniformly continuous on $(0, 1)$.
3. f is uniformly continuous on $(0, 1)$.
4. $\lim_{x \to 0+} f(x)$ exists.

76. Let $p_n(x) = a_n x^2 + b_n x + c_n$ be a sequence of quadratic polynomials where $a_n, b_n, c_n \in \mathbb{R}$, for all $n \ge 1$. Let $\lambda_0, \lambda_1, \lambda_2$ be distinct real numbers such that $\lim_{n\to\infty} p_n(\lambda_0) = A_0$, $\lim_{n\to\infty} p_n(\lambda_1) = A_1$ and $\lim_{n\to\infty} p_n(\lambda_2) = A_2$. Then
1. $\lim_{n\to\infty} p_n(x)$ exists for all $x \in \mathbb{R}$.
2. $\lim_{n\to\infty} p_n'(x)$ exists for all $x \in \mathbb{R}$.
3. $\lim_{n\to\infty} p_n\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exist.
4. $\lim_{n\to\infty} p_n'\left(\frac{\lambda_0+\lambda_1+\lambda_2}{3}\right)$ does not exist.

77. Define $f : \mathbb{R}^2 \to \mathbb{R}^2$ by
$f(x, y) = (x + 2y + y^2 + |xy|, 2x + y + x^2 + |xy|)$
for $(x, y) \in \mathbb{R}^2$. Then
1. f is discontinuous at (0,0).
2. f is continuous at (0,0) but not differentiable at (0,0).
3. f is differentiable at (0,0).
4. f is differentiable at (0,0) and the derivative $Df(0,0)$ is invertible.

78. Let $A = \{(x, y) \in \mathbb{R}^2 : x + y \ne -1\}$. Define $f : A \to \mathbb{R}^2$ by
$f(x, y) = \left(\frac{x}{1+x+y}, \frac{y}{1+x+y}\right)$. Then,
1. the Jacobian matrix of f does not vanish on A.
2. f is infinitely differentiable on A.
3. f is injective on A.
4. $f(A) = \mathbb{R}^2$.

Unit - 2

79. Which of the following are compact?
    1. $\{(x,y)\epsilon\mathbb{R}^2 : (x-1)^2 + (y-2)^2 = 9\} \cup \{(x,y)\epsilon\ \mathbb{R}^2 : y = 3\}$.
    2. $\left\{\left(\frac{1}{m},\frac{1}{n}\right) \epsilon\mathbb{R}^2 : m,n \epsilon\ \mathbb{Z}\setminus\{0\}\right\} \cup \{(0,0)\} \cup \left\{\left(\frac{1}{m},0\right) : m \epsilon\ \mathbb{Z}\setminus\{0\}\right\} \cup \left\{\left(0,\frac{1}{n}\right) : n \epsilon\ \mathbb{Z}\setminus\{0\}\right\}$.
    3. $\{(x,y,z)\epsilon\mathbb{R}^3 : x^2 + 2y^2 - 3z^2 = 1\}$.
    4. $\{(x,y,z)\epsilon\mathbb{R}^3 : |x| + 2|y| + 3|z| \le 1\}$.

80. Let f be an entire function. Suppose, for each $a\epsilon\ \mathbb{R}$, there exists at least one coefficient c_n in $f(z) = \sum_{n=0}^{\infty} c_n(z-a)^n$, which is zero. Then
    1. $f^{(n)}(0) = 0$ for infinitely many $n \ge 0$.
    2. $f^{(2n)}(0) = 0$ for every $n \ge 0$.
    3. $f^{(2n+1)}(0) = 0$ for every $n \ge 0$.
    4. there exists $k \ge 0$ such that $f^{(n)}(0) = 0$ for all $n \ge k$.

81. Let $K \subseteq \mathbb{C}$ be a bounded set. Let $H(\mathbb{C})$ denote the set of all entire functions and let $C(K)$ denote the set of all continuous functions on K. Consider the restriction map $r : H(\mathbb{C}) \to C(K)$ given by $r(f) = f_{|K}$. Then r is injective if
    1. K is compact.
    2. K is connected.
    3. K is uncountable.
    4. K is finite.

