Load sharing

STEP lecture B16
H.J. Blass
Delft University
of Technology

Objectives

To develop an understanding of the phenomenon of probabilistic load sharing in parallel structural systems and to quantify the effect on the load-carrying capacity of systems.

Prerequisite

A4 Wood as a building material

Summary

The lecture presents examples of parallel structural systems, where the positive correlation between the strength and the stiffness of timber members increases the load-bearing capacity of systems compared with that of single members. The influence of the material behaviour and the variation of timber strength and stiffness on the load sharing effect is discussed. For common structural systems the load sharing effect is quantified.

Introduction

EC5: Part 1-1: 5.4.6

EC5, like other design codes, includes a load sharing factor for assemblies consisting of several similar members connected by a load distribution system. An example of such an assembly is a timber joist floor, where the joists are linked by panel sheathing. The load sharing factor increases the member design strength by taking into account two effects: first the reduced chance that a weaker member or part will be placed at a position where the stresses are particularly high, and second the positive correlation between strength and stiffness of timber members. This positive correlation enables a stiffer member to carry a higher proportion of the applied load. On the other hand, less stiff members, which in most cases are also weaker carry less of the load. Load sharing counteracts the material variability effects to a certain extent.

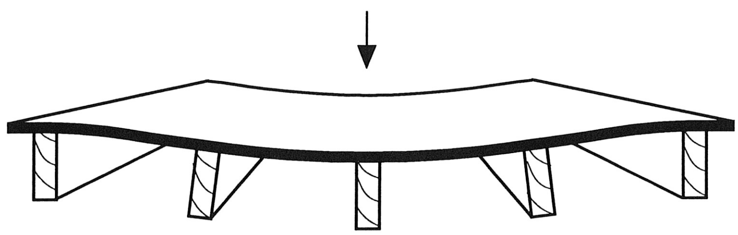

Figure 1 Floor layout under the action of a concentrated load.

The load sharing or load distribution effect improves the member behaviour in systems for both concentrated and distributed loads. For concentrated loads, the load distribution system transfers part of the load to the adjacent members, relieving the most stressed member under the concentrated load. Figure 1 shows a floor cross-section under the action of a concentrated load.

In uniformly loaded systems, the load-sharing effect is less evident. If the stiffness of all members is the same, the deformation of all members would be identical even without a load distribution system. Since in reality the member stiffness varies, softer members tend to deform more than stiffer members. Line (b) in Figure 2 shows the different joist deformations in a floor if no load distribution system is effective. If a load distribution system connects the joists, the deformations become more uniform (see line (a) in Figure 2). In this case, the load distribution system decreases the load on flexible members and increases the load on stiffer members.

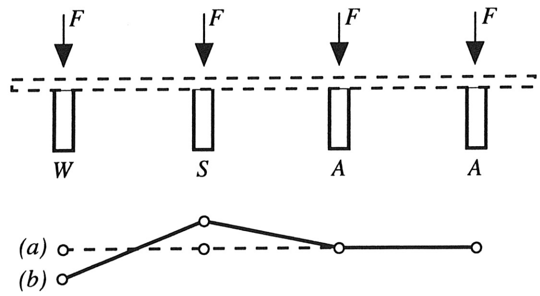

Figure 2 *Effect of the load distribution system on the joist deformations. (a) with and (b) without load distribution system. W: low stiffness member; S: high stiffness member; A: average stiffness member.*

The same situation is true where the member behaviour is no longer linear. If the stiffness of a single member under loads close to the ultimate load decreases due to microcracks or plastic deformations, the load is redistributed within the assembly and the partly damaged member is able to contribute to the load carrying capacity of the system, the total assembly load can still be increased.

Load sharing in different structural systems

Floors and flat roofs

Foschi, Folz and Yao (1989) performed a numerical study of the design of floors and flat roofs in order to derive system factors for modifying single member design equations. The structural analysis carried out in this study was restricted to linear behaviour. The loads considered were uniformly distributed dead and live loads.

The load sharing factor was derived by a reliability assessment of a single beam within the floor, i.e. the way in which the performance of a single member was affected by its insertion into the structural system was considered. Sensitivity analyses were carried out to determine the influence of different parameters. The load sharing factor k_{ls} was found to be quite insensitive to variations in the support conditions, the size, number and spacing of the joists and the ratio between dead and live load. The following factors increase the effect of load sharing:

- increasing the ratio of the stiffness of the load distribution system to the average member stiffness,
- increasing the variation of the member modulus of elasticity and
- raising the correlation between modulus of elasticity and bending strength.

The bending strength variation of the beams also significantly influences the load sharing factor. For very small and very large values of the coefficient of variation (COV), the load sharing factor is small with a maximum for COV values between 0,20 and 0,30. For a typical floor or flat roof, the load sharing factor determined was k_{ls} = 1,15. This corresponds quite well to the value of 1,10 in EC5.

Roof Trusses
Load distribution in roof truss assemblies has been studied by Wolfe and McCarthy (1989) and Wolfe and LaBissoniere (1991). Tests were performed on individual trusses and full-scale roof assemblies using three different truss configurations. The load distribution system was 12 *mm* plywood sheathing across the truss rafters. By measuring the load-deflection response of individual trusses independently and as part of the roof assembly, the effects of assembly interaction under uniformly distributed loads as well as line loads on individual trusses were evaluated.

