Serviceability limit states - Deformations

STEP lecture A17
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Objectives

To explain the motives for the control of deformations and to describe how deformations in timber structures can be estimated during the building lifetime within the context of EC5.

Prerequisites

A2 Limit state design and safety format
A19 Creep

Summary

Short- and long-term deformation behaviour of timber as influenced by climatic conditions and load variations is briefly described. Various reasons for the control of deformations in timber structures are discussed and criteria for serviceability design are suggested. The formal calculation method proposed in EC5 is presented and a design example for the serviceability limit state concludes the lecture.

Introduction

The overall performance of structures should satisfy two basic requirements. The first is safety, usually expressed in terms of load bearing capacity, and the second is serviceability, which refers to the ability of the structural system and its elements to perform satisfactorily in normal use.

It is generally understood that violation of the safety criteria may cause risk to human life and substantial damage, whereas violation of serviceability requirements rarely leads to risks for humans and usually involves lower economical losses. On the other hand, the overwhelming majority of structural defects actually observed in practice are related to serviceability. For this reason, the question of serviceability is very important in structural design.

In the case of horizontal timber elements, serviceability requirements with regard to deflections and vibrations are very often decisive for the structural dimensions. This lecture deals with serviceability related to deformations in timber structures.

Deformations in timber structures during the building lifetime

The fact that variable loads (such as imposed loads on floors and snow loads) often dominate in timber structures means that the deflection will vary considerably during the lifetime of the structure. This has to be considered in a rational serviceability design.

Figure 1 illustrates the deflection history of a beam loaded with permanent load and snow load (see Mårtensson, 1992; Thelandersson and Mårtensson, 1992). The total deflection can be subdivided into one part δ_1 due to permanent loads immediately after loading and one part δ_2 which is variable during the lifetime of the structure. The variable part δ_2 consists of a reversible portion $\delta_{2,inst}$ which is present only during limited periods when the variable load is high, and a continuously increasing portion δ_{creep}, which for all practical purposes may be considered as irreversible (Mårtensson, 1992). Short duration load peaks, such as those illustrated in Figure

1, occur both for snow loads and imposed loads in most common types of buildings.