82. For $z \epsilon\ \mathbb{C}$, define $f(z) = \frac{e^z}{e^z - 1}$. Then
    1. f is entire.
    2. the only singularities of f are poles.
    3. f has infinitely many poles on the imaginary axis.
    4. each pole of f is simple.

83. Let $D = \{z \epsilon\ \mathbb{C} : |z| < 1\}$. Then there exists a holomorphic function $f : D \to \overline{D}$ with $f(0) = 0$ with the property
    1. $f'(0) = 1/2$.
    2. $|f(1/3)| = 1/4$.
    3. $f(1/3) = 1/2$.
    4. $|f'(0)| = \sec(\frac{\pi}{6})$.

$f'(0)=0$

$|f(z)| \le c$

84. Let $f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$ and let p be a prime. Let $f_p(x)$ denote the corresponding polynomial with coefficients in $\mathbb{Z}/p\mathbb{Z}$. Then
    1. $f_2(x)$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$.
    2. $f(x)$ is irreducible over $\mathbb{Q}$.
    3. $f_3(x)$ is irreducible over $\mathbb{Z}/3\mathbb{Z}$.
    4. $f(x)$ is irreducible over $\mathbb{Z}$.

85. Suppose $(F,+,\cdot)$ is the finite field with 9 elements. Let $G = (F,+)$ and $H = (F\setminus\{0\},\cdot)$ denote the underlying additive and multiplicative groups respectively. Then
    1. $G \cong (\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/3\mathbb{Z})$
    2. $G \cong (\mathbb{Z}/9\mathbb{Z})$
    3. $H \cong (\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$
    4. $G \cong (\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/3\mathbb{Z})$ and $H \cong (\mathbb{Z}/8\mathbb{Z})$

86. Consider the multiplicative group G of all the (complex) 2^n-th roots of unity where $n = 0,1,2 \ldots$ Then
    1. Every proper subgroup of G is finite.
    2. G has a finite set of generators.
    3. G is cyclic.
    4. Every finite subgroup of G is cyclic.

87. Let R be the ring of all entire functions, i.e. R is the ring of functions $f : \mathbb{C} \to \mathbb{C}$ that are analytic at every point of $\mathbb{C}$, with respect to pointwise addition and multiplication. Then

$4 \times 4 \times 3$

1. The units in R are precisely the nowhere vanishing entire functions, i.e., $f: \mathbb{C} \to \mathbb{C}$ such that f is entire and $f(\alpha) \neq 0$ for all $\alpha \in \mathbb{C}$.
2. The irreducible elements of R are, up to multiplication by a unit, linear polynomials of the form $z - \alpha$, where $\alpha \in \mathbb{C}$, i.e., if $f \in R$ is irreducible, then $f(z) = (z - \alpha)g(z)$ for all $z \in \mathbb{C}$ where g is a unit in R and $\alpha \in \mathbb{C}$.
3. R is an integral domain.
4. R is a unique factorization domain.

88. We are given a class consisting of 4 boys and 4 girls. A committee that consists of a President, a Vice-President and a Secretary is to be chosen among the 8 students of the class. Let a denote the number of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let b denote the number of ways of choosing the committee in such a way that the number of girls is greater than or equal to that of the boys. Then

1. $a = 288$
2. $b = 168$
3. $a = 144$
4. $b = 192$

89. Pick the correct statements:

1. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as $\mathbb{Q}$- vector spaces.
2. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as fields.
3. $Gal_{\mathbb{Q}}(\mathbb{Q}(\sqrt{2})/\mathbb{Q}) \cong Gal_{\mathbb{Q}}(\mathbb{Q}(i)/\mathbb{Q})$
4. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are both Galois extensions of $\mathbb{Q}$.

