Portal frames and arches

STEP lecture B14
M.H. Kessel
Labor für Holztechnik LHT
Hildesheim

Objectives

To develop an understanding of the limit state design verification of portal frames and arches including lateral buckling and to illustrate the design procedure of EC5 by showing an example.

Prerequisites

A4 Wood as a building material
A17 Serviceability limit states - Deformations
B2 Tension and compression
B3 Bending
B6 Columns
B7 Buckling lengths

Summary

After introducing different types of portal frames and arches, the ultimate limit state design is demonstrated in two ways. First, a simplified analysis is shown considering in plane and lateral buckling. Secondly the application of a second order linear analysis is explained using a curved frame as example.

Introduction

Frames and arches often form the main structural elements in three dimensional structures covering halls of rectangular or circular ground surface, typically used in gymnasia, swimming pools or stores of bulk goods. Span dimensions vary between 20 and 100 *m* and in rectangular buildings the length is usually two to three times the span. Construction height is normally between 10 and 30 *m*. For fabrication and transportation reasons frames are normally three-hinged with one hinge at each support and one hinge at the top ridge. The width of the glued laminated timber cross-sections can be up to 240 *mm* and the depth up to 2 *m*. Larger arches can use built-up sections of glulam.

In Figure 1 six different construction types of frames are shown. Figure 1a details a system consisting of a two-piece column to resolve the corner moment into tension and compression elements. In Figure 1b the shape of the frame is achieved by means of a finger jointed haunch linking the rafter and column units. In Figure 1c the single rafter units are enclosed by two glulam columns which are linked along their length and designed as spaced columns. The haunch connection is effected by circular groups of dowels. The curved frame in Figure 1d takes advantage of the ability to curve glulam and leads to arches which are shown as a three-hinged variant in Figure 1e. The two-hinged arch in Figure 1f is necessary for flatter roofs but is greatly restricted in overall height and span by transportation requirements.

EC5: Part 1-1: 2.3.1

In general it has to be verified for all roof members that no relevant limit state is exceeded taking into account load actions in three dimensions.

This lecture, however, is restricted to the verification of frames and arches due to loads acting in their plane. Hence it follows that members forming frames and

EC5: Part 1-1: 5.4.4

arches are stressed mainly in compression and in-plane bending. In addition, lateral bending due to buckling effects has to be considered. The stresses caused by geometrical and structural imperfections, i.e. deviations between the geometrical axis and the elastic centre of the cross-section due, for example to material inhomogenities, and induced deflections shall be taken into account.

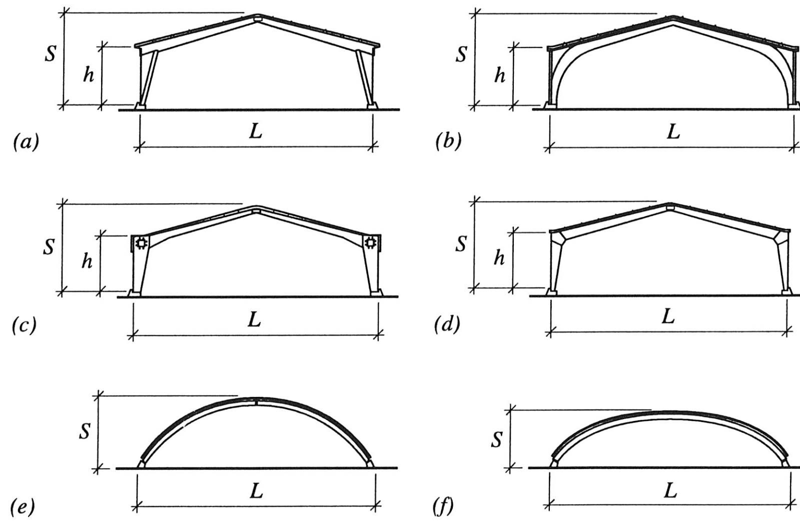

Figure 1 *Some typical frames: (a) V - shaped, (b) curved haunch, (c) dowelled haunch, (d) finger-jointed haunch. Some typical arches: (e) three-hinged, (f) flat two-hinged.*

Simplified analysis

Frames and arches can be verified by a simplified analysis in the same manner as columns and beams. The calculation of stresses due to external design loads is based on a linear theory considering equilibrium of the undeformed static system. Stresses caused by geometrical and structural in-plane and lateral imperfections and induced in plane deflections are taken into account by multiplying the compression and bending strength values by reduction factors such as k_c and k_{crit}. These factors have to be determined from the appropriate critical compression and bending stresses of in plane and lateral buckling. The deviation from straightness measured midway between the supports shall be limited for frame members to 1/500 of the length for glued laminated members.

