Bending

STEP lecture B3
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Objectives

To develop an understanding of the behaviour of timber beams, including lateral torsional buckling and to illustrate the procedures for the design of simple beams to EC5 by way of examples.

Summary

This lecture begins with an introduction to the behaviour and design of simple, solid timber or glulam beams in accordance with the requirements of EC5. It goes on to describe the factors which influence the lateral torsional buckling/instability behaviour of beams. The principles described are illustrated by a design example.

Introduction

Beams, in general, are horizontal structural elements which span at least two supports and transmit loads principally by bending action. The bending moments on the beam are due to loads which act in the plane of bending of the beam. The standard design procedure for timber beams, where the direction of grain in the wood is parallel to the span, is to ensure that:

- the design bending strength is not reached or exceeded and that the bending stresses do not cause lateral torsional buckling of the beam leading to a premature instability failure
- the design shear strength, (see STEP lecture B4), is not reached or exceeded
- the design compression strength perpendicular to the grain (bearing strength) is not reached or exceeded at supports and at concentrated load points
- the beam's deflection meets the serviceability deflection criteria (see STEP lecture A17)
- vibration (see STEP lecture A18) would not be a problem.

This lecture is concerned primarily with simple beams, i.e. beams without notches, tapers or curves. The effects of notches in beams and strength reduction in curved and tapered glulam beams are covered in STEP lectures B5 and B8, respectively. In so far as bending stresses are concerned, it is necessary to check that there is adequate capacity at the critical cross section (which may e.g. be rectangular, T or L shaped) which for a simple beam will be at the point of maximum bending moment in the beam. EC5 also requires that the influence of initial curvature, eccentricities and induced deflections are taken into account.

Simple beams

If the dimensions and support conditions of the beam are adequate to prevent instability i.e. deflections occur only in the loading plane, then it can be shown according to the theory of elasticity that the bending stresses in the beam are given by

$$\sigma = \frac{M\,y}{I} \qquad (1)$$

where M is the bending moment acting on the beam,

I	is the second moment of area of the beam cross-section,
y	is a distance from the neutral axis, and
σ	is the stress at distance y.

In general, this equation may be used to describe the behaviour of beams if

- the section is bent only about its minor principal axis or,
- when bent about its major principal axis, where closely spaced, discrete bracing is provided so that the slenderness is low.

EC5: Part 1-1: 3.1.4

EC5: Part 1 1: 2.2.3.2

Since EC5 allows the design of timber structures to be carried out on the assumption that they behave elastically, the above expression may be used for design purposes. The design bending strength , $f_{m,d}$, of a beam is defined as

$$f_{m,d} = \frac{k_{mod}\, f_{m,k}}{\gamma_M} \qquad (2)$$

EC5: Part 1-1: 3.1.7

where

$f_{m,k}$	is the characteristic bending strength,
γ_M	is the partial safety factor for material properties, and
k_{mod}	is a modification factor which takes into account the influence of load duration, service class and material type.

In addition to the k_{mod} factor, it is necessary to consider other factors which affect beam strength. For example, the influence of beam size on the bending strength is taken account of by the size factor k_h (see STEP lecture B1) and, if the beam is part of a load sharing system, its bending strength may be increased by the factor k_{ls} (see STEP lecture B16).

Combined stresses

The most common use of a beam is to resist loads by bending about its major principal axis. However, the introduction of forces, which are not in the plane of bending, on the beam results in bi-axial bending (i.e. bending about both the major and minor principal axes). Additionally, the introduction of axial loads in tension or compression results in a further combined stress effect. For beams which are subjected to bi-axial bending, the following conditions both need to be satisfied:

$$k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (3)$$

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (4)$$

where the symbols are defined as follows:

$\sigma_{m,y,d}$	is the bending stress due to moments about the y axis,
$f_{m,y,d}$	is the bending strength due to moments about the y axis,
$\sigma_{m,z,d}$	is the bending stress due to moments about the z axis,
$f_{m,z,d}$	is the bending strength due to moments about the z axis, and
k_m	is the combined bending strength factor, which allows for the effects of biaxial bending stresses and the fact that the load-carrying capacity of the beam is not exhausted just because the stresses (obtained from the theory of elasticity) have reached the respective bending strengths at one corner of the beam's cross-section.

Similar equations are given in EC5 for combined bending with axial tension or compression. For a more detailed description of the design of columns i.e. structural elements subjected to both bending and axial compression see STEP lecture B6.