90. For positive integers m and n, let $F_n = 2^{2^n} + 1$ and $G_m = 2^{2^m} - 1$. Which of the following statements are true?

1. F_n divides G_m whenever $m > n$.
2. $GCD\ (F_n, G_m) = 1$ whenever $m \neq n$.
3. $GCD\ (F_n, F_m) = 1$ whenever $m \neq n$.
4. G_m divides F_n whenever $m < n$.

Unit - 3

91. Consider a particle of mass m in simple harmonic oscillation about the origin with spring constant k; then for the Lagrangian L and the Hamiltonian H of the system

1. $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
   $H(x, \dot{x}) = \frac{p^2}{2m} + \frac{1}{2}kx^2$; p is generalized momentum
2. $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ and the generalized momentum is $p = m\dot{x}$.
3. $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ and the generalized momentum is $p = m\dot{x}$.
4. $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
   $H(x, \dot{x}) = \frac{m\dot{x}^2}{2} + \frac{1}{2}kx^2$

92. Consider the boundary value problem

$-u''(x) = \pi^2 u(x),\ x \in (0,1),$

$u(0) = u(1) = 0.$

If u and u' are continuous on $[0,1]$ then

1. $\int_0^1 u^3(x)\, dx = 0$
2. $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$
3. $u'^2(x) + \pi^2 u^2(x) = u'^2(1)$
4. $\int_0^1 u^2(x)\, dx = \frac{1}{\pi^2} \int_0^1 u'^2(x)\, dx$

93. Let $y_1(x)$ and $y_2(x)$ form a complete set of solutions to the differential equation $y'' - 2xy' + \sin(e^{2x^2})y = 0,\quad x \in [0,1]$ with $y_1(0) = 0,\ y'_1(0) = 1, y_2(0) = 1,\ y'_2(0) = 1$. Then the Wronskian $W(x)$ of $y_1(x)$ and $y_2(x)$ at $x = 1$ is

1. e^2
2. $-e$
3. $-e^2$
4. e

94. Let λ_1, λ_2 be the characteristic numbers and f_1, f_2 the corresponding eigen functions for the homogeneous integral equation

$\varphi(x) - \lambda \int_0^1 (xt + 2x^2)\, \varphi(t)dt = 0.$

Then

1. $\lambda_1 = -18 - 6\sqrt{10}, \lambda_2 = -18 + 6\sqrt{10}.$
2. $\lambda_1 = -36 - 12\sqrt{10}, \lambda_2 = -36 + 12\sqrt{10}.$
3. $\int_0^1 f_1(x) f_2(x) dx = 1.$
4. $\int_0^1 f_1(x) f_2(x) dx = 0.$

95. Consider the function

$f(x) = \sqrt{2+x}$ for $x \geq -2$

and the iteration

$x_{n+1} = f(x_n);\ n \geq 0$ for $x_0 = 1.$

What are the possible limits of the iteration?

1. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$
2. $-1.$
3. 2
4. 1

96. The PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\, \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ is

1. parabolic and has characteristics $\xi(x,y) = x + 2y, \eta(x,y) = x - 2y.$
2. reducible to the canonical form $\frac{\partial^2 u}{\partial \xi^2} = 0,$ where $\xi(x,y) = x + 2y.$
3. reducible to the canonical form $\frac{\partial^2 u}{\partial \eta^2} = 0,$ where $\eta(x,y) = x + y.$
4. parabolic and has the general solution $u = (x-y)f_1(x+y) + f_2(x-y),$ where f_1, f_2 are arbitrary functions.

97. Let $u(x,y)$ be an extremal of the functional

$$J(u) = \iint_D \left[\frac{1}{2}u_x^2 + \frac{1}{2}u_y^2 + e^{xy}u\right] dxdy,$$

where D is the open unit disk in $\mathbb{R}^2$. Then u satisfies

1. $u_{xx} + u_{yy} - e^{x+y} = 0.$
2. $u_{xx} + u_{yy} = e^{xy}.$
3. $u_{xx} + u_{yy} = -e^{xy}.$
4. $\iint_D \left[u_{xx} + u_{yy} - e^{xy}\right] h(x,y)\, dxdy = 0$ for every smooth h vanishing on the boundary of D.