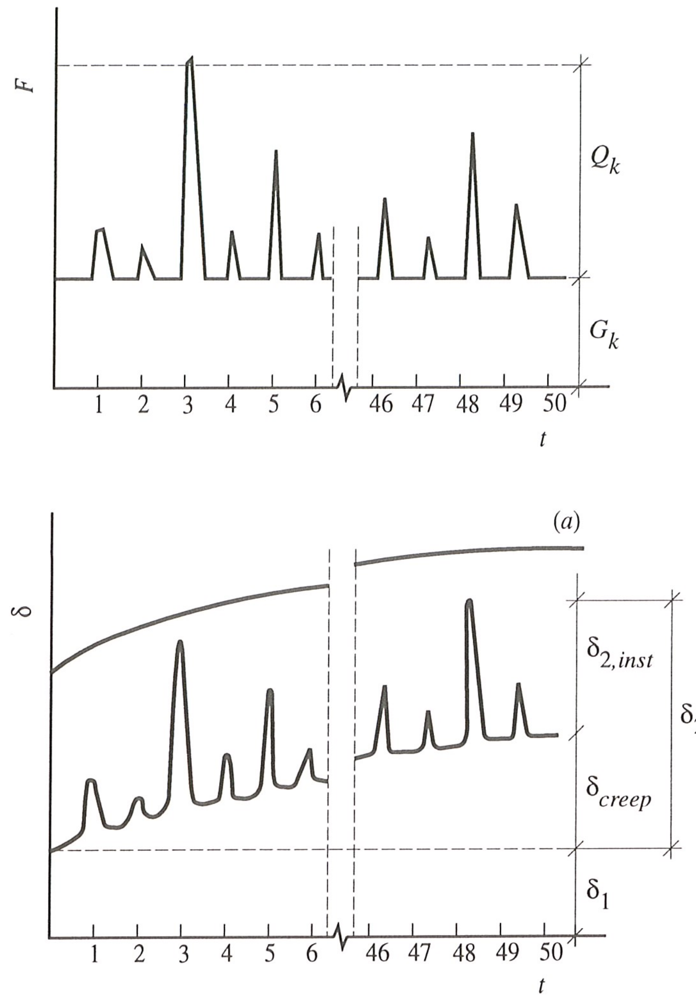

Figure 1 *Time variation in principle for deflection of a beam with permanent (G) and variable (Q) loads. Curve A shows the deflection if the beam is loaded with the characteristic loads $G_k + Q_k$ during the whole period. F is the load, δ the deflection and t the time in years.*

Hence, for design purposes, the following deflection components may be defined with reference to Figure 2[1]:

δ_o is the precamber of the beam in the unloaded state (0).

δ_1 is the beam deflection due to permanent loads immediately after loading (state 1).

δ_2 is the deflection of the beam due to variable loads plus any time dependent deflection due to permanent loads (state 2).

δ_{net} is the sagging of the beam relative to the straight line joining the supports.

1 The notation δ for deflection is used here in a conceptual sense. In EC5, where the notation *u* is used, the definitions of deflection components are slightly different from those in Figure 2.

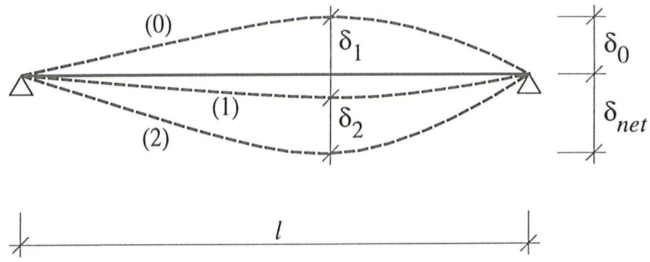

Figure 2 Deflection components for a simply supported beam.

Normally both δ_0 and δ_1 are fixed when the construction work is completed and do not change during the lifetime of the structure (unless the permanent load is changed). The components δ_2 and δ_{net}, however, will vary during the lifetime of the structure.

Load combinations for the serviceability limit state

EC1: Part 1: 9.5

The basis of design common to all Eurocodes, specifies different types of load combinations which may be used for verification in the serviceability limit state. Two of these, the characteristic (rare) combination and the frequent combination are of interest in connection with timber structures.

The *characteristic combination* is intended for use mainly in those cases where exceeding the limit state causes significant damage or unacceptable irreversible deformation. The symbolic definition of this combination is:

$$\sum_{j \geq 1} G_{k,j} + Q_{k,1} + \sum_{j > 1} \psi_{0.i}\, Q_{k.i} \tag{1}$$

The *frequent combination* is intended for use mainly in those cases when exceeding the limit state is associated with minor damage or reversible deformations. The symbolic definition of the frequent combination is:

$$\sum_{j \geq 1} G_{k,j} + \psi_{1,1} Q_{k,1} + \sum_{i > 1} \psi_{2.i}\, Q_{k.i} \tag{2}$$

In the above expressions $G_{k,j}$ and $Q_{k,i}$ are characteristic values of permanent and variable loads, respectively. The terms $\psi_{0,i}\, Q_{k,i}$, $\psi_{1,i}\, Q_{k,i}$ and $\psi_{2,i}\, Q_{k,i}$ represent the combination, the frequent and the quasi-permanent values of variable load $Q_{k,i}$, respectively.

Limitation of deformations

The most common reasons for the limitation of deformations in structures are:

- general utility and appearance (e.g. to limit annoying visual effects and to avoid sloping floors),
- structural requirements (e.g. to avoid damage to non-structural elements such as partitions, doors, windows and claddings and to guarantee smooth assembly, water tightness, drainage of roofs),
- equipment requirements (e.g. to guarantee proper functioning of machinery, pipes, cables, ducts and their supports).

