Limit state design and safety format

Original by:-
H.J. Larsen
Danish Building
Research Institute

Update by:-
Colin Short
Chartered Civil Engineer
NRO Ireland

Objective

To describe the Eurocode principles[i] for safety, serviceability of structures[ii] and the basis of design common to all materials, together with the special rules for timber structures necessitated by, for example, the effects of load duration and moisture content.

Prerequisites

A3 Actions on structures

A4 Wood as a building material

Summary

The requirements in the normal design situations are described, followed by the special requirements related to accidental design situations.

Design models

Before starting formal calculations it is necessary to analyse the structure and set up an appropriate design model. In doing this there may be a conflict between simple, but often conservative, models which make the calculations easy, and more complicated models which better reflect the behaviour but with a higher risk of making errors and overlooking failure modes.

The geometrical model must be compatible with the expected workmanship. For structures sensitive to geometrical variations it is especially important to ensure that the structure is produced as assumed during design. The influence of unavoidable deviations from the assumed geometry, displacements, and deformations during loading should be estimated.

Joints often require large areas of contact and this may give rise to local eccentricities which may have an important influence. Often there is a certain freedom regards the modelling as long as a consistent set of assumptions is used. Figure 1 gives an example for the heel joint; one possibility is to assume that the load is transferred in the joint line, in which case both members and both fastener groups are eccentrically loaded, at A e.g. with the moment Fe_A. Another possibility is to assume the top chord centrally loaded, in which case the bottom chord and its fastener group must be designed for $F(e_A + e_B)$.

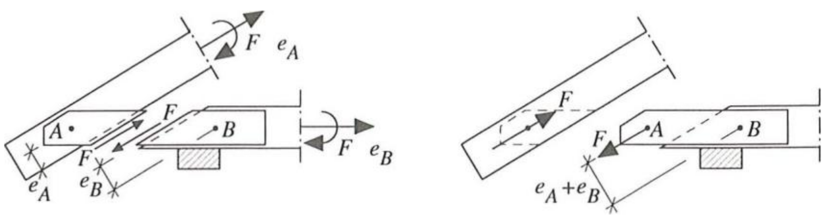

Figure 1: Heel joint

Limit state codes

The Eurocodes are limit state codes, meaning that the requirements concerning structural reliability are linked to clearly defined states beyond which the structure no longer satisfies specified performance criteria. In the Eurocode system only two types of limit state are considered: ultimate limit state and serviceability limit states.

Ultimate limit states are those associated with collapse or with other forms of structural failure. Ultimate limit states include: loss of equilibrium; failure through excessive deformations; transformation of the structure into a mechanism; rupture; loss of stability.

Serviceability limit states include: deformations which affect the appearance or the effective use of the structure; vibrations which cause discomfort to people or damage to the structure; damage (including cracking) which is likely to have an adverse effect on the durability of the structure.

Safety verification - The partial factor method

In the Eurocodes the safety verifica[i]ion is based on the partial factor method described below.

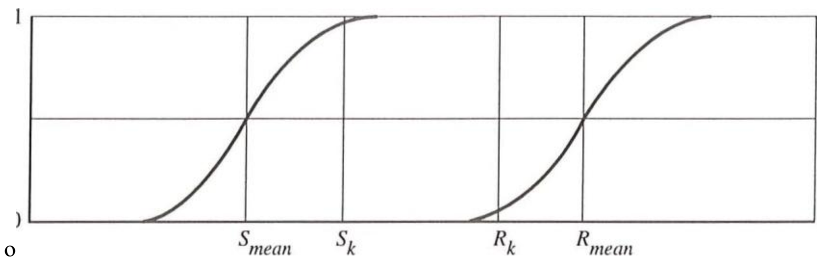

Figure 2: Statistical distributions (idealised) for action effects (S) and resistance (R). The cumulative probability is denoted P.

The main parameters are the actions, the material properties and the geometrical data. Normally, these parameters are stochastic variables with distribution functions as shown in principle in Figure 2 for the action effects (*S*) and the corresponding resistance (*R*): e.g. bending stresses and bending strength or the axial force in a centrally loaded column and the buckling load. The distributions have the mean values S_{mean} and R_{mean} and they can be assigned characteristic values S_k and R_k defined as fractiles in the distribution. For actions an upper fractile is normally used; in some cases, a lower value may be appropriate, e.g. for counteracting uplift. For resistance a lower fractile or the mean value is normally used; in exceptional cases an upper resistance value may be appropriate.