98. Consider the iteration

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right), \qquad n \geq 0$$

for a given $x_0 \neq 0$. Then

1. x_n converges to $\sqrt{2}$ with rate of convergence 1.
2. x_n converges to $\sqrt{2}$ with rate of convergence 2.
3. The given iteration is the fixed point iteration for $f(x) = x^2 - 2.$
4. The given iteration is the Newton's method for $f(x) = x^2 - 2.$

99. If $y: [0,\infty) \to [0,\infty)$ is a continuously differentiable function satisfying

$$y(t) = y_0 - \int_0^t y(s)ds$$

for $t \geq 0$, then

1. $y^2(t) = y^2(0) + (\int_0^t y(s)ds)^2 - 2y(0)\int_0^t y(s)ds$
2. $y^2(t) = y^2(0) + 2\int_0^t y^2(s)\, ds$
3. $y^2(t) = y^2(0) - \int_0^t y(s)ds$
4. $y^2(t) = y^2(0) - 2\int_0^t y^2(s)ds$

100. Let $u(t)$ be a continuously differentiable function taking non-negative values for $t > 0$ satisfying $u'(t) = 3\,u(t)^{2/3}$ and $u(0) = 0$. Which of the following are possible solutions of the above equation?

1. $u(t) = 0$
2. $u(t) = t^3$
3. $u(t) = \begin{cases} 0 \text{ for } 0 < t < 1 \\ (t-1)^3 \text{ for } t \geq 1 \end{cases}$
4. $u(t) = \begin{cases} 0 \text{ for } 0 < t < 3 \\ (t-3)^3 \text{ for } t \geq 3 \end{cases}$

101. Let $u(x,t)$ be the solution of the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

which tends to zero as $t \to \infty$ and has the value cos(x) when $t = 0$.

Then

1. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-nt}$, where a_n, b_n are arbitrary constants.
2. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 t}$, where a_n, b_n are non-zero constants.
3. $u = \sum_{n=1}^{\infty} a_n \cos(nx + b_n)e^{-nt}$, where a_n are not all zero and $b_n = 0$ for $n \geq 1$.
4. $u = \sum_{n=1}^{\infty} a_n \cos(nx + b_n)e^{-n^2 t}$, where $a_1 \neq 0,\ a_n = 0$ for $n > 1, and\ b_n = 0$ for $n \geq 1$.

102. Let $xyu = c_1$ and $x^2 + y^2 - 2u = c_2$, where c_1 and c_2 are arbitrary constants, be the first integrals of the PDE $x(u + y^2)\frac{\partial u}{\partial x} - y(u + x^2)\frac{\partial u}{\partial y} = (x^2 - y^2)u$. Then the solution of the PDE with $x + y = 0, u = 1$ is given by

1. $x^3 + y^3 + 2xyu^2 + 2x^2u = 0.$
2. $x^3 + yx^2 + (x^2 + xy)u = 0.$
3. $x^2 + y^2 + 2(xy - 1)\,u + 2 = 0.$
4. $x^2 - y^2 - u(x + y - 2) - 2 = 0.$

Unit - 4

103. Consider the following primal Linear Programming Problem.

max $z = -3x_1 + 2x_2$

subject to

$x_1 \leq 3,$

$x_1 - x_2 \leq 0,$

$x_1, x_2 \geq 0.$

Which of the following statements are true?

1. The primal problem has an optimal solution. +
2. The primal problem has an unbounded solution.
3. The dual problem has an unbounded solution.
4. The dual problem has no feasible solution.

