Trusses

STEP lecture B12
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Objectives

To understand plane trusses as part of three dimensional structures, to show the different types of trusses and to present the general and simplified analysis and the strength verification of members and joints of EC5.

Prerequisites

A17 Serviceability limit states - Deformations
B2 Tension and compression
B3 Bending
B6 Columns
B7 Buckling lengths

Summary

Proceeding from three dimensional trussed structures the shape and the appropriate load-bearing behaviour of plane trusses is discussed. The lay-out of various types of trusses is shown and indications of the selection of the web system are given. The principles and rules of EC5 for a general and simplified analysis are described. Strength verification rules and limits of deflection complete the lecture. Examples are included at various stages.

General

Trusses are built to cover spaces (living rooms, In general, the members are statically represented by three dimensional straight rods which have six degrees of freedom (three displacements and three rotations) at each end. For static and fabrication reasons, very often, the three dimensional truss structure is built up of two dimensional vertical trusses (truss A) which are erected parallel or concentrically and joined together by two dimensionally inclined trusses (trusses B1-B4) between them (see Figure 1).

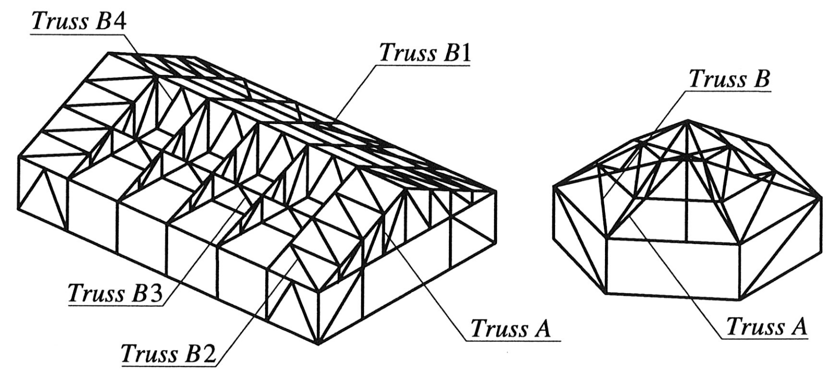


Figure 1 Three dimensional trussed structure: parallel (left), concentrical (right).

EC5: Part 1-1: 5.4.1.1

Truss A is intended to carry only the loads which act in the plane of the truss and hence it follows that statically truss *A* is a plane problem and consists of members (plane rod elements) which have three degrees of freedom only (two displacements and one rotation) at each end. In EC5 these elements are called beam elements.

EC5: Part 1-1: 5.4.1.3

Under certain circumstances members of two degrees of freedom (two displacements) at each end (pin-jointed elements) can be used.

EC5: Part 1-1: 5.4.1.1

The members of two dimensional trusses are designated in two categories: external members (top chord, bottom chord) and internal or web members (all interior vertical or diagonal members between the top (upper) and bottom (lower) chord). Joints at which members intersect and connect are called nodes or panel points. The following statements may be used to describe two dimensional trusses:

- Unless a more general model is used, trusses shall be represented for the purpose of analysis by beam elements set out along system lines and connected together at nodes (e.g. as shown in Figure 2).
- The system lines for all members shall lie within the member profile, and for external members shall coincide with the member centre line.
- A more general load-bearing model could be to present truss members by shell elements which could be very costly, however.

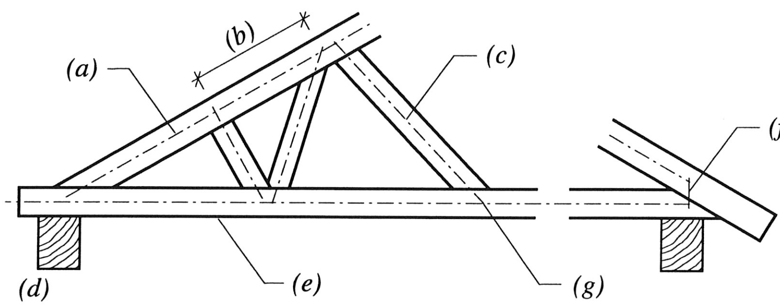


Figure 2 Two dimensional truss: (a) system line, (b) bay, (c) internal member, (d) support, (e) external member, (f) fictitious beam element, (g) node.

