Test

part B

① The eigenvalues of matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ are 5 and -1. Then the eigenvalues of $-2A+3I$ are

a) -7 and 5 b) 7 and -5 c) $-\frac{1}{7}$ and $\frac{1}{5}$

d) $\frac{1}{7}$ and $-\frac{1}{5}$

② Consider the matrix $\begin{bmatrix} 2 & a \\ b & 2 \end{bmatrix}$, where a, b are real numbers. The two eigenvalues of this matrix λ_1 and λ_2 are real and distinct $(\lambda_1 \neq \lambda_2)$ when

a) $a<0, b>0$ b) $a>0, b<0$ c) $a<0, b<0$

d) $a=0, b=0$

③ Let A be the 2×2 matrix with elements $a_{11} = a_{12} = a_{21} = 1$ and $a_{22} = -1$ Then eigenvalues of A^{25} are

a) 1024 and -1024 b) $1024\sqrt{2}$ and $-1024\sqrt{2}$

c) $4\sqrt{2}$ and $-4\sqrt{2}$ d) None.

4) Let A be the matrix $A = \begin{pmatrix} a & c \\ 0 & a \end{pmatrix}$ with $a, c \in \mathbb{R}$ and $c \neq 0$. Then there is a 2×2 matrix P such that PAP^{-1} is diagonal

a) $\forall\, a \in \mathbb{R}$ b) $\nexists\, a \in \mathbb{R}$

c) iff $a = c$ d) iff $a = 0$

5) Let A be a 5×5 nilpotent matrix. Then

a) $A - I$ is invertiable

b) $A^3 = 0$ c) A is diagonalizable

d) A^2 is need not be nilpotent.

6) Let A be a 3×3 real matrix $\ni A^3 = I$ and $A \neq I$, $A^2 + A \neq -I$. Then

a) $tr(A) = 0$ b) $tr(A) = 3$ c) $tr(A) = 1$

d) $tr(A) = -1$

7) Let A be an $n \times n$ real matrix. It is known that there are two distinct n-dimensional real column vectors $v_1, v_2 \ni Av_1 = Av_2$. Which of the following can we conclude about A??

a) All eigenvalues of A are non-negative

b) A is not full rank

c) A is not the zero matrix

d) None of the above.

8) For the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 4 & 0 & 1 \\ 3 & 1 & 5 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The eigen values of A are

a) 3, 2, 3, 4 b) 4, 5, 1, 0

c) 0, 3, 4, 5 d) -1, 2, 4, 5.

9) If A is a 6x6 real symmetric matrix of rank 5, Then, the rank of $A^2 + A - I$ is

a) 6 b) 5 c) 1 d) 0

10) Let $X = C([0,1])$ define $T : C([0,1]) \to C([0,1])$

$$T(f) = \int_0^x f(t)\,dt. \quad \forall f \in C([0,1]) \text{ Then}$$

a) T is 1-1 but not onto b) T is onto

c) T is onto but not 1-1 d) T is neither 1-1 nor onto.

11) Let $\mathbb{P}_3(\mathbb{R}) \longrightarrow \mathbb{P}_2(\mathbb{R})$

$$T(p(x)) = p''(x) + p'(x) \text{ Then the}$$

matrix of T w.r.t bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$ of $\mathbb{P}_3(\mathbb{R})$ and $\mathbb{P}_2(\mathbb{R})$ respectively is

a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{bmatrix}$

12) which of the followings are ~~true~~ not true?

a) $\exists$ a L.T $T : P_4(\mathbb{R}) \longrightarrow P_5(\mathbb{R}) \ni$
$\dim(\ker(T)) = \dim(\mathrm{Im}(T))$

b) $\exists$ a L.T $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^4 \ni T^4 = I$

c) $\exists$ a L.T $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \ni T^7 \equiv 0$

~~Then~~

d) $\exists$ a L.T $T : \mathbb{R}^6 \longrightarrow P_5(\mathbb{R}) \ni T$ is
1-1 and onto.