Beam instability

When designing beams, the prime concern is to provide adequate load carrying capacity and stiffness against bending about its major principal axis, usually in the vertical plane. This leads to a cross-sectional shape in which the stiffness in the vertical plane is often much greater than that in the horizontal plane. It is shown in STEP lecture B6 on columns that whenever a slender structural element is loaded in its stiff plane (axially in the case of the column) there is a tendency for it to fail by buckling in a more flexible plane (by deflecting sideways in the case of the column). Figure 1 illustrates the response of a slender simply supported beam, subjected to bending moments in the vertical plane; the phenomenon is termed lateral-torsional buckling as it involves both lateral deflection and twisting. This type of instability is similar to the simpler flexural buckling of axially loaded columns in that loading the beam in its stiffer vertical plane has induced a failure by buckling in a less stiff direction.

The bending moment at which such instability takes place is termed the critical moment. The formulae for critical moments for beams are given in standard text books such as that by Timoshenko and Gere (1961). It is usually assumed that the beam material has ideal elastic isotropic properties. Nevertheless, it was shown by Hooley and Madsen (1964) that the theory is also applicable to timber beams where the material is not isotropic.

The critical moment for the beam shown in Figure 1 which is simply supported at both ends in both the y and z axes, and is torsionally restrained about the x axis at the supports is given by

$$M_{crit} = \frac{\pi}{l_{ef}} \sqrt{\frac{E I_z \, I_{tor} G}{1 - \frac{I_z}{I_y}}} \tag{5}$$

where I_y and I_z are the second moments of area about the respective axes,
E is the modulus of elasticity of the material,
G is the shear modulus of the material,
I_{tor} is the torsional second moment of area for the beam cross-section, and
l_{ef} is the unrestrained length.

For a beam of rectangular cross-section $b \times h$, the corresponding simplified critical bending stress is given by

$$\sigma_{crit} = E \frac{\pi}{l_{ef}} \frac{b^2}{h} \sqrt{\frac{G}{E}} \sqrt{\frac{1 - 0{,}63\frac{b}{h}}{1 - \frac{b^2}{h^2}}} \tag{6}$$

It should be noted that the right hand square root term varies from 0,94 to 1,05 for b/h ratios of 0,1 and 0,7, respectively, which represent the realistic range of rectangular timber beams. It is therefore conservative to replace the square root with 0,94.

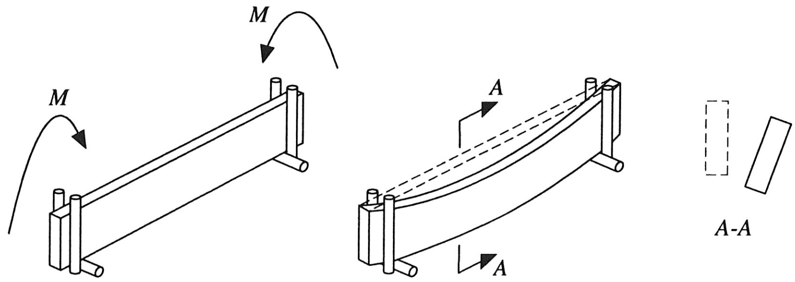


Figure 1 *Lateral-torsional buckling of a simply supported beam showing displacement at centre of beam under uniform moment. (a) simply supported beam, (b) buckled beam.*

For a homogeneous material there is only one value for E and G. In wood, the values of E and G depend on the angle between the direction of stress and the grain. In general, the E value (parallel to the grain) should be used and G is conservatively taken as $E/16$. This results in a critical stress of

$$\sigma_{crit} = \frac{0{,}75\ Eb^2}{hl_{ef}} \tag{7}$$

Similar expressions for the critical stress may be obtained for a variety of load cases, load positions and support conditions. The expression for M_{crit} given in Equation (5) is for the basic case where a simply supported beam is subjected to constant in-plane moments. If the beam is subjected to a central load acting at the level of the centroidal axis of the beam, a similar expression is obtained in which the term π is replaced by a constant 4,24. The ratio of $\pi/4{,}24$ is often referred to as the "equivalent uniform moment or m factor" and is a measure of the severity of a particular pattern of moments relative to the basic case. The values of the m factor for a number of load cases are given in Table 1. In general, lateral stability improves as the moment pattern becomes less uniform.

