1: Which of the following can't be class eqn of given gp G of order ?

a) $5 = 1+1+1+1+1$

b) $6 = 1+1+2+2$

c) $15 = \underbrace{1+\cancel{3}+1+\cdots+1}_{15\text{-times}}$

d) $33 = \underbrace{1+1+\cancel{3}+1+1+\cdots+1}_{33\text{-times}}$

② Let $x, y \in [0, 10]$ be such that $x \neq y$. Which of the followings statements are true?

a) $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall n \geq n_0 \ni |x-y| < e^{n} \cdot \varepsilon^2$

b) $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} \ni \forall n \geq n_0 \ni \frac{1}{n}\varepsilon < |x-y| < 3$

c) $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} \ni \forall n \geq n_0 \ni |x-y|^2 > \frac{1}{n \cdot \varepsilon} + \varepsilon^2$

d) $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N} \ni \forall n \geq n_0 \ni |x+n^2-y+y^2| > \varepsilon/2^n + 2^n$

③ The last two digits of 9^{9^9} will be

a) 29 b) 49 c) 69. d) 89.

④ The number of elements of order 3 in the non-abelian gp of order 58

a) 0 b) 28 c) 2 d) 1

4. Pick out the cases where the given ideal is a maximal ideal?

a) The ideal $15\mathbb{Z}$ in $\mathbb{Z}$

b) The ideal generated by x^3+x+1 in $\mathbb{Z}_3[x]$

c) The ideal generated by x^2-1 in $\mathbb{R}[x]$

d) The ideal generated by $x^3-3x^2+6x+12$ in $\mathbb{Q}[x]$.

5. Let $\mathbb{R}[x]$ be the ring of real polys in the variable x. The number of ideals in the quotient ring $\frac{\mathbb{R}[x]}{\langle x^2-3x+2 \rangle}$ is

a) 2 b) 3 c) 4 d) 6

6. Let $S = \{a+ib \mid a, b \in \mathbb{Z}, b \text{ is even}\}$

Then S is

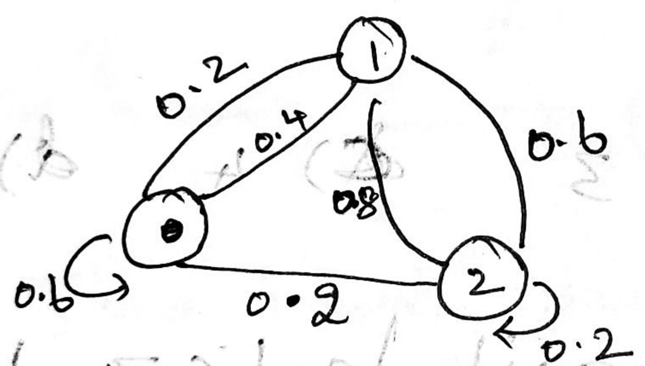
a) A subring of $\mathbb{Z}[i]$ but not ideal

b) An ideal of $\mathbb{Z}[i]$ but not prime

c) An ideal of $\mathbb{Z}[i]$ but not maximal

d) S is an ideal generated by $2+2i$

7. Consider the Markov-chain with state space $\{0, 1, 2\}$ and transition diagram



Find $F(X_2 = 3 \mid X_0 = 1)$

a) 0.27 b) 0.28 c) 0.29 d) 0.30

8) The differential eqn.

$$y'' - 3y' + 2y = 4\sin(x)$$ with

the initial condition $y(0)=1$, $y'(0)=-2$

is converted to $\phi(x) = f(x) + \int_0^x K(x,\xi)\phi(\xi)\,d\xi$

where $K(x;\xi)$ is the Kernel. Then

a) $K(1,1) = -1$, b) $K(x;\xi)$ is symmetric

c) $K(0,0) = 1$, d) $K(x;\xi)$ is not symmetric

(9) $u_t - u_{xx} = 0, \ x \in \mathbb{R}, t > 0$

$u(x,0) = e^{-x}, \ x \in \mathbb{R}$

Then $u(0,1)$ is

a) e b) e^2 c) e^3 d) None of these

10) Let $\Omega = \{(x,y) \mid x^2 + (y-2)^2 < 4\}$ with its bddry $\partial\Omega$. Consider the bddry value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ in } \Omega,$$

$u = x^2 - y^2$ on $\partial\Omega$. Then

$\operatorname{Max}\{u(x,y) \mid (x,y) \in \Omega \cup \partial\Omega\}$ is

a) 0 b) 1 c) 2 d) 3.

