Bracing - Design

STEP lecture B15
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Objectives

To develop an understanding of how compression or bending members need to be braced in order to avoid instability, to identify the governing parameters and to present the procedure of EC5 as a workable design method.

Prerequisites

A4 Wood as a building material
B2 Tension and compression
B3 Bending
B6 Columns
B7 Buckling lengths

Summary

The lecture starts with a non-mathematical introduction to the mechanisms of bracing structures. It presents the principal factors influencing the actions on bracing members and shows how the equations offered in EC5 have been derived. A practical example showing how the actions on bracing structures are evaluated complements the lecture.

Introduction

Columns require a stability calculation to check against failure or unacceptable deformations. Often it is advisable to restrain one or more points (between the main supports) from lateral deflection by bracing. This is done in an analogous way to that used for slender beams to prevent lateral buckling. Columns or beams could be part of a combined structure, for example an upper chord of a truss. The actions on the bracing structure may be derived by using a second order analysis whereby the equilibrium of moments and forces is analysed by considering the deformed shape of the respective structure. The stiffness of all members concerned and the slip of built- in joints is taken into account. However, EC5 presents a simplified method based on the above approach.

Factors influencing actions on bracing members

It is necessary to differentiate between compression and bending members to be braced. Furthermore either a single highly loaded support or several supports which form part of a bracing structure, e.g. a truss, could be analysed. The actions on supporting structures depend on the general geometry of the structure to be braced, such as cross-sectional and longitudinal dimensions, support conditions and material properties determined by the choice of the strength class, the climate and the load duration class of the governing load case. The stiffness of members and the rigidity of existing joints are very important factors, not only as attributes of the structure to be braced but especially for the bracing structure itself. To perform a second order analysis, geometrical and structural imperfections should be included.

EC5: Part 1-1: 7.2

In EC5 the initial curvature of the member axis is limited by deviation from straightness to $l/500$ of the length for glued laminated timber and to $l/300$ of the length for structural timber.

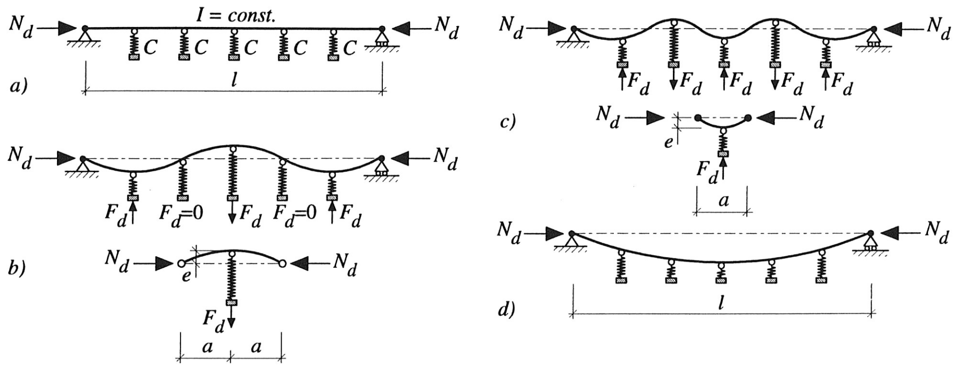

Figure 1 System and deflections of braced members.

Compression members of the length l regularly braced by elastic supports to avoid buckling produce big spring forces if the deflected shapes shown in Figure 1b and c are assumed. Möhler and Schelling (1960) showed that the minimum spring stiffness should be

$$C = k_s \pi^2 \frac{EI}{a^3} \tag{1}$$

where

$$k_s = 2\left(1 + \cos\frac{\pi}{m}\right) \tag{2}$$

and a the length, m the number of waves, so that $l = m\ a$ to guarantee a deformed line of member axis with two hinged ends and with $k_s = 2$ for one wave shape or $k_s = 4$ for an infinite number of waves. The spring force F_d (see Figure 2).

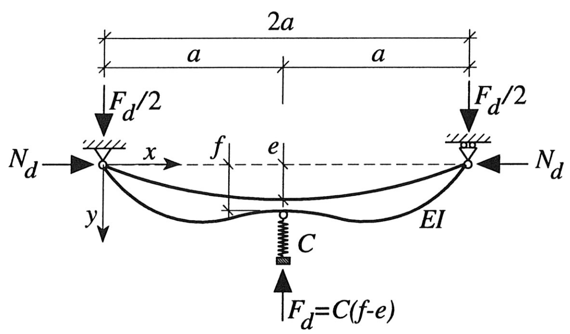

Figure 2 Shape and forms of an elastically supported member.

can be calculated conservatively by a second order analysis to be:

$$F_d \le 5{,}2\, N \frac{e}{2a} \tag{3}$$

where e is the maximum deviation of straightness.

Figure 3 shows that spring forces of $F_d = N_d\ /58$ and $F_d = N_d\ /96$ could be included, if deviations of straightness in an unloaded situation of $l/300$ or $l/500$ are assumed for solid timber, or glued laminated timber, respectively. The results have been approximated in EC5 to $N_d\ /50$ or $N_d\ /80$.