For fabrication reasons in most trusses do not consist of members which articulate perfectly along the centreline. Firstly, chords are fabricated from one timber and therefore are continuous over several bays. Secondly, rectangular or circular shaped plate connectors always cause a certain rotational stiffness. Member forces and architectural considerations determine the type of connections to be used and can result in rotationally fixed, semifixed or pinned joints. Thirdly, the depths of members and the dimensions of connectors lead to eccentricities in the joints between adjacent members.

This last case determines a general application rule:

Fictitious beam elements (see Figure 2) may be used to model eccentric connections or supports. The orientation of fictitious beam elements should coincide as closely as possible with the direction of the force in the member. This rule which gives an estimation of the complex stress distribution at eccentric connections is to allow an economic analysis.

Part 1-1: 2.3.1

Corresponding to the general design requirements it shall be verified that no EC5: relevant limit state is exceeded. In verifying assemblies like trusses, distinction has to be made between necessary global and local limit states. In both states second-order effects due to initial global and local curvatures, eccentricities and induced deflections shall be taken into account, in addition to those due to lateral loads. A very close approximation of the global geometric non-linear behaviour of trusses is intended by the application rule:

In the analysis the geometric non-linear behaviour of a member in compression (buckling instability) may be disregarded if it is taken into account in the strength verification of the individual member.

This means that only the influence of global imperfections on the displacements and rotations of truss nodes has to be taken into account. This is done by using the node coordinates of the imperfect (initially deformed) truss. The influence of local imperfections of each truss member between its nodes can be neglected in the analysis, i.e. by assuming that members remain straight between nodes, if it is taken into account in the strength verification. This procedure simplifies the analysis significantly and offers an economic use of finite element programs.

Concerning global limit states it has to be emphasized that, in general, trusses are three dimensional structures as mentioned earlier. Nevertheless, frequently trusses A and B in Figure 1 and Figure 3 are treated as two dimensional systems loaded in their plane without any mutual influence.

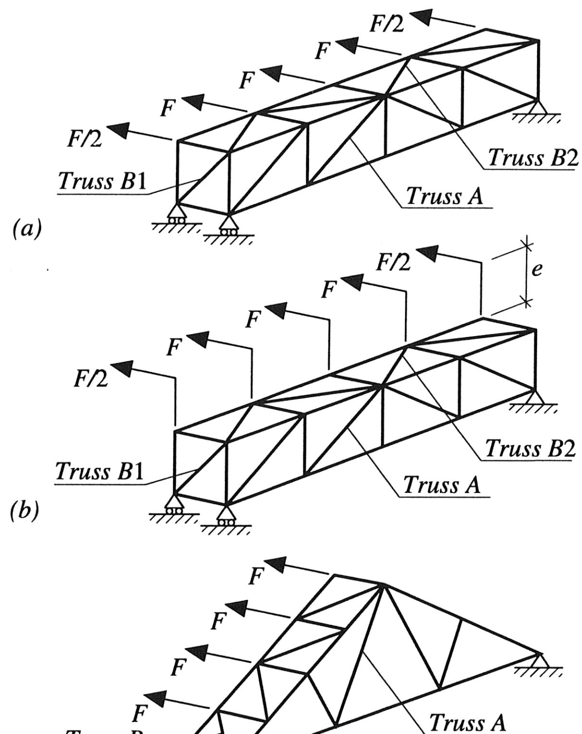


Figure 3 *Three dimensional load-bearing behaviour. (a) truss A unstressed by lateral loads, (b, c) truss A stressed.*

However, this is valid only for system (a) in Figure 3, where truss A carries the vertical loads independently and trusses B1 and B2 the lateral loads (external loads, i.e. wind, seismic loads, internal loads due to buckling of compression and bending members), B2 supporting B1. In the case of systems (b) and (c) in Figure 3 truss A again carries the vertical loads independently but trusses B1/B2 and B need the cooperation of trusses A to form a three dimensional load-bearing system to provide sufficient lateral resistance (Kessel 1986). Due to that fact it becomes evident that bracing leads to stresses in trusses A resulting from loads F which do not even have a component in the plane of truss A.