11) Consider the differential equation $y' = -2(y-3)(y+7)$ and let $y = y(x)$ be its soln such that $-1 < y(x_0) < 1$, $y'(x_0) < 0$

Then

a) $-7 < y < 7$ b) $-7 < y < 3$

c) $-\infty < y < -7$ d) No such y exists

12) Consider the matrices $A = \begin{bmatrix} 3 & 3 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ Then choose the false?

a) $A \sim B$ over $\mathbb{Q}$ b) A is diagonalizable over $\mathbb{Q}$ but not B

c) B is J.C.F of A

d) $M_A(x) = Ch_A(x)$, where $M_A(x)$ - minimal poly of A, $Ch_A(x)$ - char poly of A

13. Consider the Quadratic forms

$Q_1 = 2xy$

$Q_2 = x^2 + 6xy + 2y^2$

$Q_3(x,y) = 3x^2 + 2xy + 5y^2$ on $\mathbb{R}^2$.

Then

a) Q_1 and Q_2 are equivalent

b) Q_1 and Q_3 are equivalent

c) Q_2 and Q_3 are equivalent

d) All the equivalent

14. Let $A_{10\times10}$ - matrix with real entries such that whose char poly is $(x-4)^4(x-3)^6$ and minimal poly is $(x-4)^2(x-3)^3$. Then the number of ~~J.C.F~~ possible J.C.F of A is

a) 7 b) 11 c) 13 d) infinite

15. The system of eqn

$$\left.\begin{aligned} 2x+4ky-z &= 1 \\ x-8y-3z &= -2 \\ 2x-z &= 1 \end{aligned}\right\}$$ has

unique soln for all the values of k, lying in

a) $(-1,1)$ b) $[0,1]$ c) $(-1,2]$

d) $(1,8]$

16. The radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \sin\left(\frac{n\pi}{4}\right)$

a) $R \geq 1$ b) $R < 1$ c) $R = 1$ d) None of these

17. Let $f(z) = \frac{e^{z^2}}{z}$ on $D = \{z \in \mathbb{C} \mid 1 \le |z| \le 2\}$

Then Maximum value and Minimum value of $|f(z)|$ are M and m respectively, Then

a) $M = \frac{e^4}{2}$, $m = \frac{1}{e}$

b) $M = e$, $m = \frac{1}{e}$

c) $M = \frac{e^4}{2}$, $m = \frac{1}{2e^4}$

d) None of these

18. If C is closed curve enclosed Origin in +ve sense, Then

$$\int_C \cos\left(\cos\left(\frac{1}{z}\right)\right) dz$$ is

a) 0 b) $2\pi i$ c) πi d) $-\pi i$

19. Let S be the set of all conts ~~fns~~ $f : [0,1] \to \mathbb{R}$ such that

$$|f(x)| \le \int_0^x f(t)\,dt \quad , \forall x \in [0,1]$$

Then

a) $|S| = 1$ b) $|S| > 1$

c) $|S|$ is countably infinite

d) $|S|$ is uncountable

20. The value of

$$\lim_{n\to 0} \left\{ \left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \cdots \left(\frac{n+1}{n}\right)^n \right\}^{1/n}$$ is

a) e b) π c) $1/e$ d) $1/\pi$

21.) which of following fns are uniformly conts $(1, \infty)$

a) $\frac{1-x^2}{1-x}$ b) $\sin\left(\frac{1}{1-x}\right)$

c) $x-1 \sin\left(\frac{1}{x-1}\right)+x^2$, d) $\frac{e^x}{x^2+1}+x^2+1$

22) The number of subfield of a finite field of order 3^{10} is equal to

a) 4 b) 5 c) 3 d) 10

23. Let $y(x)$ be the soln of the differential eqn $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=0$ for $x>0$. Satisfying the condition $y(1)=2$ and $y'(1)=0$. Then the value of $e^4y(e^2)$ is

a) 2 b) 4 c) 6 d) 10

24. For a Volterra type integral equation $y(x) = x + \int_0^x (\sin(x-t)\, y(t))\, dt$

choose the incorrect ?