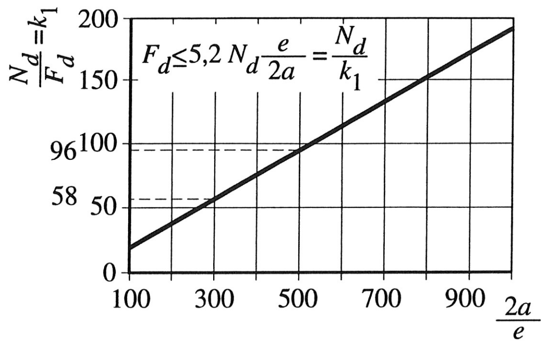

Figure 3 *Coefficient of bracing force as a function of deviation of straightness.*

Single supports of bending members

Burgess (1989) proposed the substitution N_d in equations 5.4.5.2 d and e of EC5 with

$$N_d = M_d \frac{N_{crit}}{M_{crit}} \tag{4}$$

where N_{crit} and M_{crit} are the critical forces calculated according to the classical theory of stability. EC5 proposes an alternative approximation

$$N_d = (1 - k_{crit}) \frac{M_d}{h} \tag{5}$$

where k_{crit} is calculated from equations 5.2.2 c to e of EC5 for the unbraced length of the member. Here the torsional rigidity of a beam is taken into account. No bracing is required if k_{crit} = 1. The procedure only works if the beam is braced along the compression edge.

Bracing of beam or truss systems

To achieve maximum actions on a bracing structure the imperfections of the compression or bending members to be horizontally supported by a bracing structure of the stiffness $(EI)_{ef}$ is assumed to be a single wave sine curve as shown in Figure 1d. Compression forces, N_d, produce a moment (see Figure 4)

$$M_d = n N_d y$$

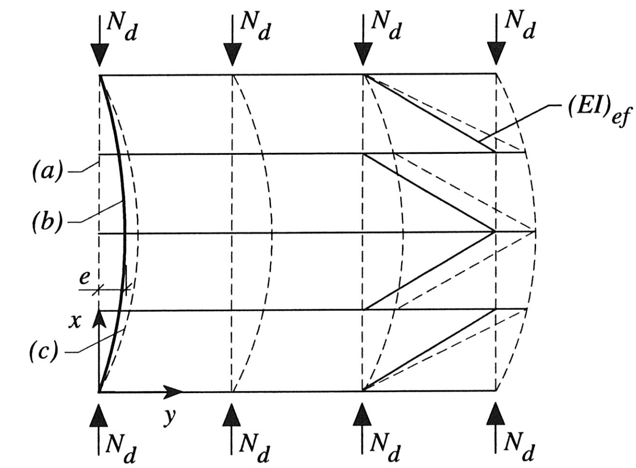

Figure 4 *Braced compression members and deflections; (a) straight line, (b) imperfect axis, (c) deflected axis. $(EI)_{ef}$ is the stiffness of the bracing structure.*

The resistance component, neglecting the stiffness EI_z of the slender members to be stiffened, is given by the differential equation of elastic line with regard to the predeflection

$$\frac{M_d}{(EI)_{ef}} = -\frac{d^2(y-y_0)}{dx^2} = -(y-y_0)'' \qquad (6)$$

which equation may be combined such that

$$y'' + \frac{n\,N_d}{(EI)_{ef}}\,y = -e\left(\frac{\pi}{l}\right)^2 \sin\left(\frac{\pi}{l}x\right) \qquad (7)$$

where the predeflection is

$$y_o = e \sin\left(\frac{\pi}{l}x\right) \qquad (8)$$

and

$$M_d = n\,N_d\,y \qquad (9)$$

The solution of the preceding differential equation leads to

$$q_d = \frac{e\left(\frac{\pi}{l}\right)^2 n\,N_d}{1 - \frac{n\,N_d}{\left(\frac{\pi}{l}\right)^2 (EI)_{ef}}} \sin\left(\frac{\pi}{l}x\right) \qquad (10)$$

see also Brüninghoff (1983).

The evaluation of this equation requires knowledge of the bracing stiffness $(EI)_{ef}$ to be calculated, taking into account not only the elastic behaviour of all members, e.g. chords and diagonals of a truss, but also the contribution related to any joint slip. To provide a simplification for common design situations, EC5 limits the maximum deformations of the bracing structure caused by q_d to $l/700$.

$$\max y = q_d \frac{l^4}{\pi^4} \frac{1}{ef(EI)} \leq \frac{l}{700} \qquad (11)$$

Then the elimination of $(EI)_{ef}$ out Equations (10) and (11) above and the conversion of a sine shaped load q_d into a constant form give

$$q_d = k_l \frac{n\,N_d}{30\,l} \qquad (12)$$

where $k_l = 1$.

For spans of more than 15 m a particular accuracy of workmanship may be expected to limit deflections so that it is reasonable to reduce the span-related imperfection by the factor

$$k_l = \sqrt{\frac{15}{l}} \qquad (13)$$

where l is given in m.