104. Suppose that system 1 has 2 components C_1 and C_2 in series while system 2 has 2 components C_3 and C_4 in parallel. The components C_1, C_2, C_3 and C_4 have independent and identically distributed life times each being exponential with mean 1. Suppose $S_i(t)$ and $h_i(t)$ are the survival and hazard rate function, respectively, for the i-th system, $i = 1,2$. Then which of the following statements are true?

1. $S_1(t) < S_2(t)$ for all $t > 0$.
2. $h_1(t) < h_2(t)$ for all $t > 0$
3. The expected life time of the system 1 is $\frac{1}{2}$.
4. The expected life time of the system 2 is 1.

105. Consider an experiment using a balanced incomplete block design with $v = 4$ treatments, $b = 6$ blocks and block size $k = 2$. Let t_i $(i = 1,2,3,4)$ be the effect of the i-th treatment and σ^2 be the variance of an observation. Which of the following statements are true?

1. The variance of the best linear unbiased estimator (BLUE) of $\sum_{i=1}^{4} p_i t_i$ where $\sum_{i=1}^{4} p_i = 0$ and $\sum_{i=1}^{4} p_i^2 = 1$ is $\sigma^2/2$.
2. The covariance between the BLUEs of the contrasts $\sum_{i=1}^{4} p_i t_i$ and $\sum_{i=1}^{4} q_i t_i$ where $\sum_{i=1}^{4} p_i q_i = 0$ is zero.
3. The degrees of freedom associated with the error sum of squares is 3.
4. The efficiency factor of the design relative to a randomized block design with 3 replicates is 2/3.

106. Consider a finite population containing $N = nk$ units, $n \geq 2$, $k \geq 2$ being integers and let these units be numbered 1 to N in some order. In order to select a sample of n units, a unit is selected at random from the first k units and every k-th unit thereafter. Under this scheme, let π_i be the probability that the i-th unit is included in the sample and π_{ij} be the probability that both i-th and j-th units are included in the sample. Also, let $\bar{y}$ denote the sample mean of a study variable, say y. Which of the following statements are true?

1. $\pi_i = \frac{n}{N}, \pi_{ij} = \frac{n(n-1)}{N(N-1)}$ for all $i, j = 1,2, \ldots, N, i \neq j$.
2. $\pi_i = \frac{n}{N}$, for all $i = 1,2, \ldots, N$ and $\pi_{ij} = 0$ for at least one pair $(i,j), i,j = 1,2, \ldots, N, i \neq j$.
3. $\pi_i = \frac{1}{N}$ for all $i = 1,2, \ldots, N$ and $\pi_{ij} > 0$ for all $i \neq j, i, j, = 1,2, \ldots, N$.
4. $N\bar{y}$ is an unbiased estimator of the population total.

107. Aerial obervations Y_1, Y_2, Y_3 and Y_4 are made on angles $\theta_1, \theta_2, \theta_3$ and θ_4 respectively, of a quadrilateral on the ground. If the observations $\{Y_i, i = 1,2,3,4\}$ are subject to normal errors with mean 0 and variance σ^2, then which of the following statements are true?

1. The best linear unbiased estimator of θ_i is $\hat{\theta}_i = Y_i - \bar{Y} + \frac{\pi}{2}, i = 1,2,3,4$, where $\bar{Y} = \frac{1}{4}\sum_{i=1}^{4} Y_i$.
2. The best linear unbiased estimator of θ_i is $\hat{\theta}_i = Y_i, i = 1,2,3,4$.
3. The error sum of squares is $4(\bar{Y} - \frac{\pi}{2})^2$.
4. The error sum of squares is $\sum_{i=1}^{4}(Y_i - \frac{\pi}{2})^2$.

108. For any set of data which of the following statements are NOT possible? (Notations have their usual significance)

1. $r_{1.234} = 0.47, r_{1.23} = 0.52$.
2. $r_{1.23} = -0.32, r_{12.3} = -0.23$.
3. $r_{12} = 0.3, r_{13} = 0.2, r_{12.3} = -0.23$.
4. $r_{1.234} = 0.47, r_{12} = 0.73$.