13) which of the followings are ~~not~~ true?

a) Let $V = \mathbb{R}^4$ and $S = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
be sub set of $\mathbb{R}^4$ ~~and~~ $\ni$ span$(S) = \mathbb{R}^4$
Then S-consisting ~~of~~ basis elts of $\mathbb{R}^4$.

b) let $V = \mathbb{R}$, $\mathbb{F} = \mathbb{Q}$.
$S = \{\pi, e, \pi^2, 1+\pi\}$ is L.I

c) Let $V = \mathbb{R}$, $\mathbb{F} = \mathbb{R}$
Suppose $S = \{\alpha \in \mathbb{R} \mid x^3 + x^2 + x + 1 = 0\}$
Then Span$(S) = \mathbb{R}$.

d) $\exists$ a L.T $T : \mathbb{R} \rightarrow \mathbb{R} \ni \sqrt{2} \in \mathrm{Im}(T)$
Then $\mathrm{Im}(T) \neq \mathbb{R}$.

14) Which of the followings are not linear transformation

a) Let $V = \mathbb{C}$, $\mathbb{F} = \mathbb{C}$.
$T : \mathbb{C} \to \mathbb{C}$ defined by $T(z) = \bar{z}$

b) Let $V = \mathbb{R}^3$, $\mathbb{F} = \mathbb{R}$.
$T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x, y, 0)$

c) Let $V = \mathbb{P}_2(\mathbb{R})$
$T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$
$T(p(x)) = p(x+1)$

d) Let $V = M_2(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$
$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + bx + dx^2 + cx^3$.

15) Which of the followings are true?

a) $\exists$ $T : \mathbb{P}_4(\mathbb{R}) \to \mathbb{R}^5$ be a L.T $\ni$ T is 1-1 but not onto

b) $\exists$: linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$
$\ni$ $T^3 = I$ but $T \neq I$.

c) $\exists$: L.T $T : \mathbb{R}^{2023} \to \mathbb{R}^{2023}$ $\ni$
T has NO real eigenvalue

d) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a L.T Then T^2 is linear transformation.

part C

16) which of the followings are true?

a) Every linear Transformation from $T: \mathbb{R}^{2023} \longrightarrow \mathbb{R}^{2023}$ is diagonalizable over $\mathbb{R}$

b) Every L.T $T: \mathbb{R}^{2023} \longrightarrow \mathbb{R}^{2023}$ has atleast one non-zero eigen vectors

c) Every L.T $T: \mathbb{R}^{2023} \longrightarrow \mathbb{R}^{2023}$ $\ni T(L) = \lambda L$ for some $\lambda \in \mathbb{R}$ and L - for some straight line in $\mathbb{R}^{2023}$

d) There doesnot exist a L.T $T: \mathbb{R}^{2023} \longrightarrow \mathbb{R}^{2023}$ $\ni \dim(\ker(T)) = 100$

17) which of the followings are diagonalizable over $\mathbb{R}$?

a) $A = \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 4 & 8 \\ 8 & 1000 \end{pmatrix}$

c) $A = \begin{pmatrix} 999 & 1 \\ 1 & 2019 \end{pmatrix}$ d) $A = \begin{pmatrix} \pi & \pi \\ \pi & \pi \end{pmatrix}$

18) Let $A \in M_n(\mathbb{R}) \ni A$ is diagonalizable over $\mathbb{R}$. Then

a) A^2+A+I is diagonalizable over $\mathbb{R}$.

b) If $AA^T = I$ then A^{-1} is also diagonalizable over $\mathbb{R}$.

c) If $A^2 = 0$ Then $Adj(A)$ is diagonalizable over $\mathbb{R}$.

d) If $A^2 = I$ Then A^{-1} has same eigen values of A.

