Shear and torsion

STEP lecture B4
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Objectives

To explain the two phenomena shear and torsion on beams with rectangular or circular cross-sections. To present the design methods given in EC5 and the governing conditions.

Summary

The presence of vertical and horizontal shear in a horizontal beam subjected to vertical loading is stated. The shear stress distribution over the cross-section is presented and also the governing shear strength (shear parallel to the grain).

An introduction to torsional stresses caused by torsional loading is given. The governing criteria and the EC5 design method are presented. Shear stresses and torsional stresses may well occur simultaneously. The combined action which is not covered in the EC5 is, nevertheless, briefly commented upon.

Design methods are illustrated by examples.

Introduction

When bending is produced by transverse loading, shear stresses will be present according to the theory of elasticity. Shear stresses transverse to the beam axis will always be accompanied by equal shear stresses parallel to the beam axis.

In glued thin-webbed I-beams and box beams there will be shear stresses in the web (panel shear) and in the contact surface between the web and the flanges (planar shear). The planar shear strength is normally less than the panel shear strength, but either one may be critical and have to be considered. Similar considerations have to be made in the case of glued thin-flanged beams. Shear also has an effect on the buckling of the web or panel (see STEP lecture B9).

For timber (and glulam) the shear strength parallel to the grain is considerably lower than the shear strength across the grain (cutting off the fibres), thus the former is critical and has to be considered in the design of solid timber and glulam beams.

Research has indicated that the shear strength depends on the stressed volume (Barrett and Foschi, 1980), but so far a possible volume effect concerning shear has not been introduced in EC5 (see also STEP lecture B1).

This lecture only deals with solid timber and glulam beams with regard to shear.

Torsional stresses are introduced when the applied load tends to twist a member. This will occur when a beam supports a load which is applied eccentric to the principal cross sectional axis. A transmission mast may be subjected to an eccentric horizontal load, resulting in a combination of shear and torsion.

Shear

From elastic beam theory it might be recalled that the shear stress at any point in the cross section of a beam can be written, in general, as:

$$\tau_v = \frac{V\,S}{I\,b} \qquad (1)$$

where τ_v is the shear stress, V is the shear force, I is the second moment of the area about the neutral axis, b is the width of the shear plane at the level of consideration and S is the first moment of the area above the shear plane taken about the neutral axis.

For a rectangular section the maximum value is:

$$\tau_v = \frac{3\,V}{2\,A} \qquad (2)$$

The shear stress distribution is parabolic as shown in Figure 1 for a rectangular section with the maximum value at the neutral axis.

For a circular section the maximum value is:

$$\tau_v = \frac{4\,V}{3\,A} \qquad (3)$$

where A is the area of the cross-section.

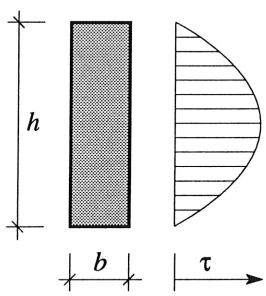

Figure 1 Shear stress distribution.

It has been found by several researchers (e.g. Keenan, 1978) that the shear stresses due to point loads near the supports are less than those calculated according to elastic beam theory. This has led to the introduction of the so called reduced shear force.

The contribution to the total shear force of a point load F within a distance $2h$ of the support can be reduced according to the influence line given in Figure 2.

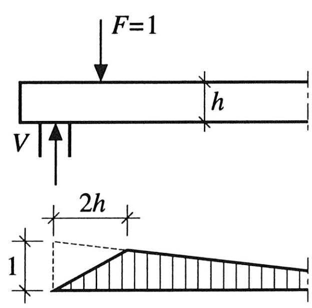

Figure 2 Reduced influence line for point loads.

EC5: Part 1-1: 5.1.7.1

The shear stress should satisfy the following condition:

$$\tau_{v,d} \le f_{v,d} \qquad (4)$$

$f_{v,d}$ is the design value of the shear strength.

Design example

Glulam beam with span l = 16 m and cross-section b x h = 190 x 655 mm with solid timber decking nailed to top suface of the beam. Strength class GL32 according to prEN 1194 "Timber structures - Glued laminated timber - Strength classes and determination of characteristic values", with loading as shown in Figure 3.