Truss Types

The type of timber truss most commonly built is triangular, i.e. double pitched (see Figure 4). The web system should be selected for convenience of connection and resulting member stresses. For instance, in some cases space for ventilation tubes is required. Web locations and node spacings may be dedicated by selection of secondary purlin framing so as to minimize chord bending stresses and buckling lengths of chord members in compression. Web directions may be chosen in a way that short internal members are in compression and long members in tension to avoid additional web bracing.

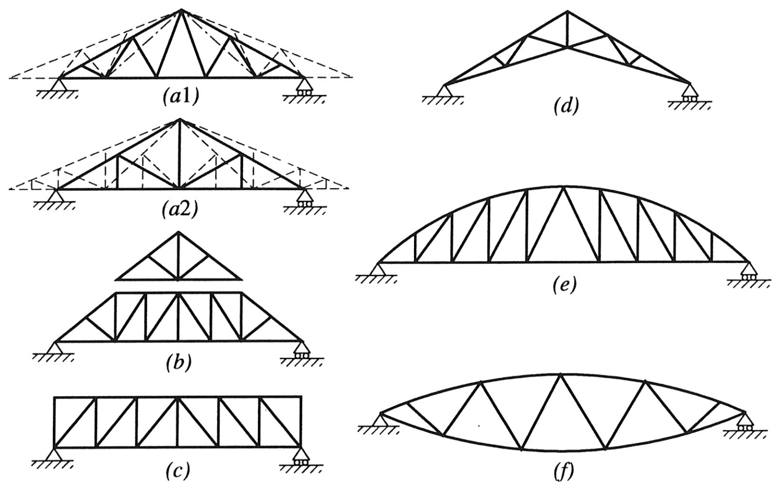


Figure 4 *Types of trusses. (a1, a2) triangular (b) compound (c) parallel (d) scissors (e) bowstring (f) fish-bellied.*

By varying geometric parameters a large variety of trusses can be developed for nearly all kinds of application. Although the dimensions of trusses are restricted by prefabrication procedure and transport, the height of triangular trusses can be very large. In such cases it may be necessary to use a trapezoidal load-bearing truss completed by a small triangle (see Figure 4b) to produce the desired triangular roof shape.

Sometimes the bottom chord shape is dictated by architectural considerations concerning interior decoration. Instead of a straight chord its centre point with correspondence to the supports is raised up (see Figure 4d) or layered down (see Figure 4f). While the first case is of no particularity the special feature of the load-bearing behaviour of the second one should be mentioned. If the top chord is flat, i.e. the top chord approaches a straight line (see Figure 5a and 5b), a stability problem arises due to the fact that the centre point of the bottom chord can deflect laterally (Kessel 1988).

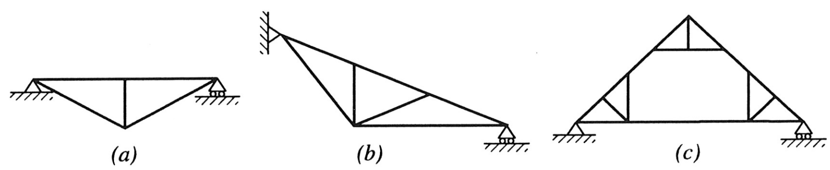


Figure 5 *Kings trusses (a, b) and attic truss (c).*

A further important variant of the triangular truss is the attic truss of domestic roofs (see Figure 5c) which however, is no longer a truss in the original sense. Due to the lack of webs the external loads cause, in addition to the axial forces, important bending moments in the chords. Particular attention must be paid to the joints in the lower chord. These joints are stressed axially with respect to roof loads, transversally with respect to ceiling and floor loads and rotationally with respect to unavoidable eccentricities of connection members.

In order to minimise deflection, the span-to-depth ratios of trusses should decrease with increasing span. Large deflections not only may cause serviceability problems, they can also create substantial secondary stresses in continuous chords. According to Ozelton and Baird (1976) truss deflections can be minimised by

- using timber of lower strength classes and consequently larger member sizes,
- keeping the number of joints and mechanically jointed splices to a minimum, and
- using fastenings with stiff slip characteristics.