The design engineer should check the deformations of the system if the deflection limitations are likely to be exeeded.

Using beams (instead of compression members) in structures the compression edge should be supported so that the equations in EC5 may be taken for bracing analysis.

To take into consideration the torsional capacity of beams as described above, the compression force can be reduced to

$$N_d = (1 - k_{crit}) \frac{M_d}{h} \tag{14}$$

where h is the depth of the beam

Design example

A hangar 60 *m* length, 20 *m* wide and 8 *m* height is to be constructed, using glulam beams of strength class GL 28 according to prEN 1194, "Timber structures - Glued laminated timber - Strength classes and determination of characteristic values". The beams span 20 *m*, are 1200 *mm* deep and 160 *mm* wide and are spaced at 6 *m* centres.

Design values of permanent and varable load for the governing load case:
permanent load: g_d = 5,4 *kN/m* (line load, permanent)
variable load: q_d = 6,0 *kN/m* (line load, short term)

EC5: Part 1-1: 3.1.7

Service class 1: k_{mod} = 0,9

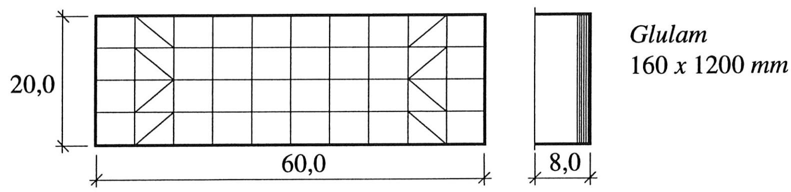

Figure 5 Design example.

Characteristic material properties

prEN 1194: 1993

The characteristic values of strength as well as modulus of elasticity for bending and torsion are taken from prEN 1194 "Timber structures, Glued laminated timber, strength classes and determination of characteristic values". The 5-percentile values are used in the designwork since an ultimate limit state is considered.

$f_{m,k}$ = 28 N/mm^2 $E_{0,05}$ = 9600 N/mm^2
E_{mean} = 12000 N/mm^2 G_{mean} = 700 N/mm^2

cross-section values:

$$W_y = \frac{h^2 b}{6} = 38{,}4 \cdot 10^6 \; mm^3 \tag{15}$$

$$I_z = \frac{h b^3}{12} = 410 \cdot 10^6 \; mm^4 \tag{16}$$

$$I_{tor} = \eta_3 h b^3 = 1500 \cdot 10^6 \; mm^4 \tag{17}$$

$$\eta_3 \approx \frac{1}{3}(1 - 0{,}63 \frac{b}{h}) = 0{,}305 \tag{18}$$

The critical moment is

$$M_{crit} = \frac{\pi}{l} \sqrt{E_{0,05}^2 \frac{G_{50}}{E_{50}} I_z I_{tor}} = 286 \; kNm \tag{19}$$

and the critical stress is

$$\sigma_{m,crit} = \frac{M_{crit}}{W_y} = 7{,}44\ N/mm^2 \tag{20}$$

The relative stress slenderness can be calculated

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} = 1{,}94 \geq 1{,}4 \tag{21}$$

and the buckling coefficient

$$k_{crit} = \frac{1}{\lambda_{m,rel}^2} = 0{,}266 \tag{22}$$

The compression force of the glulam beam is to be determined

$$N_d = (1 - k_{crit})\ \frac{M_d}{h} = 349\ kN \tag{23}$$

where

$$M_d = (g_d + q_d)\ \frac{l^2}{8} = 570\ kNm \tag{24}$$

The bracing load is then given with

$$q_d = k_l\ \frac{n\,N_d}{30\,l} = 5{,}04\ kN/m \tag{25}$$

where the imperfection factor is:

$$k_l = \sqrt{\frac{15}{l}} = 0{,}866 \tag{26}$$

n = 10 for 9 fully loaded beams and two 50% loaded gable walls
N_d = 349 kN as shown above.

The limitation for horizontal deflection is $l/700$ for bracing actions, $l/500$ for the combination of wind and bracing loads. These are normally fulfilled if the bracing structure is properly designed connected and the relationship of span l and spacing e_l is less than 6, here

$$\frac{l}{e_l} = 3{,}33 < 6 \tag{27}$$

Concluding summary

- Bracing structures are needed to restrain slender compression or bending members from lateral buckling.

- The major factors influencing the bracing actions are dimensions of the system and the beams, geometric and structural imperfections and material properties such as strength and modulus of elasticity for bending and torsion.

- The procedure offered in EC5 is based on a simplified second order analysis, such that an additional check of lateral deflections is generally required.

References

Möhler, K. and Schelling, W. (1968). Zur Bemessung von Knickverbänden und Knickaussteifungen im Holzbau. Der Bauingenieur, 43. Jahrgang, Heft 2.

Burgess, H.J. (1989). Suggested Changes in Code Bracing Recommendations for Beams and Columns. In: Proc. of the CIB-W18 Meeting, Berlin, Germany, Paper 22-15-1.

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Notation

a length between elastic supports

e_l spacing of beams