The location of the load is important - loads located at the top of a slender beam have a destabilising effect on its behaviour whilst loads located at the bottom of a beam have a stabilising effect. Clearly, support conditions are also important, in that lateral support conditions which inhibit the development of buckling deformations, i.e. against twisting of the beam at the supports in both the x and y axes, will improve a beam's lateral stability. The improvement in stability due to support conditions is generally reflected in smaller values of effective lengths. Lateral torsional buckling of beams is a complex subject outside the scope of this lecture and reference should be made to standard textbooks such as Timoshenko and Gere (1961).

Summarising the above details the main factors which influence lateral stability include:

- the unbraced span of the compressive portion of the beam (i.e. the distance between points at which lateral deflection is prevented,
- the beam's lateral bending stiffness (EI_z),
- the beam's torsional stiffness (GI_{tor}), and
- the restraints at the beam ends.

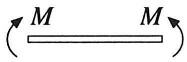
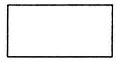
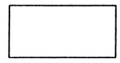
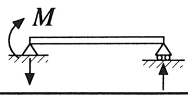
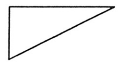
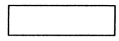
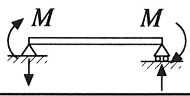
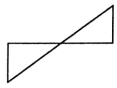
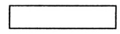
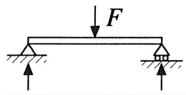
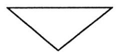
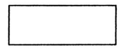
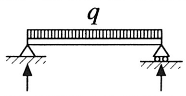
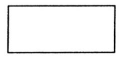
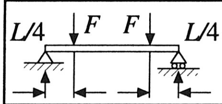
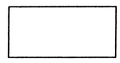
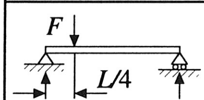
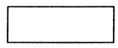
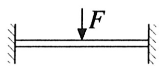
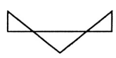
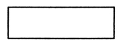
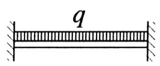
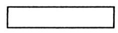
Beam & Loads	Actual bending moment	m	Equivalent uniform moment
		1,00	
		0,57	
		0,43	
		0,74	
		0,88	
		0,96	
		0,69	
		0,59	
		0,39	

Table 1 Equivalent uniform moment factors (taken from Kirby and Nethercot, 1979)

The load carrying capacity of a beam which is liable to lateral-torsional instability may be improved by the provision of bracing members. The main requirements are that the bracing members are sufficiently stiff to hold the beam effectively against lateral movement and that they are sufficiently strong to withstand the forces transmitted by the beam (see STEP lectures B15 and B7).

EC5: Part 1-1:5.2.2

EC5 requires that a check is carried out for the instability condition and that the bending capacity is modified by the factor k_{crit}, such that

$$\sigma_{m,z,d} \leq k_{crit} f_{m,z,d} \qquad (8)$$

where

$$k_{crit} = 1 \qquad (for\ \lambda_{rel,m} \leq 0,75) \qquad (9)$$

$$k_{crit} = 1,56 - 0,75\ \lambda_{rel,m} \qquad (for\ 0,75 < \lambda_{rel,m} \leq 1,4) \qquad (10)$$

$$k_{crit} = \frac{1}{\lambda_{rel,m}^2} \qquad (for\ 1,4 < \lambda_{rel,m}) \qquad (11)$$

and where the relative slenderness ratio $\lambda_{rel,m}$ for bending is given by:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} \qquad (12)$$

The critical bending stress $\sigma_{m,crit}$ for Equation (12) is obtained using the 5-percentile stiffness value $E_{0,05}$. Variation of k_{crit} with $\lambda_{rel,m}$ is shown in Figure 2. The similarity to the buckling strength-slenderness ratio curves for columns as described in STEP lecture B6 should be noted.

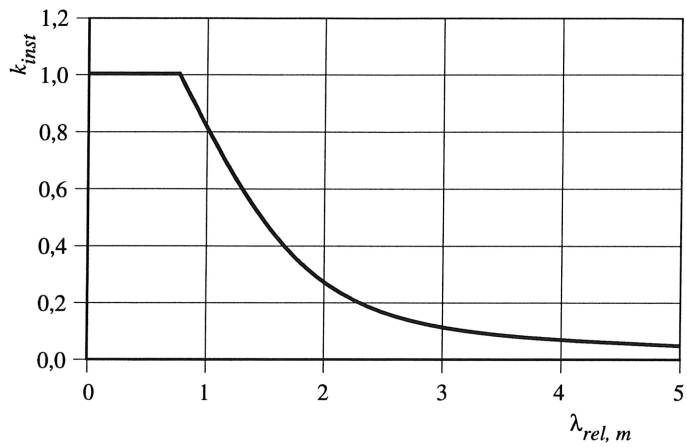


Figure 2 Variation of k_{crit} (or k_{inst}) with relative slenderness ratio λ_{rel}.

