1. Which of the followings are diagonaligable over $\mathbb{R}$?

a) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x,y) = (y,0)$

b) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x,y) = (0,x)$.

c) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x,y) = (x+y, y)$

d) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x,y) = (6x, 7y)$

② Which of the followings are true?

a) If $A_{2\times 3}(X) = b$ has $\underline{sol^n}$ Then the system has infinitly many $\underline{sol^n}$.

b) If $A_{1\times 4}(X) = b$, $b \neq 0$, ~~Then~~ has $\underline{sol^n}$ for some b, Then $AX = b'$ has infinitly many $\underline{sol^n}$ for $\forall b'$

c) If $A_{4\times 4}(X) = 0$ has infinitly many $\underline{sol^n}$ and $AX = b$, $b \neq 0$ for some b, has $\underline{sol^n}$ Then Then $AX = b$ has infinitly many $\underline{sol^n}$.

3) Which of the followings are Nilpotent Operator ?

a) $T: \mathbb{R}^3 \to \mathbb{R}^3 \quad T(x,y,z) = (0, y, z)$

b) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (y, 0)$

c) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (0, x)$

d) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (x, x)$.

4) Which of the following operator has Eigen ~~ve~~ Basis of V w.r.t T.??

a) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (x, y)$

b) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (0,0)$

c) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (x, -y)$

d) $T: \mathbb{R}^2 \to \mathbb{R}^2 \quad T(x,y) = (y, 0)$.

5. Which of the followings are not true? $\mathbb{F}=\mathbb{R}$.

a) $\mathcal{B}=\{(1,1),(2,3),(7,\pi)\}$ is L.I in $\mathbb{R}^2$

b) $\mathcal{B}=\{(\pi,7),(7,9)\}$ Then $\text{Span}\{\mathcal{B}\}=\mathbb{R}^2$

c) $\mathcal{B}=\{(3,4),(6,8)\}$ is L.D.

d) $\mathcal{B}=\{(\pi,e),(4,3),(0,0)\}$.

Then $\text{span}\{\mathcal{B}\}\supseteq\mathbb{R}^2$.

6. Which of the followings are true?

a) $\mathcal{B}=\{A,A^T\}$ are L.I in $M_2(\mathbb{R})$., $A\neq 0$.

b) $\mathcal{B}=\{A, I+A, A^2, I-A\}$ is ~~L.D~~ L.I in $M_2(\mathbb{R})$.

c) $\mathcal{B}=\{I+A, I-A\}$ is L.I, $\forall A\neq 0$ in $M_2(\mathbb{R})$

d) $\mathcal{B}=\{I, A, A^2, A^3, \ldots A^{n-1}\}$ is L.I in $M_n(\mathbb{R})$.

7). Which of the followings are false?

$\mathbb{F} = \mathbb{R}$

a) $S = \{ f \in C([0,1]) \mid f^2(x) = 0 \}$

Then $\dim(S) \neq 1$.

b) $S = \{ f \in C([0,1]) \mid f(x) > 0, \forall x \in [0,1] \}$

Then $\sin(x) \in \operatorname{Span}\{S\}$.

c) $S = \{ f \in C([0,1]) \mid f(x) = f'(x), \forall x \in [0,1] \}$

Then $\dim(S) = 1$.

d) $S = \{ f \in C([0,1]) \mid f$ is poly of degree $\leq 10 \}$

Then S has a ~~11 L.I~~ $\{v_1, v_2 \ldots v_{11}\}$ L.I in $C([0,1])$

8) $V = \mathbb{Z}_5^3(\mathbb{Z}_5)$, $\mathbb{F} = \mathbb{Z}_5$.

a) No. of two dimensional subspace of V is 1

b) No. of one dimensional subspace of V is 10

c) Every non-zero linear transformation from $T : \mathbb{Z}_5^3 \to \mathbb{Z}_5^3$ is invertiable

d) If $T : \mathbb{Z}_5^3 \to \mathbb{Z}_5^3$ is invertiable Then $T^2 = I$.

which of the followings are true??