a) It has a unique unbdd soln

b) It has a unique bdd soln

c) It has a soln such that $y(x) \to -\infty$ as $x \to -\infty$

d) It has a non-periodic soln

25. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined as

$$f(x,y) = (x^2-y^2, 2xy)$$

Then

a) f is surjective

b) f is not locally 1-1 in any ~~ngh~~ neighbourhood of (3,4)

c) f is not surjective

d) f is not diff'ble

26 WSOFAT ?

a) $\int_0^\infty \frac{1}{t^2+\sqrt{t}}\,dt$ is convergent

b) $\int_0^1 \frac{t}{e^{at}}\,dt$ is congt

c) $\int_0^\infty \frac{e^t}{4^t}\,dt$ is divergent

d) $\int_0^1 \frac{e^{t/2}}{\sqrt{1-\cos t}}\,dt$ not divgnt.

27 Let $A = A^T$ with real entries such that all the eigenvalues of A are non-negative then WOFAT?

a) $(tr(A))^n \geq n^n |A|$ c) $(tr(A))^n = n^n |A|$

b) $(tr(A))^n \leq n^n |A|$ d) $(tr(A))^n = n^n \cdot |A|$.

28) If $u(x,t)$ is the soln of the initial bdry value problem $u_{tt} = 4u_{xx}$, $0 < x < \infty$, $t > 0$

$u(x,0) = x^4$, $0 \leq x < \infty$

$u_t(x,0) = 0$, $0 \leq x < \infty$

$u(0,t) = 0$, $t \geq 0$

Then $u(1,2)$ is

a) 762 b) 267 c) 276 d) None

29. How many zeros does the complex poly $3z^9 + 8z^6 + z^5 + 2z^3 + 1$

Have in the annulus $1 < |z| < 2$?

a) 0 b) 3 c) 6 d) 9.

30. Which of the followings are not true?

a) The set of singular matrices in $M_n(\mathbb{R})$ is closed

b) the set of all nilpotent matrices in $M_n(\mathbb{R})$ is closed

c) the set of all ~~orgo~~ orthogonal matrices is closed in $M_n(\mathbb{R})$

d. $\{(x, y) \in \mathbb{R}^2 \mid xy = 1\}$ is open in $\mathbb{R}^2$ w.r.t usual topology.

part C

31. ~~If~~ ~~If~~

31. If $A = x^T y$, where $x, y \in \mathbb{R}^n$.

Then

a) A is diagonalizable if $tr(A) \neq 0$

b) A is diagonalizable iff $tr(A) \neq 0$

c) $tr(A) = 0 \Rightarrow$ A is not diagonalizable over $\mathbb{R}$.

d) A is not diagonalizable $\Rightarrow tr(A) = 0$

32. Let $A = [a_{ij}]_{m \times n}$, $a_{ij} \in \mathbb{R}$.

Let J be a J.C.F of A.

Then

a) A has rank $n \Leftrightarrow$ J has n-nonzero entries

b) A is diagonalizable $\Leftrightarrow$ J has non-entries.

c) J has n+1 non-zero entries $\Rightarrow$ A is not diagonalizable

d) If corresponding to every eigenvalue only one Jordan block in used then A may or may not be diagonalizable

33. Let V and W be two vector space over the same field F and let $T: V \to W$ be a L.T then WOFAT?

a) If S is a subspace of V, then T(S) is a subspace of W.

b) If S is a L.I set in V, then T(S) is L.I set in W

c) If S is L.D set in V, then T(S) is a L.D set in W

d) If T(S) is a L.D set in W then S is L.D set in V.

