Notched beams and holes in glulam beams

STEP lecture B5
P.J. Gustafsson
Lund University

Objectives

To develop an understanding of strength and fracture of notched beams, and beams with a hole, and to review concepts of fracture mechanics, forming the theoretical basis for the notched beam strength equation in EC 5.

Prerequisite

A4 Wood as a building material

Summary

The lecture begins with a general introduction to the performance of beams with a notch or a hole. Then, a brief review of the concepts of fracture mechanics is given. For end-notched beams a strength equation from EC 5 is included and for glulam beams with a hole an equation from literature is included. Some typical test results are indicated. Methods for reinforcement are mentioned.

Introduction

In Figure 1 beams with various types of notches or holes are shown. A notch or a hole may very significantly reduce the load bearing capacity of a beam and should preferably be avoided in design. Though not to be desired, a notch may be needed in order to bring floors to desired levels, to give clearance or to enable fit between structural members. In particular in very old timber construction, various types of notches can be observed to have been employed in the detailing of structural joints. Large holes in glulam beams can be required, for instance, for accommodating of ventilation pipes.

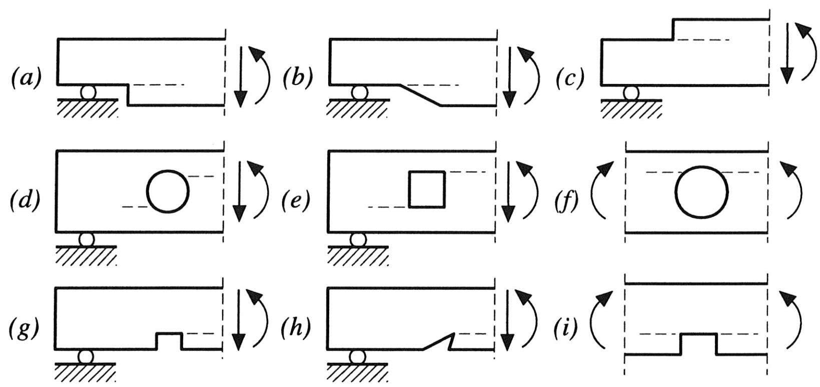

Figure 1 *Notched beams and beams with a hole. Broken line indicates probable crack propagation path.*

Fracture may develop from a notch or a hole as indicated by the broken lines in Figure 1. The fracture is often of a very sudden and brittle nature, taking place without being preceded by any large deformation or after visible warning. Depending on the geometry of the beam, the rapid crack propagation along the beam may or may not lead to a complete collapse of the beam.

The initiation of crack growth is due to perpendicular to grain tensile stress or shear stress or a combination of the two. At the tip of a notch these stresses may become very high. According to linear elastic stress analysis the stress at the tip of a sharp notch even approaches infinity, Figure 2. In such cases the magnitude of stress cannot be defined and the stress is then denoted as singular. Due to the limited strength of the material the stress at the tip of the notch does, in reality, not approach infinity. Instead, due to local damage of the material the stress distribution at the instant the crack starts to propagate may be as indicated by the broken curve in Figure 2.

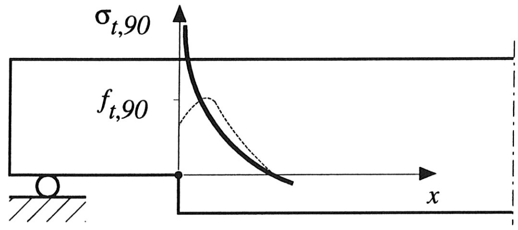

Figure 2 *Stress at the tip of a notch according to linear elastic theory and as estimated in practice, respectively.*

Drying of the wood can give a very significant addition to the local high stress and also itself cause the development of a crack at a notch or a hole. The effect of drying is twofold. As end-grain is exposed bare at a notch or hole, the rate of drying may locally become high. Moreover, the non-uniform character of the geometry at a hole or a notch adds to the magnitude of the moisture gradients and to the prevention of free shrinkage of the material. To reduce the risk of fracture caused by drying it is stated in design recommendations that end-grain surfaces at a notch or hole must be painted, or finished in some other way, so that moisture transfer is prevented. The general recommendation to avoid notches and holes is of particular relevance if the climate and relative humidity may vary.