109. Let $X_1, X_2, \ldots, X_n$ be a random sample from

$$f_\theta(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & otherwise. \end{cases}$$

Consider the problem of testing $H_0: \theta = 1$ against $H_1: \theta > 1$.

Define

$$\varphi_1 = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} x_i > c_1 \\ 0 \text{ if } \sum_{i=1}^{n} x_i \leq c_1 \end{cases} \quad \text{and}$$

$$\varphi_2 = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} x_i < c_2 \\ 0 \text{ if } \sum_{i=1}^{n} x_i \geq c_2 \end{cases}$$

where c_1 and c_2 are such that φ_1 and φ_2 are of size α. Which of the following statements are true?

1. φ_1 is more powerful than φ_2.
2. P-value of the uniformly most powerful test for testing H_0 against H_1 is given by $P_{\theta_0}[\sum_{i=1}^{n} X_i > \text{observed } \sum_{i=1}^{n} x_i]$

3. Power function of φ_2 is monotonically increasing.
4. φ_1 is unbiased.

110. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with common continuous distribution function F, which is symmetric about the median μ. Consider the problem of testing $H_0: \mu = 0$ against $H_1: \mu > 0$. Define
$R_i^+ =$ Rank of $|X_i|$ among $|X_1|, |X_2|, \ldots, |X_n|, i = 1, 2, \ldots, n$.
$L =$ Sum of the R_i^+'s, for which $X_i < 0, i = 1, 2, \ldots, n$.
$G =$ Sum of the R_i^+'s, for which $X_i > 0, i = 1, 2, \ldots, n$.
Which of the following statements are true?
1. Left tailed test based on L is appropriate for testing H_0 against H_1.
2. Right tailed test based on G is appropriate for testing H_0 against H_1.
3. Maximum possible value of L is $n(n+1)$.
4. $E_{H_1}(L+G) = \frac{n(n+1)}{2}$.

111. Let $X_1, X_2, \ldots, X_n$ be a random sample from

$$f_\theta(x) = \begin{cases} e^{-(x-\theta)}, & x > \theta \\ 0, & x \le \theta. \end{cases}$$

Define $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$. Which of the following are confidence intervals for θ with confidence coefficient $(1-\alpha)$?
1. $[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)}]$.
2. $[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)} - \frac{1}{n}\log_e \alpha]$.
3. $[X_{(1)} + \frac{1}{n}\log_e(\frac{\alpha}{2}), X_{(1)} + . \frac{1}{n}\log_e(1-\frac{\alpha}{2})]$
4. $[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)} - \frac{1}{n}\log_e(1-\frac{\alpha}{2})]$.

112. Suppose $X_1, X_2, \cdots, X_n$ are independent and identically distributed as geometric random variables with parameter p. Let f denote the number X_i's equal to 1. Then which of the following statements are true?
1. $\frac{f}{n}$ is the maximum likelihood estimate of p.
2. $\frac{f}{n}$ is an unbiased estimator of p.
3. $\frac{n-1}{\sum_{i=1}^n x_i}$ is the maximum likelihood estimator of p.
4. $Var\left(\frac{f}{n}\right) = \frac{p(1-p)}{n}$.

113. Consider the following random sample of size 11 from uniform $(\theta - 1, \theta + 1)$ distribution: -0.71, 0.3, -0.4, -0.63, -0.81, -0.7, 0.1, -0.01, 0.02, -0.96, -0.92. Which of the following are maximum likelihood estimates of θ?
1. -0.96
2. 0.3
3. 0.02
4. -0.54

114. Let X and Y be two independent $N(0,1)$ random variables. Define $U = \frac{X}{Y}$ and $V = \frac{X}{|Y|}$. Then
1. U and V have the same distribution.
2. V has t distribution.
3. $E\left(\frac{V}{U}\right) = 0$.
4. U and V are independent.