The purpose of the design is to get a low probability of failure[3][iii], i.e. a low probability of getting action values higher than the resistances. This, in the partial factor method, is achieved by using design values found by multiplying the characteristic actions and dividing the characteristic strength parameters respectively, by partial safety factors.

In all relevant design situations, it must be verified that the limit states are not reached when design values for actions, material properties and geometrical data are used in the design models. In particular, it must be verified that

- the effects of design actions do not exceed the design resistance at the ultimate limit states, and that
- the effects of design actions do not exceed the performance criteria for the serviceability limit states.

In symbolic form, for ultimate limit states corresponding to rupture, it must be verified that

$$E_d \leq R_d \tag{1}$$

For ultimate limit states related to static equilibrium or to gross displacement of the structure as a rigid body, the corresponding expression is

$$E_{d,dst} \leq R_{d,stb} \tag{2}$$

For serviceability limit states it shall be verified that

$$E_d \leq C_d \tag{3}$$

E_d is the design value of the effect of actions

R_d is the design value of the corresponding resistance

$E_{d,dst}$ is the design value of the effect of destabilising actions,

$R_{d,stb}$ is the design value of the effect of stabilising actions,

C_d is a prescribed value.

Representative and characteristic values

Actions In this article, actions are only treated to the extent necessary to describe the safety principles; for a detailed treatment reference is made to STEP article A3. For normal design situations, a distinction is made between permanent actions (denoted G) and variable actions (denoted Q). Actions may be direct (loads, forces) or indirect (resulting from imposed or restrained deformations, e.g. from moisture variations).

The main representative value is the characteristic value: G_k or Q_k. Other representative values are:

- combination values given as $\psi_0 Q_k$ (4)
- frequent values given as $\psi_1 Q_k$ (5)
- quasi-permanent values given as $\psi_2 Q_k$ (6)

Table 1 gives examples of ψ-factors. For details refer to EN 1990. For permanent actions, the characteristic value is in most cases the mean value. However, in two cases, it is necessary to use both an upper ($G_{k,sup}$) and a lower value ($G_{k,inf}$): when the coefficient of variation is greater than 0, 1 and if the structure is very sensitive to variations in G.

Action	ψ_0	ψ_1	ψ_2
Imposed load in buildings	0,7-1,0	0,5-0,9	0,3-0,8
Snow loads	0,6	0,2	0,0
Wind loads	0,6	0,5	0,0

Table 1: ψ-factors

Material properties

The material properties correspond either to the mean value or to the 5-percentile determined by standardised tests under reference conditions: duration of test 5 minutes at 20 °C and relative humidity 65%. The mean values are used for serviceability limit state verifications. The 5-percentiles are used for all properties (strength, stiffness and density) related to ultimate limit states.

Geometrical data

The characteristic geometrical values, such as spans, dimensions of cross sections, deviations from straightness, usually correspond to the values specified in the design or to nominal values.

Design values

Actions

The design actions may be different for the different limit states and are found as described below. Firstly, the possible load cases are identified, i.e. compatible load arrangements, sets of deformations and imperfections. A load arrangement identifies the position, magnitude and direction of an action.

Secondly, the actions are combined according to the following symbolic expression:

$$\Sigma\gamma_{Gj}G_{kj} + \gamma_{Q,1}Q_{k,1} + \Sigma\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad (7)$$

where γ are partial factors (load factors) for the action considered, taking account of: the possibility of unfavourable deviations of the actions, the possibility of inaccurate modelling of the actions and uncertainties in the assessment of effects of actions. Values of the load factors are given in

. Reduced partial factors may be applied for single-storey buildings of moderate span that are only occupied occasionally (storage buildings, sheds, greenhouses, and buildings and small silos for agricultural purposes), lighting masts, light partition walls, and sheeting.

Recommended factors are given in the Eurocode but the designer and verifier shall remember to check the appropriate National Application Document for the country in which the subject structure is to be erected. Those values may be different.

The representative values multiplied by the γ values - $\gamma_G G_k$,, $\gamma_Q Q_k$, $\gamma_Q\psi_0 Q_k$ - are called design actions. The principle is thus that one variable action with its characteristic value in turn is combined with the permanent actions and all other variable actions with their combination value $\psi_0 Q_k$.