Concepts of fracture mechanics

Background
As the very high stress is often concentrated in a very small region it is difficult, and in the case of theoretically infinite stress even meaningless, to try to determine by any conventional stress criterion the load bearing capacity of a beam with a hole or a notch. According to a conventional failure criterion the magnitude of stress in the most highly stressed point is compared to the fracture stress, i.e. the strength of the material. To determine load carrying capacity one has instead to rely either solely on tests or else, in addition to tests, on concepts of fracture mechanics other than conventional stress criteria.

Fracture mechanics - general
Fracture mechanics is a part of the science of the strength of materials. A solid body responds to extreme loading by undergoing large deformation or fracture. The phenomenon of fracture, i.e. separation, loss of contact, between parts of the body, is the topic of primary interest in fracture mechanics. From an engineering point of view, the calculation of the magnitude of load that causes fracture is of the most interest.

In cases when there is no or only minor stress concentration, e.g. in the case of a structural member in homogeneous tension or bending, the calculation of the fracture load can be carried out by a conventional stress criterion. On the other hand in the case of a very high stress concentration, e.g. at the tip of a sharp notch or crack, some other approach is needed. Then, within the framework of linear elastic theory, a rational calculation of fracture load can be based on either analysis of the *stress intensity* at the tip of the notch or else on analysis of the *energy release rate* when a crack is propagating. Although these two alternatives are formally different, they are basically quite analogous. Here only the latter approach will be further discussed.

Analysis of cracks within the framework of linear elastic theory is often called *linear elastic fracture mechanics.* By other models attempts are made to consider explicitly the non-linear performance of the material in the vicinity of the tip of the crack. This refers in particular to the fracture softening and damage that takes place in the *fracture process region* in front of the open crack. In linear elastic fracture mechanics this energy dissipating fracture process region is assumed to be very small when compared with the size of the actual structural detail and is mathematically regarded as a point, i.e. a region of zero size.

Energy release analysis - an example

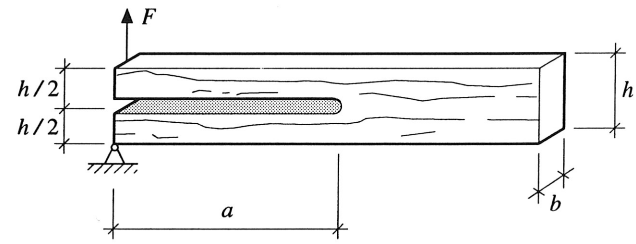

Figure 3 A specimen with an end-crack.

A beam with a considerable longitudinal crack and loaded according to Figure 3 is considered. It is assumed that stress and strain within the beam are zero when the external load, *F*, is zero. According to linear elastic theory the potential energy of the actual system, consisting of the beam and the load, is

$$W = -\frac{Fu}{2} \tag{1}$$

where *u* is the displacement of the point of loading. By elementary theory of bending of beams

$$u = \frac{2Fa^3}{3EI} \tag{2}$$

where *E* is the modulus of elasticity and $I = b(h/2)^3/12$ the second moment of area of the cross-section of each cantilever part. With *u* from Equation (2), Equation (1) gives

$$W = \frac{-F^2 a^3}{3EI} \tag{3}$$

The change of the potential energy, *dW*, during a small propagation, *da*, of the crack is then obtained by derivation:

$$dW = \frac{-F^2 a^2}{EI} da \tag{4}$$

This decrease of the potential energy corresponds to a positive energy release, - *dW*, and to a simultaneous increase of the fractured area by *b d a*. The energy release, -*dW*, per fracture area, *b d a*, is usually denoted *G* (after A.A. Griffith, who in the 1920s presented pioneering works on fracture mechanics)

$$G = \frac{-dW}{bda} = \frac{F^2a^2}{bEI} \qquad (5)$$

When the load F is so large that the crack starts to propagate, G has reached its critical value, G_c. This value corresponds to the energy dissipating ability of the material and is regarded as a material property. For European softwoods, depending on the density of the wood, G_c is roughly in the order of 150 - 600 J/m^2 for perpendicular to grain tensile fracture (Larsen and Gustafsson, 1990). Thus, the fracture criterion is

$$G = G_c \qquad (6)$$

which together with the expression for G, Equation (5), gives the fracture load F_c:

$$F_c = \frac{\sqrt{G_c bEI}}{a} = \frac{\sqrt{EG_c}}{\sqrt{h}}\frac{bh^2}{\sqrt{96}a} \qquad (7)$$

