Linear Algebra

1. Which of the followings are Subspace of $\mathbb{R}^3$?

a) $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 0\}$

b) $S = \left\{(x,y,z) \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} x+y+z=0 \\ x-y+z=0 \end{array}\right\}$

c) $S = \{(x,y,z) \in \mathbb{R}^3 \mid x=0,\ y=z\}$

d) $S = \{(x,y,z) \in \mathbb{R}^3 \mid x(x^2+y^2)=0\}$

2. Which of the followings are L.D?

a) $S = \{p(x) \mid p(0)=0\}$ in $\mathbb{P}_2(\mathbb{R})$.

b) $S = \{p(x) \mid p(1)=1\}$ in $\mathbb{P}_2(\mathbb{R})$

c) $S = \{p(x) \mid \deg(p(x)) \geq \deg(p'(x))\}$

d) $S = \left\{p(x) \;\middle|\; \frac{\deg(p(x))}{\deg(p'(x))} > 1\right\}$

(3) which of the followings are subspace of $M_2(\mathbb{R})$?

a) $\{ A \in M_2(\mathbb{R}) \mid tr(A^T A) = 0 \}$

b) $\{ A \in M_2(\mathbb{R}) \mid A = A^T \text{ and } A^2 = 0 \}$

c) $\{ A \in M_2(\mathbb{R}) \mid |A| = 0 \}$

d) $\{ A \in M_2(\mathbb{R}) \mid A + A^T = 0 , A - A^T = I \}$

(4) which of the followings are subspace of $\mathbb{R}[x]$

a) $\{ p(x) \mid p(0) = p'(0) = p''(0) = 0 \}$

b) $\{ p(x) \mid deg(p(x)) = deg(p(x+1)) \}$

c) $\{ p(x) \mid \int_0^x p(t)\,dt = 2 \}$

d) $\{ p(x) \mid \int_0^x p(t^2)\,dt = 0 \}$.

5. Let $V = \mathbb{R}^2$, $\mathbb{F} = \mathbb{R}$. be vector space
Let U, V be subspace of $\mathbb{R}^2$.
Then WOFAT?

a) $U+V \subset U \cap V$

b) $U \cap V + V \subset U+V$

c) $U \subset U+V$

d) $U \cap (U+V) \subset U \cap V + V$

6. Which of the followings are L.I in $P_3(\mathbb{R})$.

a) $\{1, 1+x^2, 1+x^3\}$

b) $\{1\ \ x+x^3, x+x^2\}$

c) $\{x, x+x^3, x+x^2\}$

d) $\{x^3+x^2+x+1, x^2+x+1, x+1, 1\}$

7. consider the following subspace of $\mathbb{R}^3$

$$W = \{(x, y, z) \mid 2x + 2y + z = 0,\ 3x + 3y - 2z = 0,\ x + y - z = 0\}$$

Then $\dim(W) =$

a) 0 b) 1 c) 2 d) 3

8. Which of the followings are subspace of $\mathbb{R}^2$?

a) $W_1 = \{(x, y) \mid 2x + y = 0\}$

b) $W_2 = \{(x, y) \mid xy = 0\}$

c) $W_3 = \{(x, y) \mid \sin^2(x) + \cos^2(y) = 0\}$

d) $W_4 = \{(x, y) \mid \sin^2(x) + \cos^2(x) = 1\}$

9. Let $\mathbb{F}$ be a finite field of order q.

What is the number of 2-dimensional subspace of the vector space $\mathbb{F}^2$ over $\mathbb{F}$?

a) q^3 b) q^2 c) $\frac{(q^3-1)}{(q-1)}$ d) q^3-1

10. Let V be the set of all real $n \times n$ matrices $A = (a_{ij})$ with the property that $a_{ij} = -a_{ji}$ for all $i, j = 1, 2, 3, \dots n$.

Then

a) V is a vector space of dimension $n^2 - n$

b) For every A in V, $a_{ii} = 0$ for all $i = 1, 2, \dots n$

c) V consists of only diagonal matrices

d) V is a vector space of dimension $\frac{n^2-n}{2}$.

11. Let V be the vector space of all complex poly $p(x)$ with degree $\leq n$.

Let $T: V \to V$ be the map $T(p(x)) = p'(1)$

WOFAT?

a) $\dim(\ker(T)) = n$ b) $\dim(\operatorname{Im}(T)) = 1$

c) $\dim(\ker(T)) = 1$ d) $\dim(\operatorname{Im}(T)) = n+1$

12. $V = M_3(\mathbb{R})$ be vector space.

Let $T: M_3(\mathbb{R}) \to M_3(\mathbb{R})$

$T(A) = BA$, B - fixed non-singular matrix

Then

a) $\text{Trace}(T) = 3\sum_{i=1}^{3} b_{ii}$, $B = (b_{ij})$, $1 \leq i \leq 3$, $1 \leq j \leq 3$.

b) $\text{Trace}(T) = \cancel{3} \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij}$

c) $\text{Rank}(T) = 3^2$

d) T is non-singular

13) Let $V = M_{7\times 10}(\mathbb{R})$ be v.s over $\mathbb{R}$.

a) $\exists\, A \in V \ni \text{rank}(A) = 7$.

b) $\exists\, A \in V \ni \text{nullity}(A) = 7$.

c) $\exists\, A \in V \ni \text{rank}(A) = 3,\ \text{nullity}(A) = 3$

d) $\exists\, A \in V \ni \text{nullity}(A) = 10$ and $\text{rank}(A) = 7$.

