Real Number System.

1. let $A_1, A_2, \ldots A_n$ be sets, where n is a fixed natural number. consider the following statements.

(i) if $A = \bigcap_{i=1}^{n} A_i$ is countable infinite, then $\exists$ atleast one A_i for $i = 1, 2, \ldots n$ which is countable.

(ii) If $A = A_1 \times A_2 \times \cdots \times A_n$ is countable infinite then each A_i for $i = 1, 2, \ldots n$ is countable.

(a) 1 is correct and 2 is incorrect.

(b) ~~The set~~ 2 is correct and 1 is incorrect.

(c) Both are correct

(d) Neither 1 nor 2 are correct.

2. If there are injective maps

$f : A \longrightarrow C$ and $g : C \longrightarrow A$ then

(a) A and C both are uncountable

(b) A and C are countable

(c) A and C are finite sets

(d) A and C have the same cardinality

3. let A and B are infinite sets. Let f is a map from A to B such that the collection of pre images of any non-empty subset of B is non-empty. Then choose the incorrect?

(a) If A is countable then B is countable

(b) such map f is always onto

(c) A and B are similar

(d) B may be countable even if A is not countable.

4. let A an infinite set of disjoint open sub-intervals of $(0,1)$. let B be the power set of A. Then

(a) A and B are Cardinally equal.

(b) A is similar to $(0,1)$.

(c) B is similar to $(0,1)$.

(d) A and B both are uncountable.

5. If f be a fuution with domain A and range B then which of the following is correct.

(a) B coutable $\Rightarrow$ A countable

(b) A coutable $\Rightarrow$ B coutable

(c) A uncoutable $\Rightarrow$ B uncoutable

d) All of the above

6. let A be any infinite set, B is subset of A then

(a) $A-B$ countable if A and B countable

(b) $A-B$ is uncountable if A uncountable and B countable.

(c) $A-B$ is countably infinite and $A \neq B \Rightarrow A$ is countably infinite.

(d) $A-B$ is countably infinte and B contable $\Rightarrow A$ countable

7. If S be a countable subset and T be an uncountable subset of $\mathbb{R}$. Which of the following is/are True?

(a) $S \cup T$ is uncountable

(b) $S \cap T$ is at most countable.

(c) $S - T$ is at most countable

(d) $T - S$ is uncountable.

8. Which of the following sets are uncountable.

(a) $\mathbb{Q}^c \cap (a,b)$, $\forall\, a,b \in \mathbb{R}$ where $a < b$

(b) Every subsets A of $\mathbb{R}$ such that $A \cap \mathbb{Q} = \phi$.

(c) $\mathbb{R} - A$, where $A \cap \mathbb{Q}^c = \phi$.

(d) If $A = \{a \mid p(a) \neq 0, \forall\, p(x) \in \mathbb{Q}[x]\}$ then $A \cap \mathbb{Q}^c$.

9. Which of the following statement is incorrect?

a) A set "A" is infinite iff A contains countable infinite set.

b) A set 'A' is infinite iff A is not similar to any of its proper subsets.

c) A set 'A' is infinite iff A is similar to a proper subset of A.

D None of these.

10. If A and B are non-empty define

$B^A = \{ f \mid f : A \longrightarrow B \text{ functions} \}$ and

$A^B = \{ f \mid f : B \longrightarrow A \text{ functions} \}$, then

(a) If B^A and A^B comtable then A and B both countable.

(b) If A^B comtable $\Rightarrow$ A finite and B comtable.

(c) If A countable and B finite $\Rightarrow$ A^B is countable.

(d) If A finite and B comtable $\Rightarrow$ A^B is comtable.

11. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a fuution, $S \subseteq \mathbb{R}$, S is countable and $T \subseteq \mathbb{R}$ T is uncountable set, then

(a) $f(S)$ is countable if f is one-one.

(b) $f(S)$ is countable if f is not one-one.

(c) $f(T)$ is countable if f is one-one.

(d) $f(T)$ is countable if f is not one-one.

12. A fuction $f: \mathbb{R} \to \mathbb{R}$ is given. Whenever we choose real numbers $a<b$ and set $\{f(x) : a<x<b\}$ has a biggest element, we call this element local maximum of function. Then set of all local maximums of fuction f is countable or uncontable?

13. let $A = \{x^2 : 0 < x < 1\}$ and $B = \{x^3 : 1 < x < 2\}$

which of the following statements is true.

(a) There is a one to one, onto funtion from A to B.

(b) There is no one to one, onto funtion from A to B taking rationals to rationals.

(c) There is no one to one funtion from A to B which is onto

(d) There is no onto funtion from A to B which is one to one.

14. Which of the following sets are uncountable?

(a) The set of all functions from $\mathbb{R}$ to $\{0,1\}$

(b) The set of all functions from $\mathbb{N}$ to $\{0,1\}$

(c) The set of all finite subsets of $\mathbb{N}$.

(d) The set of all subsets of $\mathbb{N}$.

15. An algebric number is a root of a polynomial whose coefficients are rational. The set of algebric number is

(a) finite b) countably infinite

(c) uncountable d) none of these.

16. let A be any set then

(a) A is similar to $A \times A$ for any A.

(b) A is similar to $A \times A$ for some finite A.

(c) A is similar to $A \times A$ if A is infinite

(d) A is not similar to $A \times A$ for some infinite A.

17. let A be any set. let $P(A)$ be the power set of A, that is, the set of all subsets of A; $P(A) = \{B : B \subseteq A\}$. Then which of the following is/are true about $P(A)$?

(a) $P(A) = \phi$ for some A

(b) $P(A)$ is a finite set for some A.

(c) $P(A)$ is countable set for some A

(d) $P(A)$ is uncountable set for some A.

18. Which of the following sets of functions are uncountable?

(a) $\{f \mid f : \mathbb{N} \to \{1,2\}\}$

(b) $\{f \mid f : \{1,2\} \to \mathbb{N}\}$

(c) $\{f \mid f : \{1,2\} \to \mathbb{N},\ f(1) \le f(2)\}$

(d) $\{f \mid f : \mathbb{N} \to \{1,2\},\ f(1) \le f(2)\}$

19. consider the following sets of funtions on $\mathbb{R}$

W = The set of constant funtions on $\mathbb{R}$

X = The set of polynomial funtions on $\mathbb{R}$.

Y = The set of all continous funtions on $\mathbb{R}$.

Z = The set of all funtions on $\mathbb{R}$.

Which of these sets has the same cardinality as that of $\mathbb{R}$.

a) only W b) only W and X c) only W, X and Z

d) all of W, X, Y and Z.

20. let A_n and B_n, $n \in \mathbb{N}$ be non-empty subsets of $\mathbb{R}$ such that $A_1 \supseteq A_2 \supseteq \ldots$ and $B_1 \subseteq B_2 \subseteq B_3 \subseteq \ldots$. let the cardinality of A_n be a_n and cardinality of B_n be b_n. Then the cardinality of

(a) $\bigcup_{n=1}^{\infty} B_n$ is $\lim_{n\to\infty} b_n$ (b) $\bigcup_{n=1}^{\infty} B_n$ is $\max_n b_n$

(c) $\bigcap_{n=1}^{\infty} A_n$ is $\lim_{n\to\infty} a_n$ (d) $\bigcap_{n=1}^{\infty} A_n$ is $\max_n a_n$