Finally, the effects (S) of actions - for example internal forces and moments, stresses, strains and displacements - are determined from the design values of the actions, geometrical data and, where relevant, material properties (X):

$$S_d = S(F_{d,1}, F_{d,2}, \ldots.a_{d,1}, a_{d,2}, \ldots., X_d, \ldots.) \qquad (8)$$

As a simplification it is permitted instead of (7) to use the more adverse of the following combinations[iv] .

For limit states with only one variable action: $\Sigma\gamma_{G,j}G_{k,j} + 1{,}5Q_{k,1}$ *(9)*

For other cases: $\Sigma\gamma_{G,j}G_{k,j} + 1{,}35\Sigma Q_{k,i}$ *(10)*

Table 2 Partial safety factors for ultimate limit states for buildings

		normal	reduced
Failure due to loss of static equilibrium			
Unfavorable permanent actions [b]	$\gamma_{G,sup}$	1,10	1,10
Favorable permanent actions [b]	$\gamma_{G,inf}$	0,90	0,90
Unfavorable variable actions	γ_Q	1,50	1,35
Failure governed by strength of structural material [a]			
Unfavorable permanent actions [b]	$\gamma_{G,sup}$	1,35	1,20
Favorable permanent actions [b]	$\gamma_{G,inf}$	1,00	1,00
Unfavorable variable actions	γ_Q	1,50	1,35

a Other partial factors apply when the failure is governed by strength of ground.
b Permanent actions caused by structural or non-structural components or ground and ground water.
c In this verification the unfavourable part of the characteristic value of the permanent action is multiplied by the factor 1,10 and the favourable part by the factor 0,90.
d In this verification the characteristic values of all permanent actions from one source are multiplied by 1,35 (1,20) if the total resulting action effect is unfavourable and by 1,0 if it is favourable.

In EC5 the load combinations for serviceability limit states are given as:

$$\Sigma G_{k,j} + Q_{k,1} + \Sigma \psi_{1,i} Q_{k,i}{}^{v} \qquad (11)$$

Resistance

The design value X_d of a material property with the characteristic value X_k is defined as:

$$X_d = k_{mod} X_k / \gamma_M \qquad (12)$$

γ_M is the partial safety factor for the material property. The EC5 values are given in Table 3,

k_{mod} is a modification factor taking into account the effect on the strength parameters of the duration of the actions and the moisture content. Examples of k_{mod} for solid timber and glulam are given in Table 4.

Table 3 Partial factors for materials in normal design

Ultimate limit states	
timber and wood-based materials	1,3
steel used in joints	1,1
Serviceability limit states	1,0

The factor k_{mod} depends on the Service Class to which the structure belongs and the Load-duration Class.

There are three Service Classes, denoted 1, 2 and 3. The classes 1 and 2 are characterised by the moisture content of the surrounding air. In Service Class 1 the average equilibrium moisture content in most softwoods will not exceed 12%; in Service Class 2 it will not exceed 20%. There are no limits for Service Class 3.

There are five Load-duration Classes. They are characterised by the order of accumulated duration of the characteristic load, see Table 4, where also examples of loading are given.

It is generally assumed that the relationship between the resistance (R) and the strength parameters (f), the stiffness parameters (E) and the geometrical data (a) is known. If this is the case, design values should be used to determine the design resistance:

$$Rd = R(f_{1,d}, f_{2,d}, \ldots E_{1,d}, \ldots a_{1,d}, a_{2,d}, \ldots) \tag{13}$$

The design value R_d can also be determined directly from characteristic values (R_k) determined from tests:

$$R_d = k_{mod} \, R_k / \gamma_M \tag{14}$$

For structures where the resistance depends on more than one material - e.g. timber and steel or wood-based panels - it can be difficult to select the right value of k_{mod}. It is of course always on the safe side to use the lowest value for the materials used.

Geometrical data

The geometrical design values correspond generally to the characteristic values, i.e. to the values specified in the design. In cases where the influence of deviations are critical the geometrical design values are defined by

$$a_d = a_k + \Delta a \tag{15}$$

where Δa takes account of the possible deviations from the characteristic values. Values of Δa are given in the appropriate clauses of EC5.

Table 4 Load-duration Classes and k_{mod} for solid timber and glulam.