Modern codes like EC5 only give functional requirements in general terms stating that structures should be designed in such a way that serviceability aspects such as those listed above are considered. Specific numerical limits of deflection or slope should in principle be decided by the structural engineer from case to case, depending on the actual situation and the demands of the client.

Deflection criterion to avoid significant damage

A typical case is when excessive deformations may cause significant damage to partitions, installations, fixtures and finishes. In this case the risk of exceeding the deflection limit should be kept at a low level. Therefore, the deflection should be calculated using the characteristic load combination defined by Equation (1). The damage is normally caused by deformations occurring after the construction work is completed and a deflection criterion for situations where significant damage can be expected can be written as:

$$\delta_2 \leq \delta_{2,crit} \qquad (3)$$

where δ_2 is defined in Figure 2 and $\delta_{2,crit}$ is the critical value of the deflection causing damage. The limit $\delta_{2,crit}$ generally depends on the nature and detailing of the elements which could suffer damage. In the absence of more precise information $\delta_{2,crit}$ could be taken as a fixed value, say 20 *mm*, or a certain fraction of the span ℓ, say $\ell/300$ for a simply supported beam and $\ell/150$ for a cantilevered beam. These values are often recommended for beams in floors and roofs which are in contact with partitions and non-structural elements.

Deflection criterion related to appearance and general utility

From the point of view of appearance and general utility it may often be desirable to avoid excessive deflections which are permanent or occur over long periods. Occasionally exceeding the deflection limit may, however, be acceptable if the deflections are reversible and limited to short periods of time. In this case a somewhat higher risk of passage of the limit can be accepted and the deflection may be calculated on the basis of the frequent load combination defined in Equation (2). An appropriate criterion for this case is:

$$\delta_{net} \leq \delta_{acc} \qquad (4)$$

where δ_{net} is defined in Figure 2 and δ_{acc} is the acceptable deflection limit with respect to appearance and general utility. The value of δ_{acc} depends on a number of factors such as type of building, type of structure, whether the beam is visible or not, the attitudes of the building users, etc. For instance, the requirements are normally much higher in residential buildings than in industrial buildings. As a general recommendation the value $\delta_{acc} = \ell/250$ may be given.

Deflection limits recommended in EC5

EC5: Part 1-1: 4.3

EC5 gives some recommendations for limits of deflection which may be used in the absence of more precise information. All limits given in EC5 are related to the characteristic load combination, Equation (1), with ψ_o in the last term replaced by ψ_1. In cases where it is appropriate to limit the instantaneous deflection $u_{2,inst}$ due to variable actions the criterion $u_{2,inst} \leq \ell/300$ is recommended for a beam on two supports with length ℓ. This criterion can be relevant for example when excessive deformation may cause damage to non-structural elements.

In cases where it is appropriate to limit the final net deflection $u_{net,fin}$, the criterion $u_{net,fin} \leq \ell/200$ is recommended. This criterion can be relevant when the deflection control is motivated by requirements of appearance and general utility. This limit is more liberal than that given above, since in this case the criterion is related to a more severe load combination.

Calculation of load induced deflections based on EC5 principles

The instantaneous bending deflection u_m can usually be calculated from elementary beam theory using formulas given in standard textbooks and manuals. Since the shear stiffness for wood is comparatively low, shear induced deflections may sometimes be significant. The deflection u_v due to shear can be calculated by the well established theory for shear deformations of beams. The total instantaneous deflection u_{inst} is the sum of u_m and u_v.

To get an idea of the significance of shear deformations, consider a simply supported rectangular timber beam with uniformly distributed load. For this case, the ratio between shear deflection u_v and bending deflection u_m at the mid span is approximately given by:

$$\frac{u_v}{u_m} = 0{,}96\,\frac{E}{G}\left(\frac{h}{l}\right)^2 \tag{5}$$

The ratio E/G is approximately 15 for timber and glulam. Thus u_v/u_m is roughly 0,15 for $\ell/h = 10$ and less than 0,05 for $\ell/h = 20$. For a concentrated load at the mid span the corresponding ratio is about 20% higher.