19). Let $T \in L(\mathbb{R}^3, \mathbb{R}^3)$ be a L.T. $\ni$ $T^3 = I$ and ~~$T \neq I$~~. Then

a) If T is diagonalizable ~~over~~ then T^2+T+I is diagonalizable over $\mathbb{R}$.

b) If T is diagonalizable Then x^2+x+1 is minimal poly

c) T has atleast one eigenvector

d) $|T| = 1$

20) Which of the followings are true?

~~@~~ suppose $\exists$ L.T $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\ni E_{\lambda_1} = \{(x,0) \mid x \in \mathbb{R}\}$, $E_{\lambda_2} = \{(0,y) \mid y \in \mathbb{R}\}$

Then (a) T is diagonalizable.

(b) T^2 is diagonalizable.

(c) $T^2 + T + I$ diagonalizable

(d) char poly of T is $(x-1)^2$

21) Which of the followings are true?

suppose $T: \mathbb{Z}_{2019}^{2019} \longrightarrow \mathbb{Z}_{2019}^{2019}$ be

linear has eigenvalue $\lambda = 1, 2, 3, \cdots 2019$

over $\mathbb{Z}_{2019}$. Then

a) $Tr(T) = 0$ b) T is diagonalizable

c) $Tr(T) = 40234532$

d) $|T - 2I| = 0$ but $|T - 8I| \neq 0$.

22) Suppose $T: \mathbb{P}_{21}(\mathbb{R}) \longrightarrow \mathbb{P}_{21}(\mathbb{R})$ be a L.T $\ni T^{21} = I$. Then

a) T is diagonalizable

b) T is never diagonalizable over $\mathbb{R}$

c) T is diagonalizable $\Leftrightarrow T = I$.

d) suppose T is diagonalizable over $\mathbb{R}$ Then deg(minimal poly) ≤ 1.

23) Let dim $(V) = 10000!$
Suppose $\exists$ $T: V \longrightarrow V$ be a L.T $\ni \lambda_1, \lambda_2 \cdots \lambda_k$ be distinct eigen values of T. Then

a) minimal poly of T is $(x-\lambda_1)(x-\lambda_2)\cdots(x-\lambda_k)$

b) If $\dim(E_{\lambda_i}) = n_i$ and $\sum_{i=1}^{k} n_i = \dim(V)$
Then T is diagonalizable over $\mathbb{R}$.

c) T has atleast $(k+2)$ invariant subspace

d) A.M of λ_i) $=$ G.M of λ_i, $i = 1, 2 \ldots k-1$
Then T is diagonalizable over $\mathbb{R}$

24) Which of the followings are true?

a) $\forall\ T: \mathbb{R}^3 \to \mathbb{R}^3$ be L.T ~~Then~~ Then

$\exists\ S: \mathbb{R}^3 \to \mathbb{R}^3 \ni S \circ T = I.$

b) $\exists\ T: \mathbb{R}^3 \to \mathbb{R}^3 \ni T$ is 1-1 Then

$\exists\ S: \mathbb{R}^3 \to \mathbb{R}^3 \ni S \circ T = T \circ S = I.$

c) Suppose $\exists\ S, T: \mathbb{R}^3 \to \mathbb{R}^3$

$\ni S \circ T = T \circ S = I$ Then S, T is

1-1 and onto.

d) $\exists$: a L.T $T: \mathbb{R}^5 \to \mathbb{R}^5 \ni$

$\dim(\mathrm{Ker}(T)) = 3, \dim(\mathrm{Im}(T)) = 3.$

25) Which of the following are diagonali-zable over $\mathbb{R}$?

a) Suppose $A \in M_5(\mathbb{R}) \ni A^2 = 5I.$

b) Suppose $A \in M_7(\mathbb{R}) \ni A^2 - 5A + 6I = 0$

c) Suppose $A \in M_2(\mathbb{R}) \ni A^2 = I$, but $A \neq \pm I$

d) Suppose $A \in M_{10}(\mathbb{R}) \ni A^{25} = 0, A \neq 0.$

26) which of the followings are not true?