In this equation two general and important principles should be noted:

a) The material properties that are decisive for resistance to crack propagation are stiffness, here denoted by E, and fracture energy, here denoted by G_c. The perpendicular to grain tensile strength of the material is not predicted to influence F_c, at least not directly.

b) The load bearing capacity, is strongly size-dependent in the sense that the magnitude of some formal stress at failure, e.g. $F_c/(bh)$, decreases if the absolute size of the specimen is increased.

In the above example it has tacitly been assumed, by using Equation (2), that the specimen is slender, i.e. that ratio h/a is small. The above method of calculation can be applied to other cracked geometries. Then Equation (2) must of course be replaced by an equation relevant to the compliance of the actual geometry.

End-notched beams; theoretical and experimental results

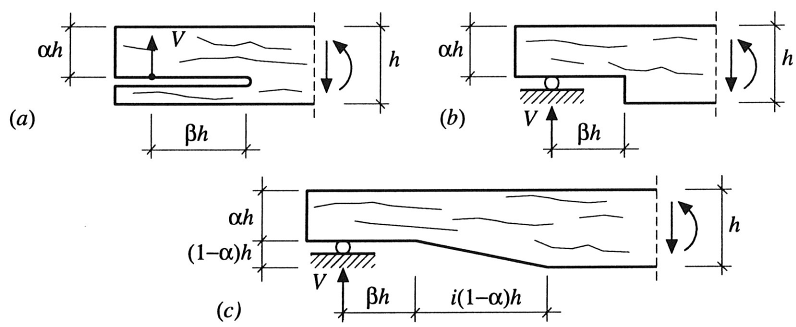

Figure 4 Geometry of end-notched beams.

By the above method of theoretical analysis, for a beam loaded and with an end-crack according to Figure 4a) (Gustafsson, 1988) the load at crack propagation is

$$V_c = \frac{b\alpha h\sqrt{G_c/h}}{\sqrt{0{,}6(\alpha-\alpha^2)/G_v} + \beta\sqrt{6(1/\alpha-\alpha^2)/E_o}} \qquad (8)$$

α and β are geometric ratios as defined in the figure. G_v and E_o are the modulus of shear stiffness and the modulus of elasticity parallel to grain, respectively. The same equation is valid for a square shaped end-notch, Figure 4b), and also for the various types of notch indicated in Figures 1g), h) and i). For small notch depth, i.e. for α close to 1,0, the resistance to notch failure is high. In that case also the risks for ordinary shear failure and bending failure of the net cross-section, αhb, must be considered.

In Table 1 a few examples of various experimental results regarding short term load bearing capacity are given. (From compilation of literature and tests: (Gustafsson, 1988) and (Riberholt et al., 1991).) The values indicated are mean values and they were obtained for dried timber with a homogeneous moisture content. The coefficient of variation for a test series is typically in the order of 20 %. Table 1 aims to illustrate the actual and low strength of notched beams even at the current favourable conditions and how various parameters influence. It is interesting that the mean value as well as the 5-percentile value of V_c has been found to be higher for specimens with a knot in the vicinity of the notch than for those without any knot (Larsen and Riberholt, 1972) and (Möhler and Mistler, 1978).

Material	*h, mm*	*b, mm*	α	β	*i*	$3V_c/(2b\alpha h)$, N/mm^2
Pine	50	44	0,75	0,5	0	4,04
	200					1,91
Glulam	300	90	0,50	0,15	0	2,16
	567	160				1,41
Spruce	120	32	0,83	0,25	0	2,90
			0,75			2,52
			0,50			2,39
			0,33			2,22
Glulam	600	100	0,92	0,42	0	3,00
			0,75			1,32
			0,50			1,13
Spruce	95	45	0,75	0,33	0	3,33
				0,66		2,94
Glulam	305	79	0,70	2,5	0	0,69
				5,5		0,36
Spruce	95	45	0,75	0,33	0	3,33
					1	3,44
					3	4,71
Glulam	300	90	0,50	0,15	0	2,16
					2	2,76
					8	4,16

Table 1 Test results. Strength (mean value) of various end-notched beams.