The long term deflection or creep under sustained loads in timber depends to a great extent on the climatic conditions, even if the rate of creep in wood at a constant high moisture content is only slightly higher than in wood at constant low moisture content. The most important factor is the intensity of variation of moisture content in the material. This means that the long term deflection is higher for timber in outdoor conditions, with rapid and frequent fluctuations in relative humidity, than for timber indoors, where the climate is controlled. For the same reason, timber with large cross sections exhibits lower creep than timber in small sizes, since the material in a heavy timber beam has a much slower response to fluctuations in the surrounding relative humidity. Surface treatment leading to increased moisture resistance at the surfaces has the same effect (Mårtensson, 1992, Taylor et al 1991).

In addition to load induced deformations, the serviceability of structural systems in timber is very much influenced by shrinkage and swelling in the material. The deformations imposed by moisture variations and moisture gradients can often be of the same order of magnitude or larger than those induced by mechanical loads. Such effects have to be controlled by appropriate structural design and detailing and by adequate moisture control of the timber before it is built into the structural system. Pure moisture induced deformations will not be considered further in this lecture.

EC5: Part 1-1: 4.1

Principles for the calculation of deformations are given in the form of application rules in EC5. According to these rules, the instantaneous deformation u_{inst} under an action should be calculated on the basis of mean values of the appropriate stiffness moduli, which are specified in standards associated with EC5 for timber and glulam as well as for those wood based materials which are classified for structural use.

The final deformation u_{fin} including long term deformation is calculated as:

$$u_{fin} = u_{inst}(1 + k_{def}) \tag{6}$$

where k_{def} is a creep factor which describes the increase in deformation with time depending on climatic conditions and the duration of the load considered. Values of k_{def} are given in EC5 (Table 4.1) for different materials and for different service classes and load-duration classes.

When the deformation is to be calculated for a load combination with actions belonging to different load duration classes, the contribution of each action to the total deflection should be calculated separately and then added.

Design example

Figure 3 shows a flat roof supported by straight glulam beams with cross section 165 x 990 *mm*, spacing 6 *m* and a span of 20 *m*. Strength class GL36 with $E_{0,mean}$ = 14500 N/mm^2. Service class 1. The dimensions of the beam have been determined on the basis of design in the ultimate limit state. The second moment of area is $I = bh^3/12 = 13{,}3 \cdot 10^9\ mm^4$.

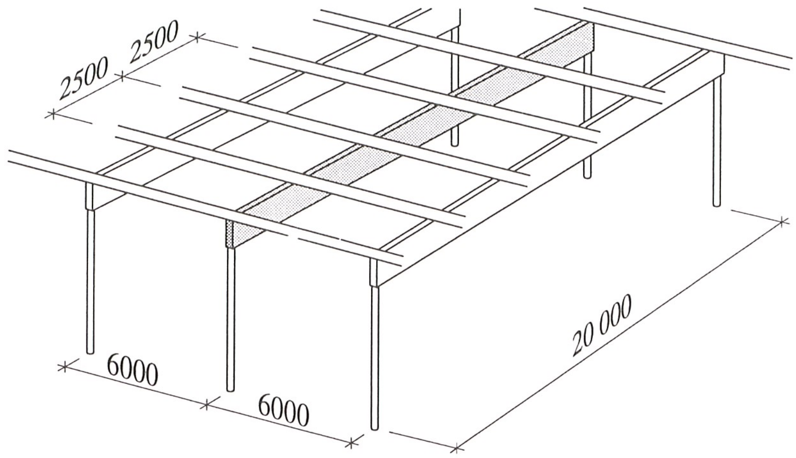

Figure 3 *Roof structure considered in design example.*

Characteristic load values:
permanent load: $G_k = 0{,}5\ kN/m^2$
snow load: $Q_k = 0{,}8\ kN/m^2$, $\psi_0 = 0{,}6$, $\psi_1 = 0{,}2$, $\psi_2 = 0$

