~~Exam~~ Complex Analysis

① Let f be an analytic fn $\mathbb{C}$. Then which of the followings statements are true?

a) If $|f(z)| \le 1 \ \forall z \in \mathbb{C}$ Then f' has infinitly many zero on $\mathbb{C}$

b) If f is onto then $f(\sin(z))$ is onto

c) ~~If~~ If f is onto ~~a~~ Then $f(e^z)$ is onto on $\mathbb{C}$

d) If f is 1-1 then $f(e^z)$ is 1-1.

② Let f be a non-constant entire fn
Then which of the followings are not true?

a) $Im(f)$ is non-empty open subset of $\mathbb{C}$

b) $\overline{Im(f)} = \mathbb{C}$

c) $(Im(f))^{\circ} = Im(f)$

d) $\overline{(Im(f))^{c}} = \phi$

3) Let $f: \mathbb{C} \to \mathbb{C}$ be an analytic fn :

Then $|f'(x+iy)|^2 = $ ___ , $(f = u+iv)$

a) $u_x^2 + u_y^2$ b) $u_x^2 + v_x^2$ c) $v_y^2 + u_y^2$ d) $v_y^2 + v_x^2$

4) Let $p(z) = \sum_{k=0}^{N_0} a_k z^k$, for $1 < N_0 < \infty$, $a_n \in \mathbb{R} \setminus \{0\}$

Let $D = \{ z \in \mathbb{C} \mid |z| < 1 \}$ Then

a) $p(D)$ is open b) $p(D)$ Connected

c) $p(D)$ is closed d) $p(D) \subseteq \mathbb{R}$.

5) Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be a power series

and suppose $f(n) = n$, $\forall n \in \mathbb{N}$ Then

a) Radius of convergence is ∞

b) $\cancel{\exists} 0 < R < \infty$ c) $\sum_{n=0}^{\infty} a_n z^n$ converges for $z \in \mathbb{R}$

d) $\sum_{n=0}^{\infty} a_n z^n = 0$, $\forall z \in \mathbb{R}$,

6) Let $u(x,y) = x^3 - 3xy^2 + 2x$. For which of the following fn v, is $u+iv$ a holomorphic fn on $\mathbb{C}$?

a) $v(x+iy) = y^3 - 3x^2y + 2y$

b) $v(x+iy) = 3x^2y - y^3 + 2y$

c) $v(x+iy) = x^3 - 3xy^2 + 2x$

d) $v(x+iy) = 0$

7) Let f be a real-valued harmonic fn on $\mathbb{C}$. that is $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$,

Define $g = \dfrac{\partial f}{\partial x} - i\dfrac{\partial f}{\partial y}$, $h = \dfrac{\partial f}{\partial x} + i\dfrac{\partial f}{\partial y}$

Then

a) g and h are both holomorphic fns

b) g is holomorphic, but h need not be holomorphic

c) h is holomorphic but g need not be holomorphic

d) Both g and h are identically equal to the zero fn

8) Let $p(z), q(z)$ be two non-zero complex poly. Then $p(z)\overline{q(z)}$ is analytic iff

a) $p(z)$ is constant b) $q(z)$ is a constant

c) $p(z)q(z)$ is constant d) $\overline{p(z)}q(z)$ is a constant

9) Let $f: D \to D$ be a holomorphic fn with $f(0)=0$, where $D = \{z \in \mathbb{C} \mid |z|<1\}$. Then

a) $|f'(0)| = 1$ b) $|f(1/2)| \le 1/2$ c) $|f(1/2)| \le 1/4$

d) $|f'(0)| \le 1/2$.

10) Let $f: \mathbb{C} \to \mathbb{C}$ be complex-valued fn ~~of the fn~~ given by $f(z) = u(x,y) + iv(x,y)$

Suppose that $v(x,y) = 3xy^2$. Then

a) f can't be holomorphic on $\mathbb{C}$ for any choice of u

b) f is holomorphic on $\mathbb{C}$ for a suitable choice of u

c) f is holomorphic on $\mathbb{C}$ for all choice of u

d) v is not differentiable as a fn of x and y

(11) The power series $\sum_{n=0}^{\infty} 2^{-n} z^{2n}$ converges if

a) $|z| \leq 2$ b) $|z| < 2$ c) $|z| \leq \sqrt{2}$ d) $|z| < \sqrt{2}$

12) Let $f(z) = z^5 - 5z + 2$ ~~Then~~ $\forall z \in \mathbb{R}$, Then

a) f has no real root b) f has exactly one real root

c) f has exactly ~~three~~ three real root

d) All roots of f are real.

13) Let f be a holomorphic fn in the open unit disc such that $\lim_{z \to 1} f(z)$ does not exist.
Let $\sum_{n=0}^{\infty} a_n z^n$ be the taylor series of f about $z=0$
and let R be its radius of convergence. Then

a) $R=0$ b) $0 < R < 1$ c) $R=1$ d) $R > 1$

14) Let $f(z) = \sum_{k=1}^{\infty} k x^k$ ~~Then~~. Let R be radius of convergence
Then

a) $R > 0$ and the series convergent on $[-R, R]$

b) $R > 0$ and the series converges at $x = -R$ but not converges at $x = R$

c) $R > 0$ and the series does not converges outside $(-R, R)$

d) $R = 0$

15) Consider the power series $\sum a_n z^n$,

where a_n = number of divisor of $n^{500!}$ then the radius

convergence of $\sum_{n=0}^{\infty} a_n z^n$ is is

a) 1 b) $500!$ c) $\frac{1}{500!}$ d) 0

16) Let $P(z) = a_0 + a_1 z + a_2 z^3 + a_3 z^4$, $a_3 \neq 0$.

consider $f(z) = \sum_{n=0}^{\infty} P(n^2) z^n$. Then radius of

convergence R is

a) 0 b) 1 c) K d) ∞.

17) Let $\sum_{n=0}^{\infty} a_n z^n$ be convergent power series such that

$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = R > 0$. Let $P(z)$ be poly such that

$\deg(P(z)) = K$. Then radius of convergence of

the power series $\sum_{n=0}^{\infty} P(n) a_n z^n$ equals

a) R b) RK c) K d) K+R