EC5 equation for end-notched beams and its background

Before the development of the EC5 equation, a few simplifying modifications of Equation (8) were made (Larsen, 1992). The ratio E_0/G_v was throughout set equal to 16. Secondly, the introduction of the "new" material parameter G_c was avoided by assuming that $\sqrt{E_o\, G_v}$ is proportional to the shear strength of the material, f_v. The constant of proportionality was found from test results. Solid wood and glulam were assigned somewhat different constants. Moreover, from test results (Riberholt et al., 1991) a factor that considers the effect of a taper, Figure 4c), was developed.

EC5: part 1-1:5.1.7.2

From the above the risk of crack propagation from a notch is taken into account in EC5 through a formal reduction by a factor k_v of the design shear strength, $f_{v,d}$, of the net cross-section $b\alpha h$:

$$\tau_d = \frac{3V_d}{2b\alpha h} \leq k_v f_{v,d} \qquad (9)$$

where the reduction factor k_v ($\leq 1{,}0$) is

$$k_v = \frac{k_n(1+1{,}1\,i^{1{,}5}/\sqrt{h})}{\sqrt{h}(\sqrt{\alpha-\alpha^2}+0{,}8\,\beta\sqrt{1/\alpha-\alpha^2})} \qquad (10)$$

For solid timber k_n is set equal to 5,0 and for glulam k_n is set equal to 6,5. Note that the beam depth, h, must be in *mm*. To avoid the risk of the development of another mode of failure, shear failure in the net cross-section, a value of k_v greater than 1,0 may not be used in Equation (9). If the notch is located on the compression side of the beam, Figure 1c), k_v may be set equal to 1,0. Figure 5 illustrates how k_v is affected by α, β, i and h.

The EC5 equation refers to beams of structural size. For very small members the non-zero size of the fracture process region can be of importance. To consider this in an approximate manner the distance β h may in the calculations be assigned a somewhat increased value, e.g. increased by 10 *mm*. Such a consideration is of particular significance if parameters α, β and h are all small.

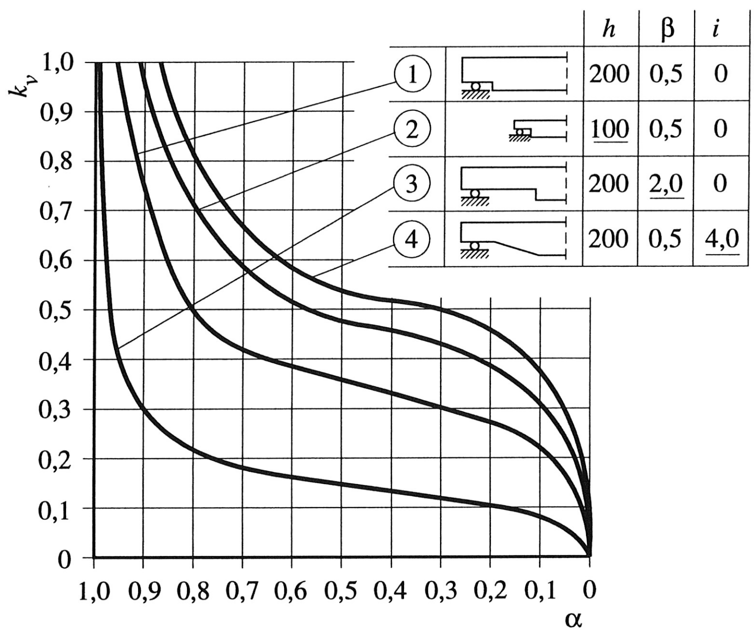

Figure 5 Factor k_v versus α for solid timber beams with various h, β and i.

Glulam beams with a hole

In EC5 no strength equation is included for beams with a hole. Therefore, the design of such a beam will involve specific consideration in each case. In the following some guidance for a preliminary estimate of the effect of a hole is given.