The use of lower strength classes may appear uneconomical, since the necessary cross-sectional dimensions increase. However, since the fastener spacings and distances often determine the size of the cross-sections, the choice of a high strength class frequently does not lead to material savings. Because the load-carrying capacity of connections using mechanical fasteners depends on the density of the timber, and the ratio of density over strength increases with lower strength classes, it is usually more economic to use timber of lower strength classes when the necessary mechanical connections determine the cross-sectional dimensions.

Preliminary design

Generally architectural considerations determine the shape and pitch of roofs. But for economic reasons the following rules concerning the depth-span ratio of trusses should be followed:

triangular or pitched	1/6 or deeper,
bowstring	1/8 to 1/6,
flat or parallel chord	1/10 to 1/8.

Once the truss geometry has been fixed, the centrelines of the members are dependent on their size. Therefore, it is usually necessary to conduct a preliminary design to determine approximate member sizes and connection types. For this purpose a simplified analysis is used with all loads placed at nodes and all joints assumed pinned. Member forces can then be determined graphically or analytically. Based on these axial forces, preliminary web and chord sizes can be selected taking into account approximate moments due to any distributed loads or concentrated loads that will not in practice be applied at nodes.

General analysis

EC5: Part 1-1: 5.4.1.2

Trusses shall be analysed as framed structures, where the deformations of the members and joints, the influence of support eccentricities and the stiffness of the supporting structure are taken into account in the determination of the member forces and moments. If the system lines for internal members do not coincide with the centre lines, the influence of the eccentricity shall be taken into account in the strength verification of these members.

The analysis should be carried out using the appropriate values of member stiffness and joint slip. Fictitious beam elements should be assumed to be as stiff as the adjacent element.

Care should be taken if the fictitious beams have to be very short, i.e. shorter than about 100 *mm*. This could lead to a nearly singular stiffness matrix and to unreasonable numerical results which could be missed. Sometimes, it is advisable to use an adapted analysis, e.g. finite element analysis which makes available rod elements with built in end eccentricities.

Example:
It can be assumed that a computer program is available for analysing trusses. For the input the member stiffness of the bottom chord and joint slip of the dowel connection is given here: Bottom chord with a rectangular cross-section b x h = 50 x 180 mm^2. Strength class C24 according to prEN338.

Member stiffness: $E_d = E_{0,50} = 11000\ N/mm^2$

Timber-to-timber connection with dowels $d = 8\ mm$
$K_{ser} = 380^{1,5} \cdot 8/20 = 3000\ N/mm$ is the instantaneous slip modulus per shear plane under service load F_{ser}.

If a geometric non-linear analysis is carried out, the member stiffness should be divided by the partial factor γ_m (given in EC5 table 2.3.3.2).

Verifying serviceability
$E_d = E_{0,50} / \gamma_M = 11000 / 1,0 = 11000\ N/mm^2$
EC5: Part 1-1: 4.1 (4) — $K_{ser,fin} = K_{ser} / (1 + k_{def})$

and verifying strength of members and joints
$E_d = k_{mod}\, E_{0,05} / \gamma_M = 0,9 \cdot 7400 / 1,3 = 5100\ N/mm^2$
EC5: Part 1-1: 6.1 (9) — $K_{u,fin} = 2K_{ser} / (3\ (1 + k_{def}))$

Joints may be generally assumed to be rotationally pinned. Translational slip at the joints may be disregarded for the strength verification unless it would significantly affect the distribution of internal forces and moments. Joints may be assumed to be rotationally stiff, if their deformation would have no significant effect upon the distribution of member forces and moments.

Simplified analysis

EC5: Part 1-1: 5.4.1.3

As an alternative to a general analysis, a simplified analysis is permitted for fully triangulated trusses which comply with the following conditions:

- there are no re-entrant angles in the external profile,
- some part of the bearing width lies vertically below the support node (see Figure 2),
- the truss height exceeds 0,15 times the span and 10 times the maximum chord depth.

The axial forces in the members should be determined assuming that every node is pin-jointed. The bending moments in single-bay members should also be determined on the basis that the end nodes are pin-jointed. Bending moments in a member which is continuous over several bays should be determined as if the member was a beam with a simple support at each node. The effect of deflection at the nodes and partial fixity at the joints should be taken into account by a reduction of 10% in the node bending moment. The reduced node moments should be used to calculate the span bending moments.

