TEST

① Let $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$ be a poly of degree of $n \geq 1$, where a_0, a_n are non-zero. Then $f(z) = \frac{1}{p(1/z)}$ which is a meromorphic fn on $\mathbb{C} \setminus \{0\}$

a) has a removable singularity at $z=0$ and is non-vanishing there.

b) has a removable singularity at $z=0$ and has a zero of order n at $z=0$

c) has a pole of order n at $z=0$

d) has an essential singularity at $z=0$.

② Let $C^1(\mathbb{R})$ be the collection of all continuously diff'ble fn on $\mathbb{R}$. Let

$S = \{ f \in C^1(\mathbb{R}) \mid f(0)=0, f(1)=1, |f'(x)| \leq 3/4 \ \forall x \in \mathbb{R} \}$

Then

a) $S = \phi$ b) $S \neq \phi$ c) S is countably infinite

d) S is uncountable.

③ ~~Suppose B is a subset of~~

③ The value of the integral $\displaystyle\int_{|z+\pi|=2} \frac{\sin(z)}{e^z - 1} \, dz$ is

a) 0 b) $2\pi i$ c) 4π d) $4\pi i$

④ Harmonic conjugate of the fn $4xy + 3$ is

a) $2(y^2 - x^2)$ b) $-2(y^2 - x^2)$

c) $2(y^2 + x^2)$ d) $(y^2 - x^2)$

(5) If $f(z) = z|z|$, for every $z \in \mathbb{C}$.

Then $f(z)$ is differentiable

a) at all points z b) only at $z=0$

c) only at $z=1$ d) nowhere.

6) The fixed points of $f(z) = \frac{2iz+5}{z-2i}$ are

a) $1 \pm i$ b) $1 \pm 2i$ c) $2i \pm 1$ d) $i \pm 1$

7) The value of $\int_{|z|=1} \frac{e^z}{(z-2)(z-3)} dz$ is

a) $2\pi i$ b) πi c) 0 d) $\pi i/6$

8) The value of the constant α for which the fn $u(x,y) = \alpha x^2 - y^2 + xy$ is harmonic is

a) 1 b) 5 c) 0 d) -1

9) If $f(z) = \frac{1}{\sin(1/z)}$, $z \neq 0$ Then

a) $Z = \pi$ is a pole of f b) $z = 1/6\pi$ is a pole of f

c) $Z = \frac{1}{\pi}$ is a pole of f d) f has no poles.

10. The maclaurin series expansion of $f(z) = \frac{z}{z^4+9}$ is

a) $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^{4n+2}$, $|z| < 2$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} z^{4n+1}$, $|z| < \sqrt{3}$

c) $\sum_{n=0}^{\infty} \frac{1}{3^{2n+1}} z^{4n+1}$, $|z| < 4$

d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+2}} z^{4n}$, $|z| < 4$.

11) If $f(z) = \dfrac{1-\cos(z)}{z^2}$ then $z=0$ is

a) a double pole b) a simple pole

c) is a removable singularity

d) is an essential singularity.

12) The radius of convergence of the power series $\sum_{k=1}^{\infty} \dfrac{z^4}{k^2}$ is

a) 1 b) 2 c) 0 d) ∞.

13) The value of the line integral $\int x\,dz$ along the straight line joing (0,0) and (1,1) is

a) $\frac{1}{2}+i\frac{1}{2}$ b) $\frac{1}{2}-\frac{i}{2}$ c) 0 d) $\frac{1}{2}$.

15) Let U denote the unit open disc centred at '0'. Let $f : U \setminus \{0\} \rightarrow \mathbb{C}$ be an analytic fn. Which of the followings are true?

a) $f'(z) \neq 0, \forall z \in U \setminus \{0\}$

b) If f is bijective then $f'(z) \neq 0$, $\forall z \in U \setminus \{0\}$ ~~$\forall z \in U$~~

c) If $f'(z) \neq 0, \forall z \in U \setminus \{0\}$, then f is bijective

d) If $f'(z) \neq 0$ for every $z \in U \setminus \{0\}$, then f is injective.

16) The number of roots of the poly

$p(z) = z^7 + z^6 + 7z^5 + 14z^4 + 31z^3 + 73z^2 + 23z + 200$

in the open left-half of the complex plane is

a) 3 b) 4 c) 5 d) 6.

17) If C is a circle $|z|=4$ and

$$f(z) = \frac{z^2}{(z^2-3z+2)^2}$$ then $\int_C f(z)\,dz$ is

a) πi b) 0 c) $-\pi i$ d) $-4\pi i$

18. Let f be an entire fn with $f(z) \to 1$ as $|z| \to \infty$. Then $f(0)$ has the value

a) 1 b) -1 c) 0 d) 2.

19. Let f be analytic in the disk $= \{z \mid |z|<1\}$ with $f(0)=0$ and $|f(z)| \leq 1$ for all z in the disk. ~~Then~~ Then which of the following statements does not hold?

a) $|f(\tfrac{1}{4})| \leq 1/4$ b) $|f'(0)| \leq 1$

c) $|f(1/2)| > 1/2$ d) $|f(-1/4)| \leq 1/2$.

20) If $\gamma(t) = 1 + 2e^{it}$, $0 \le t \le 2\pi$, Then

$\frac{1}{2\pi i} \int_{\gamma} \frac{z^2+3}{z-2} dz$ has the value

a) 0 b) 1 c) 7 d) 5.

21) The residue of $\oint \frac{1}{z^2+1}$ at $z = i$ is

a) i b) $\frac{i}{2}$ c) $-\bar{i}/2$ d) $\underline{1}$.

22) The singularity of the fn $\frac{\sin(z)}{z^2}$ at $z=0$ is

a) pole of order 2 b) A removable singularity

c) An essential singularity d) A simple pole.