14. Let $A \in M_3(\mathbb{R}) \ni A^3 = I,\ A \neq \pm I$

Then

a) A is invertiable

b) $\exists\, B \in M_3(\mathbb{R}) \ni AB = I$ but not $BA = I$

c) $\exists\, \lambda \in \mathbb{R} \ni Av = \lambda v$, for some non-zero v in $\mathbb{R}^3$.

d) $|A| \in \mathbb{R} \setminus \{0\}$.

15.) Let $S = \{ T : \mathbb{R}^3 \to \mathbb{R}^3 \mid T$ is linear

and $T(1,0,1) = (1,2,3)$,

$T(1,2,3) = (1,0,1) \}$

Then S is

a) a singleton set b) S is finite set.

c) S is countably infinite set

d) S is uncountable.

16. Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the L.T whose matrix with respect to the standard basis of $\mathbb{R}^3$ is

$$\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$, where $a, b, c \in \mathbb{R} \setminus \{0\}$ then

T is a) 1-1 b) onto c) invertiable

d) has rank 1., e) None

17. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the L.T such that $T(1,2) = (2,3)$ and $T(0,1) = (1,4)$ Then $T(5,6)$ is

a) $(6,-1)$ b) $(-6,1)$ c) $(-1,6)$ d) $(1,-6)$

18. consider the vector space V over $\mathbb{R}$ of poly fn of degree ≤ 3.
Let $T : V \to V$ be defined by $T(f(x)) = f(x) - xf'(x)$. Then

Rank (T) is

a) 1 b) 2 c) 3 d) 4

19. $V = M_n(\mathbb{R})$ be V.S over $\mathbb{R}$. WOFAT?

a) $\exists A \in M_3(\mathbb{R}) \ni$ rank$(A) =$ nullity(A)

b) $\exists A \in M_6(\mathbb{R}) \ni$ rank$(A) =$ nullity(A).

c) $\exists A \in M_7(\mathbb{R}) \ni$ rank$(A) = 1$, nullity$(A) = 6$

d) $\exists A \in M_2(\mathbb{R}) \ni$ rank$(A) = 2$
and nullity$(A) = 0$.

20) $V = \mathbb{P}_2(\mathbb{R})$, let $T: \mathbb{P}_2(\mathbb{R}) \longrightarrow \mathbb{P}_3(\mathbb{R})$ be

a L.T defined by $T(f(x)) = \int_0^x f(t)\,dt + f'(x)$

Then matrix representation of T w.r.t. basis

$\{1, x, x^2\}$ and $\{1, x, x^2, x^3\}$ is

a) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1/2 & 0 \\ 0 & 2 & 0 & 1/3 \end{bmatrix}$ b) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1/2 & 0 & 1/3 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1/2 \\ 0 & 2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$

21. Let the mapping T_1, T_2, T_3, T_4 from $\mathbb{R}^3$ to $\mathbb{R}^3$ be defined by

$$T_1(x, y, z) = (x^2 + y^2, x + y, y + z)$$

$$T_2(x, y, z) = (x + y, y + z, x + z)$$

$$T_3(x, y, z) = (x + y, xy, z + 1)$$

$$T_4(x, y, z) = (x, 2y, 5z)$$

WOFAR Linear map?

a) T_1 and T_2 b) T_2 and T_4 c) T_3, d) T_4

22. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (x+y,\ y+z,\ z+x)$$

Then

a) rank$(T) = 0$, nullity$(T) = 3$

b) rank$(T) = 2$, nullity$(T) = 1$

c) rank$(T) = 1$, nullity$(T) = 2$

d) rank$(T) = 3$, nullity$(T) = 0$

23. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a L.T, where $n \geq 2$.

For $K \leq n$. Let $E_1 = \{V_1, V_2, V_3, \ldots V_k\} \subseteq \mathbb{R}^n$

and $E_2 = \{TV_1, TV_2, TV_3, \ldots TV_K\}$ Then

WOFAT?

a) If E_1 is L.I then E_2 is L.I

b) If E_2 is L.I then E_1 is L.I

c) If E_1 is L.D then E_2 is L.D

d) If E_2 is L.I then E_1 is L.D

24. Let V be a f.d. V.S over $\mathbb{R}$.

Let $T: V \to V$ be a L.T $\ni$ rank(T^2) = rank(T)

Then

a) $Ker(T^2) = Ker(T)$

b) $Range(T^2) = \cancel{Range(T)}$ $Range(T)$

c) rank$(T) = n$

d) range $(T) = Ker(T)$

25.

25) Let $T: \mathbb{R}^7 \to \mathbb{R}^7$ defined by

$$T(x_1, x_2, x_3, \cdots x_7) = (x_7, x_6, x_5, \cdots x_2, x_1)$$

Then WOFAT

where D-diagonal

a) $|T| = 1$ b) $\exists$ B-basis in $\mathbb{R}^7 \ni [T]_B = D$,

c) $T^7 = I$ d) $\exists$ smallest $n < 7$ $\ni T^n = I$.