34. Let $V = \mathbb{P}_n(\mathbb{R})$, $\mathcal{B} = \{x^n, x^{n-1}, x^{n-2}, \dots x, 1\}$

be basis of ~~V~~. Now define a

a L.T $T : V \to V$;

$$T(p(x)) = p'(x) + \frac{1}{x}\int_0^x p(t)\,dt$$

and let A be the matrix of T

w.r.t $\mathcal{B}$. Then

a) $|A| \neq 0$ b) $|A| = \frac{1}{(n+1)!}$

c) A is diagonalizable over $\mathbb{R}$

d) A is ~~diag~~ triangularizable

over $\mathbb{R}$.

35. Let V be v.s and T be a L.T from V into V. Then

a) If Range(T) $\cap$ Nullspace$(T) = \{0\}$ and $T^2 \equiv 0$ then $T \equiv 0$

b) If $T^2 \equiv 0 \Rightarrow T \equiv 0$ then Range(T) $\cap$ Nullspace$(T) = \{0\}$

c) If V be f.d v.s and ~~Rank(T^2) = Rank(T)~~ Rank(T^2) = Rank(T) Then Range(T) $\cap$ Nullspace$(T) = \{0\}$

d) If Range(T) = Null-space(T) then Range(T^2) = Null-space(T^2)

36. Determine which of the following polys are irreducible over the indicating rings

a) $x^5 - 3x^4 + 2x^3 - 5x + 7$ over $\mathbb{R}$

b) $x^3 + 2x^2 + x + 1$ over $\mathbb{Q}$

c) $x^3 + 3x^2 - 6x + 3$ over $\mathbb{Z}$

d) $x^4 + x^2 + 1$ over $\mathbb{Z}_2$

37. Let K be an extension of the field $\mathbb{Q}$.

a) If K is a finite extension then it is an algebraic extension

b) If K is an ~~aleg~~ algebraic extension then it must be a finite extension

c) If K is an alegebraic extension then it must be an infinite extension

d) If K is a finite extension then it need not be an algebraic extension

38. Let $f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Then

a) f_x and f_y ~~are~~ bdd on $\mathbb{R}^2$.

b) For any $u \in \mathbb{R}^2$, the directional derivative $D_u f(0,0)$ has ~~ab~~solute value at most one.

c) f is <u>conts</u> on $\mathbb{R}^2$

d) f is diffble on $\mathbb{R}^2$

39. WOFAT?

a) $f(x) = e^{-x^2}$ can be approximated by sequence of polys pointwise on $\mathbb{R}$

b) $f(x) = e^{-x^2}$ can be approximated by sequence of polys uniformly on $\mathbb{R}$

c) $f(x) = \sin(x)$ can be approximated by seq of polys uniformly on $(0, e)$

d) $f(x) = 1 + x + x^2$ can be approximated by seq of polys uniformly on $\mathbb{R}$.

40. Let $f : [0,1] \longrightarrow \mathbb{R}$ be twice differential on $(0,1)$ such that $f(0) = f(1) = 0$ and $f'' + 2f' + f \geq 0$. Then which of the following values can't be attained by f

a) π b) e c) e^{π} d) π^{e}

41. Which of the following space is first Countable?

a) $\mathbb{R}$ w.r.t co-countable topology

b) $\mathbb{R}$ w.r.t co-finite top

c) $\mathbb{R}$ w.r.t discrete top.

d) $\mathbb{R}$ w.r.t usual top.

42. The given system has (2,0) and (0, 3/2) as critical points, then MOOFAT?

$$x'(t) = 2x - x^2 - xy$$
$$y'(t) = 3y - 2y^2 - 3xy$$

a) $(0, 3/2)$ is a unstable saddle point

b) $(2, 0)$ is a unstable node.

c) $(0, 3/2)$ is a unstable node

d) $(2, 0)$ is asymptotically stable node.

43. The solⁿ of the integral eqn

$$\phi(x) = e^{x}\sin(x) + \int_0^x \frac{2+\cos(x)}{2+\cos(\xi)}\,\phi(\xi)\,d\xi$$

a) satisfies $\phi(-\pi) = -e^{\pi}\log(3)$

b) $\phi(\pi) = e^{\pi}\log(3)$

c) $\phi(2\pi) = 0$

d) $\phi(n\pi) = 0$, where n-even integer.