115. Let X and Y be independent and identically distributed random variables having a normal distribution with mean 0 and variance 1. Define Z and W as follows:

$$\binom{Z}{W} = \begin{cases} \binom{X}{Y} & if\ XY > 0 \\ \binom{-X}{Y} & if\ X < 0\ and\ Y > 0 \\ \binom{X}{-Y} & if\ X > 0\ and\ Y < 0 \end{cases}$$

Then

1. Z and W are independent.
2. Z has $N(0,1)$ distribution.
3. W has $N(0,1)$ distribution.
4. $Cov\,(Z, W) > 0$.

116. A fair coin is tossed repeatedly. Let X be the number of Tails before the first Head occurs. After the first Head occurred, an additional Y Tails appear before the next Head occurs. Which of the following statements are true?

1. $(X\ is\ even, Y\ is\ even) = P(X\ is\ odd, Y\ is\ odd)$.
2. $P(X\ is\ even, Y\ is\ even) = P(X\ is\ even, Y\ is\ odd)$.
3. $P(X\ is\ even, Y\ is\ even) > P(X\ is\ even, Y\ is\ odd)$.
4. $P(X\ is\ even, Y\ is\ even) < P(X\ is\ even, Y\ is\ odd)$.

117. Let X_n be distributed as a Poisson random variable with parameter n. Then which of the following statements are correct?

1. $\lim_{n\to\infty} P(X_n > n + \sqrt{n}) = 0$.
2. $\lim_{n\to\infty} P(X_n \le n + \sqrt{n}) = 0$.
3. $\lim_{n\to\infty} P(X_n \le n) = \frac{1}{2}$.
4. $\lim_{n\to\infty} P(X_n \le n) = 1$.

118. Let $(X_n)_{n\ge 0}$ be a Markov chain on state space $S = \{-N, -N+1, \ldots, -1, 0, 1, \ldots, N-1, N\}$ with the transition probabilities given by

$p_{i,i+1} = p_{i,i-1} = \frac{1}{2}$ for all $-N+1 \le i \le N-1$

$p_{N,N-1} = p_{-N,-N+1} = p_{N,N} = p_{-N,-N} = \frac{1}{2}$.

Then

1. $(X_n)_{n\ge 0}$ has a unique stationary distribution.
2. $(X_n)_{n\ge 0}$ is irreducible.
3. $\lim_{n\to\infty} P(X_n = N | X_0 = 0) = \lim_{n\to\infty} P(X_n = -N | X_0 = 0)$.
4. $(X_n)_{n\ge 0}$ is recurrent.

119. Suppose U and V are independent and identically distributed random variables with $P(U = i) = P(V = i) = \frac{1}{4}$ for $i = 1, 2, 3, 4$. Consider the triangle T, bounded by the $x-$ axis, $y-$ axis and the line $Ux + Vy = UV$. Then which of the following statements are true?

1. $P(Area(T) < 2) = \frac{5}{16}$
2. $P(T\ is\ isosceles) = \frac{1}{4}$
3. $P(Area(T) \le 8) = 1$
4. $P(Area(T) > 1) = 1$

120. For any set of data, which of the following statements are true?

1. Standard deviation $\le \frac{1}{2}$ (range).
2. Mean absolute deviation about mean $\le$ standard deviation.
3. Mean absolute deviation about median $\le$ standard deviation.
4. Mean absolute deviation about mode $\le \frac{1}{2}$ (range).

8

P X → No. of tails./H.
Y → H/Tails./H
n+2

$\left(\frac{1}{2^{n+1}}\right)\left(\frac{1}{2^{n+2+m}}\right)$

n, +2
+1

N

$\begin{array}{c|cccc} & & & & \\ N & 1/2 & 1/2 & 0 & \cdots 0 \\ N-1 & 1/2 & 0 & 1/2 & \cdots \\ & 0 & 1/2 & 0 & 1/2 \cdots \\ 0 & & & & \\ -1 & & & & \\ -2 & & & & \end{array}$