EC5: Part 1-1: 7.2 P (1)

Design example

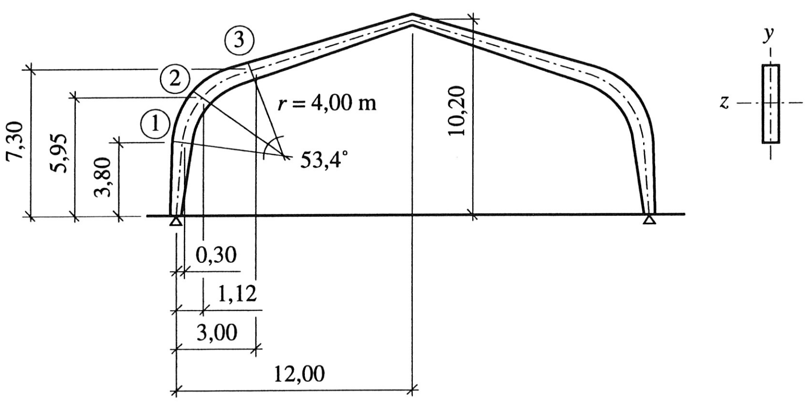

Figure 2 *Example: Curved frame of a riding-hall.*

The design values of the vertical (V) and horizontal (H) support reactions and the internal axial forces (N) and moments (M) at cross-sections 1, 2 and 3 and the ridge deflections u_{hor} and u_{vert} are given in Table 1 corresponding to the appropiate combination of actions 1,35 g + 1,5 s.

S_d	units	g	s	1,35 g + 1,5 s
V	kN	54	48	145
H	kN	23	31	78
N_1	kN	-44	-50	-134
N_2	kN	-41	-56	-139
N_3	kN	-30	-42	-104
M_1	kNm	-73	-104	-255
M_2	kNm	-91	-133	-322
M_3	kNm	-61	-92	-220
u_{hor}	mm	0	0	-
u_{vert}	mm	21	32	-

Table 1 *Design values of reactions, internal axial forces and moments and ridge deflections.*

The frames are fabricated of glued laminated timber GL 28 with the following appropriate material design properties (see prEN 1194):

design bending strength $f_{m,g,d} = \frac{0,9 \cdot 28}{1,3} = 19,4\ N/mm^2$

design compression strength $f_{c,o,g,d} = \frac{0,9 \cdot 27}{1,3} = 18,7\ N/mm^2$

moduli of elasticity $E_{0,mean,g} = 12000\ N/mm^2$
$E_{0,05,g} = 9600\ N/mm^2$

Verification of ultimate limit states in cross-section (2)

EC5: Part 1-1: 5.2.1

Compression buckling

It is assumed that cross-sections (1) and (3) are laterally supported.

$k_c = min\ (k_{c,y}, k_{c,z})$

In the case of in plane buckling the curved frame can be interpreted as an arch with sufficient accuracy. One half of the arc length is estimated as the sum of length of rafter (11 m) and column (6 m).

$\lambda_y = 1,25\ (11 + 6)\ /\ i_y = 75$

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In the case of lateral buckling, the buckling length is estimated as arc length between cross-sections (1) and (3).

$$\lambda_z = 63,4 \cdot \pi \cdot \frac{400}{180}\ i_z = 100$$

$$\lambda_{rel,z} = \frac{100\sqrt{\frac{27}{9600}}}{\pi} = 1,7 > 0,5$$

$$k_z = 0{,}5\left(1 + 0{,}1\,(1{,}7 - 0{,}5) + 1{,}7^2\right) = 2$$

$$k_c = k_{c,z} = \frac{1}{\left(2 + \sqrt{4 - 1{,}7^2}\right)} = 0{,}33$$

EC5: Part 1-1: 5.2.2

Lateral torsional buckling
In addition it is assumed that cross-sections (1) and (3) are torsionally fixed.

$$M_{crit} = \frac{\sqrt{BC}}{2r}\left[\sqrt{\frac{B}{C}} + \sqrt{\frac{C}{B}} + \sqrt{\frac{B}{C} + \frac{C}{B} + \frac{4\pi^2}{\alpha^2} - 2}\right] = 1{,}65\ MNm$$

(See Timoschenko and Gere 1961 or Pflüger 1975.)