Design values for the governing load case:
Dead load: g_d = 3 kN/m (permanent)
Variable load: F_d = 20 kN (short term)

EC5: Part 1-1: 3.1.7

Service class 3: k_{mod} = 0,7 (short term)

$$2h = 2 \cdot 655 = 1310 \; mm = 1{,}31 \, m \quad say \, 1{,}3 \, m$$

Maximum V, by using the so called reduced influence line, when the one point load is placed 1,3 m to the right of the support (see Figure 3).

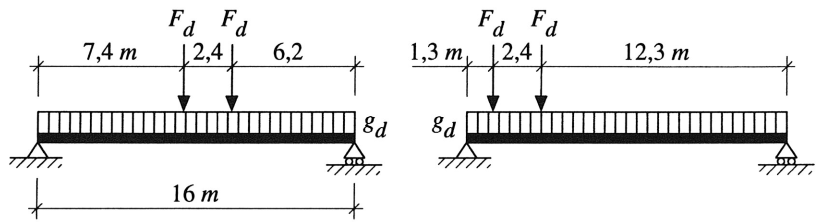

Figure 3 Critical load arrangements.

$$V_g = \frac{3 \cdot 16}{2} = 24 \, kN$$

$$V_F = \frac{20 \cdot (12{,}3 + 14{,}7)}{16} = 33{,}8 \, kN$$

$$V_d = 24 + 33{,}8 = 57{,}8 \, kN$$

Left support:

$$A_1 = \frac{20 \cdot (6{,}2 + 8{,}6)}{16} + \frac{3 \cdot 16}{2} = 42{,}5 \, kN$$

Maximum M at the point where V = 0, i.e. at a distance 7,5 m from the left support A_1.

Maximum M_d:

$$M_d = 42{,}5 \cdot 7{,}5 - \frac{3 \cdot 7{,}5^2}{2} - 20 \cdot (7{,}5 - 7{,}4) = 232 \, kNm$$

Area:

$$A = 124 \cdot 10^3 \, mm^2$$

Section modulus:

$$W = 13{,}6 \cdot 10^6 mm^3$$

$$\sigma_{m,d} = \frac{M_d}{W} = \frac{232 \cdot 10^6}{13{,}6 \cdot 10^6} = 17{,}1\,N/mm^2$$

$$\tau_{v,d} = \frac{3V}{2A} = \frac{3 \cdot 57{,}8 \cdot 10^3}{2 \cdot 124 \cdot 10^3} = 0{,}70\,N/mm^2$$

Characteristic material properties:
The characteristic values are taken from prEN 1194:

$f_{m,g,k} = 32\ N/mm^2 \quad f_{v,g,k} = 3{,}5\ N/mm^2$

The design strength values are:

$$f_{m,d} = \frac{k_{mod} f_{m,g,k}}{\gamma_M} = \frac{0{,}7 \cdot 32}{1{,}3} = 17{,}2\,N/mm^2$$

$$f_{v,d} = \frac{k_{mod} f_{v,g,k}}{\gamma_M} = \frac{0{,}7 \cdot 3{,}5}{1{,}3} = 1{,}88\,N/mm^2 \qquad (12)$$

Verification of failure condition:

$\sigma_{m,d} \leq k_{crit} \quad f_{m,d}$

EC5: Part 1-1: 5.2.2

$k_{crit} = 1{,}0$ since the beam is prevented from buckling laterally by the decking.

$17{,}1 < 17{,}2\ N/mm^2$

Verification of failure condition:

EC5: Part 1-1: 5.1.7.1

$\tau_{v,d} \leq f_{v,d}$

$0{,}70 < 1{,}88\ N/mm^2$

Torsion

According to commonly accepted elastic theory the maximum torsional stress for solid members can be written:

Circular cross-section:

$$\tau_{tor} = \frac{2T}{\pi r^3} \qquad (5)$$

where τ_{tor} is the maximum torsional stress, T is the torsional moment and r is the radius of the section.

Square/rectangular cross-section:

$$\tau_{tor} = \frac{T}{\alpha h b^2} \qquad (6)$$

where $h \geq b$ and α are numerical factors depending on the ratio h/b. Timoshenko (1955) gives the following table:

h/b	1,00	1,50	1,75	2,00	2,50	3,00	4,00	6,00	8,00	10,00	∞
α	0,208	0,231	0,239	0,246	0,258	0,267	0,282	0,299	0,307	0,313	0,333

Table 1 *α-factor.*

The torsional stress distribution along the principal axis for a rectangular section is shown in Figure 4. The maximum stress value occurs at mid point of each longer side.