Roof load carrying capacity was increased and apparent truss stiffness variability was decreased by load sharing mechanisms within the assembly. When partial damage occurred to individual trusses, a redistribution of loads away from these trusses enabled them to continue to contribute to the assembly load carrying capacity at a lower load level.

If a single truss in a system was loaded along its top chord with the design load, 40% to 70% of that load was transferred to adjacent trusses by the sheathing. The load sharing effect on the load carrying capacity of the trusses resulted in ratios of measured roof assembly strength to minimum truss strength from 1,09 to 1,47. These values depend on the effectiveness of the load distribution system and on the position of the truss in the assembly. They indicate that the design load carrying capacity of the entire truss, i.e. members and connections, can be increased by at least 10% due to the load sharing effect. For most systems, a factor of 1,10 can be considered as a safe minimum value.

Sheet piling
Load sharing effects also increase the bending capacity of planks in sheet piling or retaining walls, if they are interconnected, for example by tongue and groove joints. In this case, the load distribution system is the connection between the single planks. This connection causes a nearly uniform deflection of the individual planks under uniformly distributed loads although their stiffness values may vary considerably.

A theoretical analysis (Van der Linden et al., 1994) was carried out, based on the following properties of ekki planks (*Lophira alata*) in wet condition:

$f_{m,mean} = 103\ N/mm^2$ and $E_{0,mean} = 17600\ N/mm^2$.

The coefficient of correlation between bending strength and modulus of elasticity was 0,73 and the coefficient of variation for both bending strength and modulus of elasticity 15%. The bending stress distribution over the depth of the planks included a plastic behaviour in the compression zone, leading to a decrease in stiffness at higher stress levels. The analysis included the generation of sheet piling systems based on varying properties between the planks and constant properties within the planks and the subsequent calculation of their load carrying capacity using a nonlinear finite element model. Comparing the characteristic load carrying capacity per plank for systems with ten planks to the capacity of individual planks leads to a load sharing factor of about 1,15. This factor is applicable only to the bending strength values for the planks, since the load distribution system is not effective for axial forces.

Design example

Timber floor with beams b x h = 60 x 200 *mm* spaced at a = 0,60 *m* interval with tongued and grooved floor boards acting as load distribution system, span

l = 4,60 m. Strength class C24 according to prEN 338 "Structural timber. Strength classes".

Design values of permanent and variable load for the governing load case:
permanent load: $g_d = 1,0\ kN/m^2$ (uniformly distributed load, permanent)
variable load: $q_d = 3,0\ kN/m^2$ (uniformly distributed load, medium-term)

EC5: Part 1-1: 3.1.7

Service class 1: $k_{mod} = 0,8$

Design bending stress:

$$\sigma_{m,d} = \frac{(g_d + q_d)\ a\ l^2}{8\ W} = \frac{4,0 \cdot 0,6 \cdot 4,6^2 \cdot 10^6}{8 \cdot 400 \cdot 10^3} = 15,9\ N/mm^2$$

prEN 338: 1991

Characteristic material property:
The characteristic value of the bending strength is taken from prEN 338 "Structural timber - Strength classes": $f_{m,k} = 24\ N/mm^2$.

EC5: Part 1-1: 2.2.3.2

Design value of bending strength:

$$f_{m,d} = \frac{k_{mod}\ f_{m,k}}{\gamma_M} = \frac{0,8 \cdot 24}{1,3} = 14,8\ N/mm^2$$

EC5: Part 1-1: 2.3.2.1b

Verification of failure condition:

$$\frac{\sigma_{m,d}}{k_{ls}\ f_{m,d}} = \frac{15,9}{1,1 \cdot 14,8} = 0,98 < 1$$

Concluding summary

- Load sharing increases the characteristic load carrying capacity of members in parallel structural systems compared to single members, based on the positive correlation between strength and stiffness of timber members.

- A stiff load distribution system, a close correlation between strength and stiffness and plastic behaviour of the members are all beneficial to the load sharing effect.

- Typical assemblies where the load sharing effect increases load carrying capacity are flat roofs, floors, trusses, rafters, wall studs and sheet piling with effective load distribution systems connecting the individual members.

References

Foschi, R.O., Folz, B.R. and Yao, F.Z. (1989). Reliability-Based Design of Wood Structures. Structural Research Series, Report No. 34, Department of Civil Engineering, University of British Columbia, Vancouver, Canada, ISBN 0-88865-356-5.

Van der Linden, M.L.R., Van de Kuilen, J-W. G. and Blass, H.J. (1994). Application of the Hoffman yield criterion for load sharing in timber sheet piling. In: Proc. of the 1994 Pac. Timber Eng. Conf. Gold Coast, Australia.

Wolfe, R.W. and LaBissoniere, T. (1991). Structural Performance of Light-Frame Roof Assemblies. II. Conventional Truss Assemblies. Research Paper FPL-RP-499, Forest Products Laboratory, Forest Service, US Department of Agriculture, Madison, Wisconsin, USA.

Wolfe, R.W. and McCarthy, M. (1989). Structural Performance of Light-Frame Roof Assemblies. I. Truss Assemblies With High Truss Stiffness Variability. Research Paper FPL-RP-492, Forest Products Laboratory, Forest Service, US Department of Agriculture, Madison, Wisconsin, USA.