Load-duration Class	Duration [a]	Examples of loading	k_{mod} for Service Classes 1&2	k_{mod} for Service Classes 3
Permanent	more than 10 years	self-weight	0,60	0,50
Long-term	6 months - 10 years	storage	0,70	0,55
Medium-term	1 week - 6 months	imposed load	0,80	0,65
Short-term	less than one week	Snow [b] and wind	0,90	0,70
Instantaneous		accidental load	1,10	0,90

a The Load-duration Classes are characterised by the effect of a constant load acting for a certain period of time. For variable action the appropriate class depends on the effect of the typical variation of the load in the life of the structure. The accumulated duration of the characteristic load is often very short compared with the total loading time.

b In areas with a heavy snow load for a prolonged period of time, part of the load should be regarded as medium-term.

Accidental situations

All structures must be designed in such a way that they will not be damaged by events like explosions, impact, or consequences of human errors, to an extent disproportionate to the original event. Means of achieving this include:

- eliminating or reducing the possibility of the events mentioned,
- selecting a structural form and design which has a low sensitivity to the hazards considered or which can survive adequately the accidental removal of an individual element.

For some structures the building regulations may require that they be designed for a situation where the structure is exposed directly to a specified accidental action A. In these cases, the following load combination is used instead of (7):

$$G_{k,j} + A + \psi_{1,i} Q_{k,1} + \sum_{i>1} \psi_{2,i} Q_{k,i} \qquad (16)$$

and R_d is calculated with $\gamma_M = 1$.

Example

For the building shown in Figure 3, two load arrangements have to be considered besides self-weight (g, G):

- s: snow, fixed action with ψ_0 = 0 6.
- w: wind, fixed action with ψ_0 = (),6.

Figure 3 Load arrangements

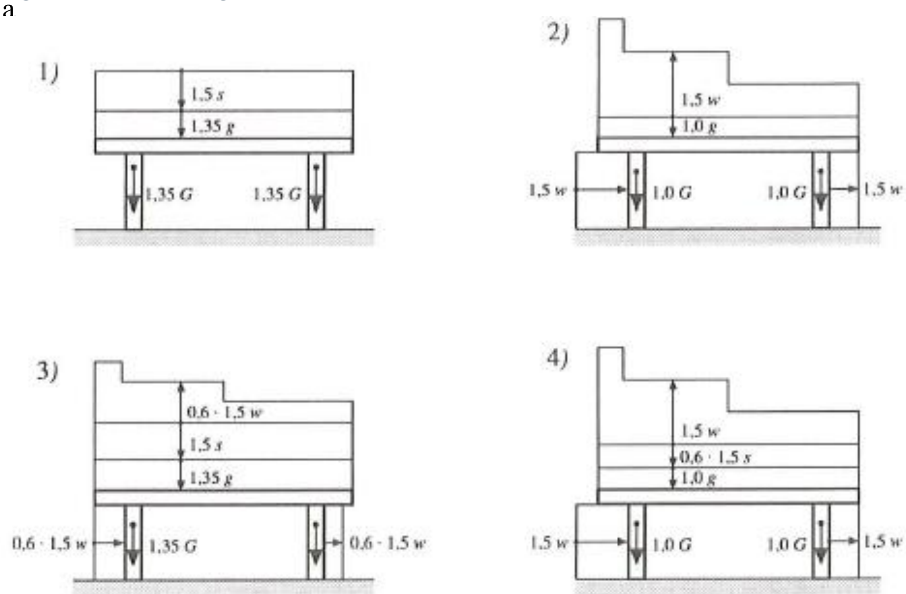

Figure 4 Load combinations corresponding to the load arrangements shown in Figure 3

The following loads/load combinations are possible, see Figure 4:

- Self-weight alone. Permanent. Due to the low value of k_{mod}' this load may be decisive in theory, but rarely in practice.

1. Self-weight + snow, short-term. This combination gives the greatest axial force in the columns.
2. Self-weight + wind, short-term. This combination may be decisive for anchoring against uplift.
3. Self-weight + snow + (wind, combination value), short-term. This combination gives the greatest axial force in the columns combined with bending in the columns.
4. Self-weight + wind + (snow, combination value), short-term. This combination gives the greatest moment in the columns.

---

[i] For details refer to Eurocode EN 1990, EN 1991-1, and ENV 1995-1-1.

[ii] In this article "structure" covers the whole structure as well as any structural part of it.

[iii] The probability of failure can be estimated by statistical methods, and in the future such methods may be used by designers. Today, they are only used for very special structures, e.g. for bridges with very large spans or for the calibration of the safety elements (e.g. partial coefficients) of the simple verification systems used in practice.

[iv] The simplified expressions are on the unsafe side for Q_2 less than 30-50% of Q_1.

[v] In Eurocode 1 ψ_0 is used instead of ψ_1