$$B = E_{0,05}\,I_z = 9600 \cdot 0{,}16^3 \cdot \frac{1{,}00}{12} = 3{,}28\ MNm^2$$

$$C = \frac{E_{0,05}}{E_{o,mean}}\,G_{mean}\,I_T = \frac{9600}{12000}\,750 \cdot 0{,}3 \cdot 0{,}16^3 \cdot 1{,}00 = 0{,}74\ MNm^2$$

$$r = 4{,}00\ m,\ \alpha = \frac{63{,}4 \cdot \pi}{180} = 1{,}11$$

$$\sigma_{m,crit} = \frac{1{,}65 \cdot 10^9 \cdot 6}{160 \cdot 1000^2} = 62\ N/mm^2$$

$$\lambda_{rel,m} = \sqrt{\frac{28}{62}} = 0{,}67 < 0{,}75 \Rightarrow k_{crit} = 1{,}0$$

EC5: Part 1-1: 5.2.4

Reduction in strength due to bending of laminations during production

$$\frac{r_{in}}{t} \geq 240 \Rightarrow k_r = 1{,}0$$

Design bending stress

$$\sigma_{m,y,d} = k_l\,\frac{6\,M_{ap,d}}{b\,h_{ap}^2}$$

$$\alpha = 0 \Rightarrow k_l = 1 + 0{,}35\,\frac{h_{ap}}{r} + \frac{0{,}6\,h_{ap}^2}{r^2}$$

$$\frac{h_{ap}}{r} = \frac{1}{4} \Rightarrow k_l = 1{,}125$$

$$\sigma_{m,y,d} = 1{,}125\,\frac{6 \cdot 322 \cdot 10^6}{160 \cdot 1000^2} = 13{,}6\ N/mm^2$$

Design compression stress

$$\sigma_{c,o,d} = \frac{139 \cdot 10^3}{160 \cdot 1000} = 0{,}9\ N/mm^2$$

Verification with respect to compression buckling

$$\frac{\sigma_{c,o,d}}{k_c f_{c,o,d}} + \frac{\sigma_{m,y,d}}{k_r f_{m,d}} \leq 1 = \frac{0,9}{0,33 \cdot 18,7} + \frac{13,6}{1,0 \cdot 19,4} = 0,15 + 0,70 = 0,85 < 1,0$$

Verification with respect to lateral torsional buckling

$$\frac{\sigma_{c,o,d}}{f_{c,o,d}} + \frac{\sigma_{m,y,d}}{k_r k_{crit} f_{m,d}} = \frac{0,9}{18,7} + \frac{13,6}{1,0 \cdot 1,0 \cdot 19,4} = 0,05 + 0,70 = 0,75 < 1,0$$

EC5: Part 1-1: 4

Verification of serviceability limit states

In this design no precamber is included. However, precamber could be necessary in other cases especially where deflections in curved members are caused by changes in moisture content.

Vertical ridge deflection, $u_{vert,1}$, due to permanent load, g, (see Table 1).

$u_{vert,1,inst} = 21\ mm$
$u_{vert,1,fin} = (1 + 0,6)\ 21 = 34\ mm$

Vertical ridge deflection u_{vert2} due to variable load s (see Table 1).

$u_{vert,2,inst} = 32\ mm$
$u_{vert,2,fin} = (1 + 0,25)\ 32 = 40\ mm$

For frames and arches, EC5 gives no recommendations on deflection limits, because such limits are related to the intended use of the construction. In the absence of special conditions that call for other requirements, EC5 recommends the final net ridge deflection to be

$u_{vert,net,fin} = 34 + 40 = 74\ mm < 24000 / 200$

EC5: Part 1-1: 5.4.4(2)

Second order linear analysis

The calculation of stresses is based on geometric nonlinear theory considering equilibrium of the deformed imperfect static system. The contribution of any joint slip to the induced deflections should be taken into account.

Besides the limit states of rupture or excessive deformation of a section, member or connection, the nonlinear calculation must be able to detect a possible limit state due to the transformation of the structure into a mechanism or due to instability. For example, firstly a king post truss with a precambered upper chord transforms into a lateral mechanism when the vertical deflection reaches the value of the precamber and secondly a flat three-hinged roof loses its stability by snap through buckling.

Nowadays, these calculations are carried out by finite element computer programs. Normally two dimensional finite rod elements (two displacements and one rotation at each node) are used. These elements are able to take into account imperfections and induced deformations in the plane of the frame. All stresses due to lateral effects have to be calculated using the simplified analysis. Otherwise, a more complex description of the frame by three dimensional finite elements is necessary (Kessel, et al. 1984 and Young and Kuo 1991).