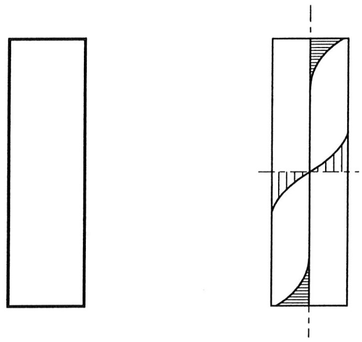

Figure 4 *Torsional stress distribution.*

EC5: Part 1-1: 5.1.8

The torsional stress shall satisfy the following condition:

$$\tau_{tor,d} \leq f_{v,d} \tag{7}$$

According to Möhler and Hemmer (1977) the above mentioned criterion is on the safe side.

Shear and torsion in combination

A combined action may occur in some cases. Little research has been carried out, and limited information is available. EC5 provides no guidence for this situation. Möhler and Hemmer (1977), however, have suggested the following governing condition:

$$\frac{\tau_{tor,d}}{f_{tor,d}} + \left(\frac{\tau_{v,d}}{f_{v,d}}\right)^2 \leq 1 \tag{8}$$

where $f_{tor,d}$ is the design torsional strength, which is considered to be different from (and higher than) the design shear strength, $f_{v,d}$.

Design example

Glulam column (pole) with cross-section b x h = 140 x 300 *mm*. Strength class GL32 according to prEN 1194.

Design values for the governing load case:

Shear force: V_d = 18 *kN* (short term)

Torsional moment: T_d = 2,4 *kNm* (short term)

EC5: Part 1-1: 3.1.7

Service class 3: k_{mod} = 0,7 (short term)

Area:

$$A = 42 \cdot 10^3 mm^2$$

$$\frac{h}{b} = \frac{300}{140} = 2{,}14 \quad \text{and } \alpha = 0{,}249 \text{ (Table 1)}$$

$$\tau_{v,d} = \frac{3V}{2A} = \frac{3 \cdot 18 \cdot 10^3}{2 \cdot 42 \cdot 10^3} = 0{,}64\,N/mm^2$$

$$\tau_{tor,d} = \frac{T_d}{\alpha h b^2} = \frac{2{,}4 \cdot 10^3 \cdot 10^3}{0{,}249 \cdot 300 \cdot 140^2} = 1{,}64\,N/mm^2$$

Characteristic material properties:
The characteristic value is taken from prEN 1194:

$f_{v,k} = 3{,}5\ N/mm^2$

The design strength value is:

$$f_{v,d} = \frac{k_{mod} f_{v,k}}{\gamma_M} = \frac{0{,}7 \cdot 3{,}5}{1{,}3} = 1{,}88\,N/mm^2$$

EC5: Part 1-1: 5.1.7.1

Verification of failure condition:

$\tau_{v,d} \leq f_{v,d}$

$0{,}64 < 1{,}88\ N/mm^2$

EC5: Part 1-1: 5.1.8

Verification of failure condition:

$\tau_{tor,d} \leq f_{v,d}$

$1{,}64 < 1{,}88\ N/mm^2$

According to Möhler and Hemmer (1977) the governing condition for the combined action is:

$$\frac{\tau_{tor,d}}{f_{tor,d}} + \left(\frac{\tau_{v,d}}{f_{v,d}}\right)^2 \leq 1$$

$$\frac{1{,}64}{1{,}88} + \left(\frac{0{,}64}{1{,}88}\right)^2 = 0{,}87 + 0{,}12 = 0{,}99 < 1$$

There is no design (or characteristic) value, f_{tor} given, and so the f_v-value is used, which is on the safe side (Möhler and Hemmer, 1977).

Concluding summary

- Shear is rarely a governing condition in beam design.
- For beams of small span-depth ratios, or subjected to concentrated loads close to the supports, the shear control might still be critical.
- It is permissible to reduce the shear force due to point loads which are located close to supports.
- For poles or masts embedded in the ground (and thus cantilevered) the shear force may be high and therefore critical.
- Concerning torsion more research is needed to confirm the torsional strength value.

- Further investigation has to be carried out in order to establish a reliable basis for design criteria in the case of combined shear and torsion.

References

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