a) Suppose $\exists\, v_1 \neq 0, v_2 \neq 0$ in V, and let $T: \mathbb{R}^5 \longrightarrow \mathbb{R}^5$ $\ni$ $Tv_1 = Tv_2$ Then T is not invertable

b) Let $A \in M_2(\mathbb{R})$, suppose $\exists\, v_1, v_2 \in \mathbb{R}^2$ $\ni$ $Av_1 = 1 \cdot v_1$, $Av_2 = -1 \cdot v_2$ Then A^2 is diagonalizable over $\mathbb{R}$.

c) Let $A \in M_2(\mathbb{R})$, Suppose $\exists\, v_1, v_2 \in \mathbb{R}^2$ $\ni$ $Av_1 = \lambda_1 v_1$, $Av_2 = \lambda_2 v_2$, where $\lambda_1 \neq \lambda_2$ then $E_{\lambda_1} \cap E_{\lambda_2} \neq \{0\}$ (v_1, v_2 are non-zero)

d) Let $A \in M_2(\mathbb{R})$. Suppose $\ker(A) \neq \{0\}$ then $\operatorname{rank}(A) \leq \dim(\ker(A))$.

27) Let $A \in M_{10}(\mathbb{R})$, suppose $Av = 2v$ for some $v \neq 0$ in $\mathbb{R}^{10}$. then

a) v is an eigenvector of $A^{10} + A^{9} + \cdots + I$

b) If A is invertiable then $-\frac{1}{2}$ is eignvalue of A^{-1}

c) suppose A^{-1} is diagonalizable then A is diagonalizable.

d) suppose $A^{2} + A + I = 0$ and $|A| \neq 0$ ~~The A~~ Then $A^{-1} = -(A+I)$

28) which of the followings are true?

a) $\exists$ L.T. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \ni \ker(T) = \{(x,x) \mid x \in \mathbb{R}\}$ and $\operatorname{Im}(T) = \{(x,-x) \mid x \in \mathbb{R}\}$

b) $\exists$ L.T $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \ni \ker(T) = \operatorname{Im}(T)$.

c) $\exists$ L.T $T: M_2(\mathbb{R}) \rightarrow M_2(\mathbb{R}) \ni$ $\ker(A) = \{A \in M_2(\mathbb{R}) \mid A = -A^T\}$ and $\operatorname{Im}(A) = \{A \in M_2(\mathbb{R}) \mid A = A^T\}$

d) $\exists$ L.T $T: \mathbb{R}[x] \rightarrow \mathbb{R}[x]$ such that $\operatorname{Im}(T) = \{p(x) \in \mathbb{R}[x] \mid p(0) = 0\}$

29) which of the followings are true?

Suppose $T: \mathbb{R}^5 \to \mathbb{R}^5$ be a L.T

and $\beta = \{v_1, v_2, --- v_5\}$ basis of $\mathbb{R}^5$

and if $T(v_1) = v_4$ $T(v_2) = v_3$, $T(v_3) = v_3$

$T(v_4) = v_2$ $T(v_5) = v_1$ Then

a) T is invertiable b) T is onto

c) T is 1-1 d) $\dim(\ker(T)) \leq 1$

30) suppose $T: \mathbb{R}^{10} \to \mathbb{R}^{10}$ be

a L.T $\ni$ $T(v_i) = \lambda v_i$, $\forall v_i \in \mathbb{R}^{10}$

for some fixed $\lambda \in \mathbb{R}$. and if

S is L.T $: \mathbb{R}^{10} \to \mathbb{R}^{10} \ni$ S is diagonalizable

Then

a) $S-T$ is diagonalizable.

b) $T-S$ is need not be diagonalizable.

c) $[T-S]_\beta = D$, for β - basis of $\mathbb{R}^{10}$

d) $T^2+S^2+S-T+I$ is diagonalizable over $\mathbb{R}$.

31) Which of the followings are not diagonaliza-ble over $\mathbb{R}$?

a) $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

c) $A = \begin{bmatrix} \pi & 1 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$