Compared to the simplified method, the advantages of a second order linear analysis are:

- no need to estimate critical stresses in determining the factors k_c and k_{crit};
- no need to estimate bracing forces if a three dimensional simulation of a set of frames or arches is carried out (Kessel 1984).

Design example

The second order linear analysis is shown for the previous example in Figure 2. A two dimensional nonlinear finite rod element is used. The initial deflections are shown in Figure 3 assuming an angle *f* of inclination

$$\phi = 0{,}005\sqrt{\frac{5}{6}} = 0{,}0046$$

which leads to a corresponding initial vertical ridge deflection

$$u_{vert,initial} = 0{,}0046 \cdot 12 = 0{,}055\ m$$

as shown in Figure 3a and a corresponding initial horizontal ridge deflection

$$u_{hor,initial} = 0{,}0046 \cdot 7 = 0{,}032\ m$$

as shown in Figure 3b.

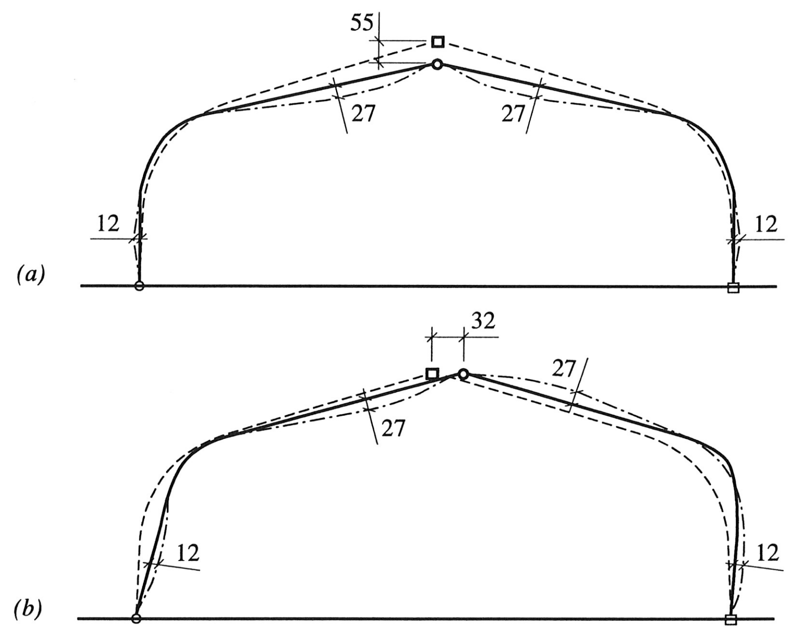

Figure 3 *Imperfect structure: (a) corresponding to symmetrical actions, (b) corresponding to non-symmetrical actions. Dashed line: initial frame; solid line: inclined frame; chain-dotted line: inclined frame including curvature.*

The combination of actions remains unchanged, but the stresses and deflections are calculated using a value of *E* of

$$E = E_{0,05}\, f_{m,d} \,/\, f_{m,k} = 9600 \cdot 19{,}4 \,/\, 28 = 6650\ N/mm^2$$

The resulting design values are given in Table 2.

S_d	units	1,35 g + 1,5 s	
		symmetric	non-symmetric
V	kN	144	146
H	kN	79	79
N_1	kN	-135	-135
N_2	kN	-140	-140
N_3	kN	-105	-106
M_1	kNm	-269	-260
M_2	kNm	-343	-327
M_3	kNm	-238	-227

Table 2 *Nonlinear analysis. Design values of reactions, internal axial forces and moments.*

Verification of ultimate limit state in cross-section (2)

Since, lateral buckling is not part of the applied nonlinear analysis the factors $k_c = k_{c,z}$ and k_{crit} remain valid therefore:

the design bending stress is

$$\sigma_{m,y,d} = 1{,}125\,\frac{6 \cdot 343 \cdot 10^6}{160 \cdot 1000^2} = 14{,}5\ N/mm^2$$

the design compression stress is

$$\sigma_{c,o,d} = \frac{140 \cdot 10^3}{160 \cdot 1000} = 0{,}9\ N/mm^2$$

and the interaction equation is

$$\frac{0{,}9}{0{,}33 \cdot 18{,}7} + \frac{14{,}5}{1{,}0 \cdot 1{,}0 \cdot 19{,}4} = 0{,}15 + 0{,}75 = 0{,}90 < 1{,}0$$

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