44. Consider the PDE $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$

$(x, t) \in \mathbb{R} \times (0, \infty)$ with initial

condition $u(x,0) = \begin{cases} -1 & ; x \le 0 \\ 2x-1 & ; 0 \le x \le 1 \\ 1 & ; x \ge 1 \end{cases}$

Then the possible soln of the given PDE is/are

a) $u(x,t) = -1$

b) $u(x,t) = 2$

c) $u(x,t) = 0$

d) $u(x,t) = \frac{2x-1}{2t+1}$

45) Let $y(x)$ be a twice continuously differentiable fn on $(0,\infty)$ satisfying $y(1)=1$ and

$y'(x) = \frac{1}{2} y\left(\frac{1}{x}\right), x>0$ Then

WOFAT?

a) $y(x) \to 0$ as $x \to 0$

b) $y(x)$ is bdd in $(0,\infty)$

c) $y(x)$ is decreasing in $(0,1)$ and increasing $(1,\infty)$

d) $y(x)$ is decreasing in $(0,\infty)$

46. Let G be a gp of order 1001, then

a) It has normal subgp of order 143

b) It has normal subgp of order 91

c) It has normal subgp of order 77

d) It has unique normal subgp for order 77, 91, 143.

47. Let be a gp of order 60 and G has a normal sub gp of order 2. Then WOFAT?

a) G has normal subgp of order 6.

b) G has normal subgp of order 10

c) G has normal subgp of order 30

d) G has a subgp of order 15

48. Let $f : [-4, \infty) \to S$ be a conts fn Then WOFAT?

a) If S is closed then f has fixed point

b) If $S = (-2, a)$ then f has fixed point

c) If $S = (-4, a)$, then $\forall a > 0$, f has a fixed point.

d) If $S = [-1, \infty)$ then f has fixed point.

49. Let $A \subseteq \mathbb{R}$ and every fn defined on A is uniformly conts then

a) A can be finite set

b) A can be infinite set

c) $A \cap A' = \phi$

d) A is closed

50) In which of the following cases, there is a cont's fn f from the set S onto the set T?

a) $S = (-\infty, 0) \cup (0, \infty)$, $T = \mathbb{R}$

b) $S = (-1, 0) \cup (0, 1)$, $T = \mathbb{R}$

c) $S = [0, 1)$, $T = (-\infty, 0]$

d) $S = [0, 1) \cup [2, 3]$, $T = (-3, \infty)$

51. For $\lambda \in \mathbb{R}$, consider the differential equ

$$y'(x) = \lambda \sin(x + y(x)),$$

$$y(0) = 1.$$

Then Initial value problem has

a) No soln in any ngh of '0'

b) a soln in $\mathbb{R}$ if $|\lambda| < 1$

c) a soln in a ngh of $(0,1)$

d) a soln in $\mathbb{R}$ only if $|\lambda| > 1$

52. Let X be the space of all real poly $a_5t^5+a_4t^4+a_3t^3+a_2t^2+a_1t+a_0$ of degree at most 5. We may think of X as a topology space via identification with $\mathbb{R}^6$ given by

$$a_5t^5+a_4t^4+\dots+a_0 \longleftrightarrow (a_5,a_4,a_3,a_2,a_1,a_0)$$

which of the following ~~are~~ subsets are connected in X?

a) set of all poly in X that do not vanish at 2.

b) All polys in X whose derivative vanishes at $t=3$

c) All poly in X that vanish at ~~at~~ $t=4$ and $t=5$

d) All polys in X that are increasing (from $\mathbb{R}$ to $\mathbb{R}$).

53. For any +ve integer,

Let $f_n(x) = \dfrac{x}{1+nx^2}$ Then WOFAT ?

a) The seq $\{f_n\}$ converges uniformly on $[0,\infty)$

b) ~~The~~ ~~seq~~ of ~~fn~~ $\{f_n\}$ converges uniformly only on $[1, b)$, $b>1$

c) The seq ~~of~~ ~~fn~~ $\{f_n\}$ has a uniformly convergent subseq in $\mathbb{R}$

d) The seq $\{f_n'(x)\}$ converges uniformly on $\mathbb{R}$.