Strength verification of members and joints

EC5: Part 1-1: 5.4.1.4

For elements in compression, the effective column length for in-plane strength verification should generally be taken as the distance between two adjacent points of contraflexure.

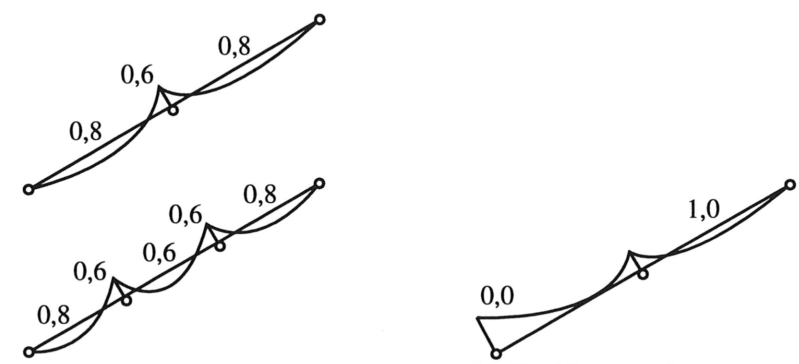
For fully triangulated trusses, the effective column length for members which are only one bay long without especially rigid end connections, and for continuous members without lateral load, should be taken as the bay length.

When a simplified analysis has been carried out, the following effective column lengths may be assumed (see Figure 6).

- for continuous members with a lateral load but without significant end moments
  - in an outer bay: 0,8 times the bay length,
  - in an inner bay: 0,6 the bay length,
  - at a node: 0,6 times the largest adjacent bay length.
- for continuous members with a lateral load and with significant end moments
  - at the beam end with moment: 0 (i.e. no column effect),
  - in the penultimate bay: 1,0 times bay length,
  - remaining bays and nodes: as described above.

For the strength verification of members in compression and connections, the calculated axial forces should be increased by 10%.

A check shall also be made that the lateral (out-of-plane) stability of the members is adequate.



(a) No significant end moments *(b) Significant end moments*

Figure 6 Effective column lengths

Limiting values of deflection

EC5: Part 1-1: 4.3.2

For trusses the limiting values of deflection for beams apply both to the complete span, and to the individual deflection of members between nodes. Referring to the truss span l, the limits are

$u_{2,inst} \leq l/300$
$u_{2,fin} \leq l/200$
$u_{net,fin} \leq l/200$

EC5: Part 1-1: 5.4.5.3

These limits are recommended unless special conditions call for other requirements, e.g. for the deflection of bracing systems (see trusses B of Figure 1). Furthermore it should be noticed that the horizontal bracing load of trusses increases with their vertical deflection.

EC5: Part 1-1: 4.1

The final deformation of a truss fabricated from members which have different creep properties should be calculated using modified stiffness moduli, which are determined by dividing the instantaneous values of the modulus for each member by the appropriate value of $(1 + k_{def})$.

The deflection u of a truss may be determined by computer program, e.g. based on finite elements, or analytically by the method of virtual work, using the relationship

$$u = \Sigma\, F_i\, F_{1i}\;\; l_i\; / \, A_i\, E_i + 2\,\Sigma\, F_i\, F_{1i}\; / \, n_i\, K_i$$

where F_i axial force of truss member i,
F_{1i} force of truss member i caused by unit load,
n_i number of fasteners at one joint of truss member i.

Trusses with punched metal plate fasteners

EC5: Part 1-1: 5.4.1.5

Additional rules for trusses with punched metal plate fasteners are given in Annex D and for joints in STEP lectures C11 and D3.

Concluding summary

- Trusses form part of three dimensional structures.
- By varying geometric parameters a large variety of trusses can be developed for nearly all kinds of application.
- In general, trusses shall be analysed as framed structure (rod elements), where deformations of members and joints and eccentricities are taken into account.
- Certain conditions allow a simplified analysis assuming pin-jointed members.
- Lateral global and local stability of trusses has to be verified.

References

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Ozelton, E.C and Baird, J.A. (1976). Timber designer's manual. 1st edn. Crosby Lockwood Staples London, United Kingdom, 518 pp.