In design recommendations the risk of crack development as shown in Figure 1 d) and e) is often considered by a reduction analogous to Equation (9) of the design shear strength:

$$\tau_d = \frac{3V_d}{2b\alpha h} \le k_{hol} f_{v,d} \tag{11}$$

Various proposals for the calculation of the reduction factor k_{hole} can be found. Based on test results, according to a Swedish glulam design manual (Carling and Johannesson, 1988):

$$\text{For} \quad D/h \le 0{,}1 : k_{hol} = 1 - 555(D/h)^3 \tag{12a}$$

$$\text{For} \quad D/h > 0{,}1 : k_{hol} = 1{,}62/(1{,}8 + D/h)^2 \tag{12b}$$

For circular holes D represents the diameter of the hole and for square and rectangular holes D is the length of the diagonal. The hole is assumed to be placed symmetrically with respect to the depth of the beam. Corners of the hole must be rounded with a radius of curvature ≥ 25 *mm*, the side length ratio of rectangular holes may not be greater than 3,0 and α, see Figure 6, may not be less than 0,5. Moreover, measures to reduce moisture variations in the timber are required.

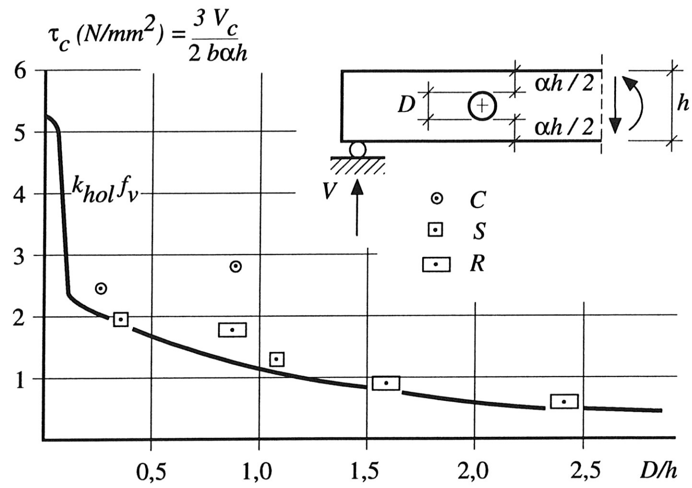

Figure 6 *Nominal shear stress at cracking, τ_c, versus hole size ratio D/h. For the square and rectangular holes D is the diagonal.*

Equation (12) is based on several sets of tests of glulam beams of one size: $b \approx 90$ *mm* and $h \approx 500$ *mm* (Johannesson, 1983). According to the actual design recommendation, if $b > 90$ *mm* k_{hol} shall be multiplied by the further reduction factor $(90/b)^{0,2}$ before inserted in Equation (11).

In Equation (12) there is no explicit consideration of bending. In the case of pure bending, cracks have been found to develop as shown in Figure 1f). According to the actual recommendation, if less than 8 lamination boards remain in the net cross-section, the design bending strength shall be reduced by 25 %.

In Figure 6 a set of the shear loading test results is shown. The centre of each mark represents the mean value of 4 tests. Maximum and minimum of the individual values are indicated by the vertical bars. The quality of glulam tested was for *D/h* = 0 estimated to have mean shear strength f_v = 5,2 N/mm^2. The corners of square and rectangular holes were rounded to r = 25 *mm*, the beam size was 88 x 495 mm^2, the distance from support to centre of the hole was 1250 *mm* and the side length ratio of the rectangular holes was 3,0. To be perfect the curve in Figure 6, representing Equation (12), should coincide with the mean value marks. Current deviations are on the safe side.

Methods of reinforcement

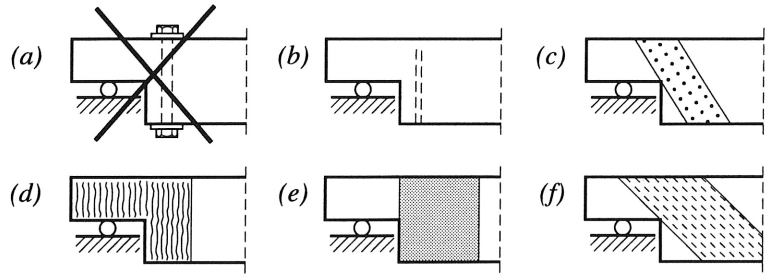

Figure 7 *Methods of reinforcement.*

Some conceivable methods of reinforcement at an end-notch are shown in Figure 7 (Möhler and Mistler, 1978). Principles of reinforcement to prevent failure at a hole are similar. Depending on the stiffness of the reinforcement and its attachment to the glulam, the reinforcement may act together with the wood and significantly increase the load at cracking, or else the reinforcement may become active only when the wood has cracked.