54. which of the following are true?

a) $\exists$ an entire fn defined on $\mathbb{C}$ such that $f(0)=1$ and $|f(z)| \le |z|^{-2}$ for $z \in \mathbb{C}$ $\ni$ $|z| \geq 10$

b) ~~If~~ $f: \mathbb{C} \to \mathbb{C}$ is non-constant entire fn then its image is dense in $\mathbb{C}$

c) $\exists$ an non-constant entire f $\ni$ $|f(z)| \le M$, $\forall z \in \{x+iy \mid y>0\}$

d) $$\int_\gamma \frac{e^{-z}}{(z-1)^2} dz = 0, \quad \gamma = |z| = 2.$$

55. Let $f: \mathbb{C} \to \mathbb{C}$ be an entire fn $\ni f(z+1) = f(z+i) = f(z)$ $\forall z \in \mathbb{C}$. WOFAT?

a) f is constant b) $f(z) = 0, \forall z \in \mathbb{C}$

~~c) $\exists z_0, z_1 \ni \forall x, y \in \mathbb{R}$,~~

~~$f($~~

c) $\exists$ a complex number z_0, z_1 in $\mathbb{C}$ and $a, b \in \mathbb{R}$, $f(x+iy) = z_0 \sin(x) + i z_1 \cos(y)$

d. f is not necessarily constant but $|f(z)|$ is constant

CSIR NET Part A Mock test - December 23

56 1)The product of two numbers is 5376 and their HCF is 16. Find the largest number.

A)112 B)212 C)256 D)128

57 2)What is the least number which when divided by 16 and 18 leaves 9 as remainder in each case, but when divided by 27 leaves no remainder.

A)639 B)729 C)541 D)873

58 3)Three numbers are in ratio 2:3:4 and their LCM is 60. Their HCF is

A)5 B)10 C)15 D)20

59 4)Ratio of money with A and B is 2:3 and that with B and C is 3:5. If the money with B is 120, how much money does C has?

A)50 B)150 C)200 D)250

60 5)Income of A and B are in the ratio 7:9, and their expenditures are in the ratio of 3:4. If each of them saves 800. Find the income of A

A)2000 B)2400 C)4800 D) 5600

61 6)Ram joined a company in 2000 and got 1500 as a salary. Salary increased by 10% for the first year, 20% for the second year and 40% for the third year. What will be the salary after 3 years?

A)2440 B)2660 C)2772 D) 3600

62 7)In an exam, if a student gets 135 then he scored 25% less than the pass mark. If the pass % is 60, then find the total mark of the exam.

A)250 B)300 C)375 D)420

63 8)10% of voters did not cast their vote in an election between two candidates. 10% of the votes polled were found invalid. The successful candidate got 54% of the valid votes and won by a majority of 1620 votes. The number of voters enrolled on the vote list was

A)21500 B)22000 C)25000 D)28000

64

9)The average age of 8 men is increased by 2 years when two of them whose ages are 21 and 23 years are replaced by two new men. The average age of the two new men is

A)25 B)30 C)35 D)40

65

10)The ratio between the presnet age of P and Q is 6:7. IF Q is 4 years older than P, what will be the ratio of ages of P and Q after 4 years?

A)7:8 B)6:9 C)10:13 D)2:3

66

11)Anna left for city A from city B at 5.20 am. She traveled at the speed of 80 km/hr for 2 hours 15 minutes. After that, the speed was reduced to 60 km/hr. If the distance between two cities is 350 km, at what time did Anna reach city A?

A)9:45 B)10:25 C)10:45 D)11:00

67

12)12 men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work?

A)2 days B)3 days C)5 days D)6 days

68

13)In what ratio must a grocer mix two varieties worth 60 a kg and 65 a kg so that by selling the mixture at 68.2 a kg he gets a profit of 10%?

A)2:3 B)3:2 C)1:3 D)3:4

69

14) The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

A)3 B)4.2 C)5.4 D)8

70

15)A girl of heigh 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

A)1.4 m B)1.6 m C)2.2 m D)2.4m