9) Let $V = \mathbb{R}^4$, $F = \mathbb{R}$.

a) $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y + z + w = 0\}$

Then $\dim\left(\frac{V}{W}\right) =$ ___

b) $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid \begin{matrix} x + y = 0 \\ y + z = 0 \end{matrix}\}$

Then $\dim\left(\frac{V}{W}\right) =$ ___

c) $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x = y, z = w\}$

Then $\dim(V/W) =$ ___

d) $W_1 = \{(x, y, z, w) \in \mathbb{R}^4 \mid x = 0, z = 0\}$

$W_2 = \{(a, b, c, d) \in \mathbb{R}^4 \mid b^2 = 0, d = 0\}$

Then $\dim(W_1 + W_2) =$ ___.

10) which of the followings are true?

$V = M_3(\mathbb{R})$, $F = \mathbb{R}$.

a) $S_1 = \{A \in M_3(\mathbb{R}) \mid A = -A^T\}$ Then $\dim(S_1) = 2k$, for some $k \in \mathbb{Z}$.

b) $S = \{A \in M_3(\mathbb{R}) \mid A^2 = 0\}$ Then $\dim(\text{span}(S))$ is atmost 2.

c) $S = \{A \in M_3(\mathbb{R}) \mid A = A^T\}$ Then $\dim(\text{span}(S)) = 3k$ for some $k \in \mathbb{Z}$.

d) $S = \{A \in M_3(\mathbb{R}) \mid A^2 + A + I = 0\}$ is subspace of $M_3(\mathbb{R})$.

11) Which of the followings are true?

a) $S = \{A \in M_2(\mathbb{R}) \mid \text{rank}(A) \leq 1\}$.

Then $\text{Span}\{S\} = M_2(\mathbb{R})$

b) $S = \{A \in M_2(\mathbb{R}) \mid \text{rank}(A) \geq 2\}$

Then $\text{span}(S) = M_2(\mathbb{R})$

c) $S = \{A \in M_2(\mathbb{R}) \mid \text{tr}(A) = 0\}$.

Then $\text{Span}\{S\} = S$

d) $S = \{A \in M_2(\mathbb{R}) \mid |A| = 0\}$

Then ~~dim(span~~ $\dim\left(\dfrac{M_2(\mathbb{R})}{\text{span}(S)}\right) = 0$

12) Which of the followings are true?

$V = M_{5\times 3}(\mathbb{R})$, $\mathbb{F} = \mathbb{R}$.

a) $\exists$, $A, B \in V \ni \text{rank}(A+B) \geq 4$.

b) $\exists A, B \in V \ni \text{rank}(AB^T) \geq 4$.

c) $\exists A, B \in V \ni \text{rank}(A) \geq \text{rank}(B) \geq 3$

d) $\exists A, B \in V \ni \text{rank}(AB^T) \leq \text{Max}\{\text{rank}(A), \text{rank}(B)\}$

13) Which of the followings are true?

a) There exists $A \in M_3(\mathbb{R})$, $A \neq I$ $\ni A^3 = I$.

b) $\exists A \in M_2(\mathbb{R}) \ni A^2 = I$

c) $\exists A \in M_3(\mathbb{R}) \ni A^2 + A + I = 0$.

d) $\exists A \in M_2(\mathbb{R}) \ni A^2 + A + I = 0$

14) Which of the followings are not true?

a) $\exists T : \mathbb{R}^2 \to \mathbb{R}^2 \ni T^{2023} \equiv I$.

b) $\exists T : \mathbb{R}^2 \to \mathbb{R}^2 \ni T^2 \neq I$,

c) $\exists T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R}) \ni T^3 \equiv 0$.

d) $\exists T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R}) \ni$ and $\ker(T) \neq \{0\}$ ~~then~~ $\ni T^2 = I$.

15) Which of the following maps are invertiable ??

a) $T : \mathbb{R}^2 \to \mathbb{R}^2 \ni T^{2023} = -I$.

b) Suppose $T : P_2(\mathbb{R}) \to P_2(\mathbb{R}) \ni$

$T^2 = T = I$ ~~Then~~

c) Suppose $T : M_2(\mathbb{R}) \to \mathbb{R}^4 \ni$

$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a\ b\ c\ d)$

d) Suppose $T : M_2(\mathbb{R}) \to P_3(\mathbb{R})$

such that $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + bx + cx^2 + dx^3$

16). Which of the following Linear transformation $\ni \lambda = 0$ has eigen value of T for some eigen vector ie EV?

a) $T : \mathbb{R}^2 \to \mathbb{R}^2 \ni T(x,y) = (y, 0)$

b) $T : P_2(\mathbb{R}) \to P_2(\mathbb{R}) \ni T(p(x)) = p'(x)$

c) $T : M_2(\mathbb{R}) \to M_2(\mathbb{R}) \ni T^2 = 0$.

d) $T : C([0,1]) \to C([0,1]) \ni T^2 = I$.

