① Which of the followings are true?

a) $\exists$ a fn $f: \mathbb{R} \to \mathbb{R} \ni f$ is disconts on $\mathbb{Q}$

b) $\exists$ a fn $f: \mathbb{R} \to \mathbb{R} \ni f$ is disconts on $\mathbb{Q}^c$

c) $\exists$ a fn $f: \mathbb{R} \to \mathbb{R} \ni f$ is conts on $\mathbb{Z}$

d) $\exists$ a fn $f: \mathbb{R} \to \mathbb{R} \ni f$ is Nowhere conts on $\mathbb{R}$ but f^2 is conts on $\mathbb{R}$.

② Which of the followings are true?

a) $\exists$ a conts fn $f: \mathbb{R} \to \mathbb{R} \ni$

$\mathrm{Im}(f) = (0,1)$

b) $\exists$ a conts fn $f: \mathbb{R} \to \mathbb{R} \ni$

$\mathrm{Im}(f) = (0,1) \cup (2,3)$

c) $\exists$ a conts fn $f: \mathbb{R} \to \mathbb{R} \ni$

$\mathrm{Im}(f^2) = \mathbb{Z}$.

d) $\exists$ a conts fn $f: \mathbb{R} \to \mathbb{R} \ni$

$\mathrm{Im}(f) = \{\frac{1}{n} \mid n \in \mathbb{N}\}$.

③ which of the followings are true?

a) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ $\ni$ $a_n \to \pi$

b) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $a_n \geq 0, \forall n \in \mathbb{N}$

~~$\nexists$~~, $\ni$ $a_n \to -1$.

c) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $a_n \to 1$

~~then~~ such that $\sum a_n$ is convergent

d) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $a_n \leq 0$

such that $a_n \to \frac{1}{2}$.

(4) which of the followings are true??

a) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $a_n \geq 0$

$\ni \sum a_n = 0$

b) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $a_n \geq 0$

$\ni \dfrac{a_{n+1} + a_n}{2} \longrightarrow \dfrac{l+l}{2}$, $l \neq 0$.

c) $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ and $\sum_{n=1}^{\infty} a_{2n} = l$,

then $\sum_{n=1}^{\infty} a_n = k$, for some k.

d) suppose $\{a_n\}$ be a seq in $\mathbb{R}$ and $\lim_{n\to\infty} \sup\{a_n\} = 1$

Then $\sum a_n$ is convergent.

⑤ Which of the followings are true?

a) Suppose $\exists$ a seq $\{a_n\}$ $\ni$ $|a_{n+1} - a_n| < \frac{1}{n}$ $\forall n \in \mathbb{N}$. Then $\{a_n\}$ is convergent.

b) Suppose $\exists$ a seq $\{a_n\}$ $\ni$ $d(a_n, a_{n-1}) < \frac{1}{n}$ $\forall n \in \mathbb{N}$. Then $\sum a_n$ is convergent.

c) Suppose $\exists$ a seq $\{a_n\}$ $\ni$ $d(a_{n+1}, a_n) < \frac{1}{n^2}$ $\forall n \in \mathbb{N}$. Then $\sum a_n$ is convergent.

d) Suppose $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$, $\ni$ $a_n \geq 0$ and $\sum a_n$ is convergent then $\sum \sqrt{a_n a_{n+1}} < \infty$.

6) Let $\{a_n\}$ be a seq of +ve real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the followings is true?

a) If $l = 1$ then $\lim_{n\to\infty} a_n = 1$

b) If $l = 1$ then $\lim_{n\to\infty} a_n = 0$

c) If $l < 1$ then $\lim_{n\to\infty} \sup\{a_n\} = 0$

d) If $l < 1$ then $\lim_{n\to\infty} \sum a_n$ is convgt

7) Which of the followings series are divergent.

a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2\left(\frac{1}{n}\right)$ b) $\sum_{n=1}^{\infty} \frac{1}{n} \log(n)$

c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin(1/n)$ d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan\left(\frac{1}{n}\right)$.

8) Let $\{a_n\}$ be a seq of +ve real numbers.

The series $\sum a_n$ converges if the series

a) $\sum_{n=1}^{\infty} a_n^2$ converges

b) $\sum_{n=1}^{\infty} \frac{a_n}{2n}$ converges

c) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges.

d) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

9) For $x \in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & ; x = 0 \end{cases}$

Then which of the followings are false?

a) $\lim_{x \to 0} \frac{f(x)}{x} = 0$

b) $\lim_{x \to 0} \frac{f(x)}{x^2} = 0$

c) $\lim_{x \to 0} x^2 f(x) = 0$

d) $\lim_{x \to 0} \frac{x^2 f(x+1)}{e^x} = 0$.

10) Let S be a subset of $\mathbb{R}$ such that 2023 is an interior point of S. Then,

a) S contains an interval

b) $\exists$ a seq $\{a_n\}$ in $S \ni a_n \to 2023$

c) $\exists$ $x \neq 2023$ in $S \ni x$ is an interior point of S

d) $\exists\, x \in S \ni |x - 2023| = 0.02023$