Using a bolt as indicated in Figure 7a) it is normally required that the screw nut must be re-tightened to avoid loss of stress due to creep and shrinkage. The method according to Figure 7a) should in general be avoided. With a rod, bolt or screw glued into a drilled hole, Figure 7b), a reinforcement with high stiffness is achieved. It must be noted that this arrangement prevents shrinkage of the wood and may therefore cause cracking if the wood is dried. A steel plate nailed to the wood, Figure 7c), may not be expected to have any very significant effect until the wood has cracked. Gluing and nailing plywood to the sides of the beam, Figure 7d) and e), is from the technical point of view probably one of the best ways to reinforce glulam at a hole or a notch. The glue is normally designed to be the active part, the main purpose of the nails is to give pressure during hardening of the glue. Glass-fibre reinforcement, Figure 7e) and f), acts in a similar way (Larsen et al., 1994). An advantage of glass-fibre reinforcement is its appearance: it is transparent and looks like a thick lacquer. A disadvantage is that practical experience is as yet very limited.

Other strength and safety improving measures are by tapering, Figure 4c), and rounding off. To achieve a proper effect by rounding off a notch, the radius of curvature must be large, say at least 25 *mm*.

Concluding summary

- Notches and holes should preferably be avoided. They often give locally very high perpendicular to grain tensile and shear stress that may cause crack propagation. The fracture can be very sudden and rapid. Moisture change increases the risk of fracture.

- In the case of very local and concentrated stressing conventional stress criteria are not applicable. Rational analysis can instead be carried out by fracture mechanics. Such an analysis based on energy release considerations shows that there is a size-effect in the strength. Moreover, the stiffness and fracture energy, together forming the fracture toughness, are found to be the decisive material properties.

- For end-notched beams a strength equation is included in EC5. For simplicity it assumes that the fracture toughness of the material is proportional to its shear strength.

- Beams with a hole or a notch can be reinforced by a bolt tightened by a nut, by a rod or a screw glued into the beam, or by nailing or gluing a steel plate, plywood or a layer of glass-fibre to the sides of the beam.

References

Carling, O. and Johannesson, B. (1988). Limträhandboken (Glulam manual). Svenskt limträ, Stockholm.

Gustafsson, P.J. (1988). A study of strength of notched beams. In: Proc. of CIB-W18A Meeting 21, Parksville, Canada, Paper 21-10-1.

Johannesson, B. (1983). Design problems for glulam beams with holes. Thesis, Chalmers University of Technology, Sweden, 73 pp., ISBN 91-7032-2.

Larsen, H.J. and Riberholt, H. (1972). Tests with not classified structural timber. Rapport nr R 31 (in Danish), Technical University of Denmark.

Larsen, H.J. and Gustafsson, P.J. (1990). The fracture energy of wood in tension perpendicular to the grain - results from a joint testing project. In: Proc. of CIB-W18A Meeting 23, Lisbon, Portugal, Paper 23-19-2.

Larsen, H.J. (1992). Latest development of Eurocode 5. In: Proc. of CIB W18A Meeting 25, Åhus, Sweden, Paper 25-102-1.

Larsen, H.J., Gustafsson, P.J. and Traberg, S. (1994). Glass fibre reinforcement perpendicular to grain. In: Proc. of the Pacific Timber Eng. Conf. 1994, Surfers Paradise, Australia.

Möhler, K. and Mistler, H.-L. (1978). Untersuchungen über den Einfluß von Ausklinkungen im Auflagerbereich von Holzbiegeträgern auf die Tragfestigkeit. Report, Lehrstuhl für Ingenieurholzbau und Baukonstruktionen, Universität Karlsruhe, Germany.

Riberholt, H., Enquist, B., Gustafsson, P.J. and Jensen, R.B. (1991). Timber beams notched at the support. Report TVSM-7071, Lund University, Sweden.