Design example

A simply supported rectangular solid timber floor beam of cross section 50 x 200 *mm*, with a clear unsupported span of 3500 *mm* is required to support a design medium-term load of 2 *kN/m* uniformly distributed, in service class 1 conditions. Check that the bending strength of the beam satisfies the requirements of EC5.

prEN 338: 1991

Assume the following characteristic values for bending strength and modulus of elasticity taken from prEN 338 "Structural timber - Strength classes".

$f_{m,k} = 16\ N/mm^2 \qquad E_{0,05} = 5400\ N/mm^2$

Modification factors

For service class 1 (medium-term), $k_{mod} = 0{,}8$. If the floor beam may be assumed to be laterally restrained throughout the length of its compression edge (e.g. by floor boards) with torsional restraints at its supports (e.g. by suitable hangers) then $k_{crit} = 1{,}0$. Since the floor beams do not span more than 6m, and assuming the attached decking is continuous over at least two spans and the joints are staggered, they may be treated as a load-sharing system, hence k_{ls} may be taken as 1,1. Finally, since the beam depth is greater than 150 mm, the size factor, k_h is 1,0.

EC5: Part 1-1: 2.2.3.2

Hence, the design value of the bending strength is:

$$f_{m,d} = \frac{k_{mod}\ k_{inst}\ k_{ls}\ k_h\ f_{m,k}}{\gamma_M} = \frac{0{,}8 \cdot 1{,}0 \cdot 1{,}1 \cdot 1{,}0 \cdot 16}{1{,}3} = 10{,}8\ N/mm^2$$

The design bending stress is:

$$\sigma_{m,d} = \frac{q_d\ l^2}{8\ W} = \frac{2 \cdot 3500^2 \cdot 6}{8 \cdot 50 \cdot 200^2} = \frac{3{,}06 \cdot 10^6}{333 \cdot 10^3} = 9{,}2\ N/mm^2$$

EC5: Part 1-1: 2.3.2.1b

Thus the beam satisfies the bending requirements of EC5 as the calculated bending stress is less than the corresponding design value. It would also be necessary to check that the beam's shear stress and bearing stresses at the supports, as well as the mid-span deflection, are within EC5 limits.

EC5: Part 1-1: 7.2

It should be noted that EC5 limits the deviation from straightness measured mid-way between supports to 1/300 and 1/500 of the length of structural timber and glued laminated beams and columns, respectively. Deviations of cross-sections from target sizes are limited by tolerance class 1 in prEN 336 for structural timber and by prEN 390 for glued laminated timber.

If the floor boards cannot be relied upon to provide the necessary lateral restraints to the compression region of the beam, the bending design strength would have to be checked for possible reduction due to lateral instability.

From Equation (7), $\sigma_{m,crit}$ is

$$\sigma_{m,crit} = \frac{0,75\ Eb^2}{h\,l_{ef}} = \frac{0,75 \cdot 5400 \cdot 50^2}{200 \cdot 3500} = 14,5\ N/mm^2$$

And from Table 1, the uniform moment factor for a uniformly loaded simply supported beam is 0,88. Thus using Equation (12), the relative slenderness ratio is

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} = \sqrt{\frac{16 \cdot 0,88}{14,5}} = 0,985$$

And from Figure 2, or using Equation (10) the instability factor k_{crit} is 0,82. Thus

$$f_{m,d} = k_{crit}\ 10,8 = 0,82 \cdot 10,8 = 8,86\ N/mm^2.$$

Since the actual bending stress is 9,2 N/mm^2, the beam would have to be enlarged or lateral restraints would have to be provided.

Concluding summary

- The "simple" beam, i.e. that which deflects only in the plane of bending, represents the great majority of beams which the engineer has to design.

- The main design requirement for simple beams is to ensure that the values of the design strengths exceed the applied stress levels as obtained using the elastic theory and that the actual deflections are within EC5 limits.

- The design strength values are obtained by applying various modification and partial safety factors on the appropriate characteristic strengths.

References

Timoshenko, S. and Gere, J.M. (1961). Theory of Elastic Stability, McGraw-Hill Book Co. Inc. New York, NY., 2nd Edition.

Hooley, R.F., and Madsen, B. (1964). Lateral Stability of Glue Laminated Beams, Journal of the Structural Division, ASCE, ST3: 201 - 218.

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