Uniformly distributed loads q_k on the beams (spacing 6 *m*) and corresponding creep factors k_{def} are given in the following table (snow load taken as medium term load):

Load	q_k (N/mm)	k_{def}
Permanent	3,0	0,6
Snow	4,8	0,25

First, the instantaneous mid-span bending deflection u_1 for a reference load q_{ref} = 1,0 *N/mm* is calculated.

$$u_1 = \frac{5\, q_{ref}\, l^4}{384 EI} = \frac{5 \cdot 1{,}0 \cdot 20000^4}{384 \cdot 14500 \cdot 13{,}3 \cdot 10^9} = 10{,}8\ mm$$

The shear component of the deflection can be estimated from Equation (5). With E/G15 and $\ell/h \approx 20$ the additional deflection due to shear is about 3,5% of the bending deflection. Thus, the total deflection due to q_{ref} becomes u_{ref} = 11,2 *mm*.

Deflection control for the case when significant damage can be expected

In this case the additional deflection occurring after the building has been erected is assumed to be of interest (δ_2 in Figure 2). The criterion given by Equation (3) is used with the characteristic load combination, Equation (1).

Creep deflection from permanent load + final deflection from snow load

$= \quad [3{,}0 \cdot 0{,}6 + 4{,}8\ (1{+}0{,}25)]u_{ref} = 7{,}8\ u_{ref} \approx 84\ mm$ (or $l/240$).

If the deflection at the mid-span needs to be limited due to structural requirements this value is usually too large. If a non-structural element connected to the beam needs to be protected against excessive deformations, the deflection at the point where the partition is placed should be checked.

The criterion $u_{2,inst} \leq \ell/300$ recommended in EC5 could possibly also be applied here. In this case $u_{2,inst} = 4{,}8\ u_{ref}\ 54\ mm < \ell/300 = 67\ mm$.

Thus, the beam performance is considered acceptable according to this criterion. It is quite clear that any suggestion of a general deflection limit can be questioned. The only way to assure a rational serviceability design is to evaluate the design situation based on the relevant circumstances in each specific case.

Deflection control with respect to appearance and general utility

From the point of view of appearance the final net deflection is usually of interest. The criterion given in Equation (4), with the frequent load combination, Equation (2), is relevant in this case.

Final deflection due to permanent load + final deflection due to $\psi_1\ Q_k$ (frequent value of snow load) $= [3{,}0 \cdot (1{+}0{,}6) + 4{,}8 \cdot 0{,}2 \cdot (1{+}0{,}25)]\ u_{ref} = 6{,}0\ u_{ref} = 65\ mm$. This value corresponds to $\ell/300$ and can usually be considered acceptable for a beam of this size.

The corresponding criterion recommended in EC5 is in principle the same, but with the characteristic load combination, Equation (1). This gives:

$$u_{net,fin} = [3{,}0 \cdot (1{+}\ 0{,}6) + 4{,}8 \cdot (1{+}\ 0{,}25)]\ u_{ref} = 10{,}8\ u_{ref} = 117\ mm$$

This value is higher than the recommended limit in EC5, which is $\ell/200 = 100\ mm$. Again, different criteria intended to check the same functional requirement give different results. In this particular case, quite large deflections may occur on very rare occasions with extreme snow loads. These large deflections are only temporary and reversible and might be accepted in many cases. A way to avoid them without increasing the beam dimensions is to apply a precamber u_o to the beam. The precamber could be chosen equal to the deflection due to permanent load + half the deflection due to the frequent value of the variable load. This gives a precamber of $4\ u_{ref} \approx 43\ mm$.

Concluding summary

- Serviceability criteria related to deflections often govern the dimensions of horizontal timber elements.
- The deformation of timber structures changes during their lifetime, due to variable loads, moisture variations and creep.
- The reasons for limitation of deformations should be clearly defined by the designer in each specific case.
- The choice of load combination for calculation of deflections should depend on the expected consequences of excessive deformations.

References

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