17) Which of the followings are diagonali -zable over $\mathbb{R}$?

a) $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 5 \\ 5 & 4 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

18) Find the characteristic roots of given transformation

a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x,-y)$

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x+y, x-y)$

c) $T ; \mathbb{R}^2 \to \mathbb{R}^2$ $T(x,y) = (0, x+y)$

d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ $T(x,y,z) = (x+y, y+z, x+z)$

19) a) S.T A is nilpotent $\iff$ all eigenvalues of A are zero

b) S.T any nilpotent matrix has trace zero

c) Let λ be an eigenvalue of A.
P.T $A+KI$ has eigenvalue $\lambda+K$.

d) Find Char poly of zero matrix $A_{n\times n}$.

20) I. Any 10th Theorems statements only to write.

II. Any 10-definition

III Any 10-Results.

21. Let $M_n(\mathbb{F}) = V$, $\mathbb{F}$ - field

Which of the followings are subspaces of V.

a) $\{A \in M_n(\mathbb{F}) \mid |A| \neq 0\}$

b) $\{A \in M_n(\mathbb{F}) \mid |A| = 0\}$

c) $\{A \in M_n(\mathbb{F}) \mid AB = BA$, for some fixed matrix B in $V\}$

d) $\{A \in M_n(\mathbb{F}) \mid A^2 = A\}$

22) No. of proper subspace of $V = \mathbb{R}$, $\mathbb{F} = \mathbb{R}$ is

a) 0 b) 1 c) 2 d) infinite

23) Let $V = \{f : \mathbb{R} \to \mathbb{R} \mid f$ is fn on $\mathbb{R}\}$

$\mathbb{F} = \mathbb{R}$

a) $W_1 = \{f \in V \mid f(-x) = -f(x), \forall x \in \mathbb{R}\}$

is subspace??

b) $W_2 = \{f \in V \mid f(x) = -f(x), \forall x \in \mathbb{R}, f(-x) = f(+x), \forall x \in \mathbb{R}$

Then W is subspace of V?

c) $W_3 = \{f \in V \mid f^2(x) \geq 0 \ \forall x \in \mathbb{R}\}$

Then W is subspace of V.

e) $W_1 \cap W_2 =$ —

f) $W_1 + W_2 =$ —

g) $W_1 \cap W_3 =$ —

24) $V = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is fn} \}$

Which of the followings are subspace of V?

a) $\{ f \in V \mid f(x^2) = f(x)^2 \ \forall x \in \mathbb{R} \}$

b) $\{ f \in V \mid f(0) = f(1) \}$

c) $\{ f \in V \mid f(3) = 1 + f(-5) \}$

d) $\{ f \in V \mid f(-1) = 0 \}$

e) $\{ f \in V \mid f \text{ is conts on } \mathbb{R} \text{ and } f(1) > 0 \}$

25) $V = \mathbb{R}^3$, $\mathbb{F} = \mathbb{R}$.

Which of the followings are subspace of V.

a) $W = \{(a,b,c) \in \mathbb{R}^3 \mid ab = 0\}$

b) $W = \{(a,b,c) \in \mathbb{R}^3 \mid a, b \in \mathbb{Q}\}$

c) $W = \{(a,b,c) \in \mathbb{R}^3 \mid a = b^2\}$

d) $W = \{(a,b,c) \in \mathbb{R}^3 \mid a+b+c = 0\}$

26) Which of the followings are true?

a) Any subset of a L.I set is L.I

b) Any set which contains $v, -v$ then the set is L.D.

c) Any L.I subset of $\mathbb{R}^3$ contains atmost ~~three~~ three elts.

d) If S is Spaning of $\mathbb{R}^4$ Then S contains atleast 4 elts.