Columns

STEP lecture B6
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Objectives

To develop an understanding of the phenomenon of in-plane buckling, to identify the governing parameters and to present the procedures of EC5 as a design method.

Prerequisites

A7 Solid timber - Strength classes
A8 Glued laminated timber - Production and strength classes

Summary

The lecture begins with a non-mathematical introduction of flexural buckling. It presents the principal factors influencing the stability of columns and shows how the buckling curves in EC5 have been derived. A practical example of the design of an eccentrically loaded column complements the lecture.

Introduction

When a slender column is loaded axially, there exists a tendency for it to deflect sideways. This type of instability is called flexural buckling. The strength of slender members depends not only on the strength of the material but also on the stiffness, in the case of timber columns mainly on the bending stiffness. Therefore, apart from the compression and bending strength, the modulus of elasticity is an important material property influencing the load-bearing capacity of slender columns. The additional bending stresses caused by lateral deflections are taken into account in a stability design.

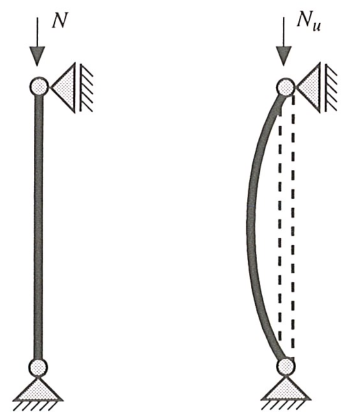

Figure 1 *Two-hinged column.*

There are two principal ways to design a compression member: the first involves a second order analysis whereby the equilibrium of moments and forces is calculated by considering the deformed shape of the respective member or structure. The second approach uses buckling curves to account for the decrease in strength of a real column compared to a compression member which is infinitely stiff in bending. Here, the stability design is carried out as a compression design with modified compression strength. The decrease in load-bearing capacity depends on the slenderness ratio of the member in question and is based on the behaviour of a two-hinged column (see Figure 1). For single members or compression members forming part of a framework, this method can be used by first determining the respective buckling (effective) length (see STEP lecture B7) and subsequently treating the structure as a two-hinged column of the same length. This lecture only deals with column design based on buckling curves.

Factors influencing column strength

The factors influencing the load-bearing capacity of a timber column may be divided into two groups. The first group involves the nominal geometry of the compression member such as its cross-section and length, its support conditions and the material properties which are determined by the choice of the strength class, the surrounding climate and the load duration class of the governing load case. The factors belonging to this first group are either determined by or known to the design engineer. The engineer is able to influence the load-bearing capacity and hence meet the design requirements by adjusting these factors.

A second group of factors also influencing column strength involves geometric and material imperfections and variations. Since real structures are never perfect, these factors have to be considered during the design of columns. However, because the design engineer in general has no information regarding these factors, their influence has to be taken into account implicitly. The influence of these factors on the load-bearing capacity of timber columns is included in the design rules in EC5.

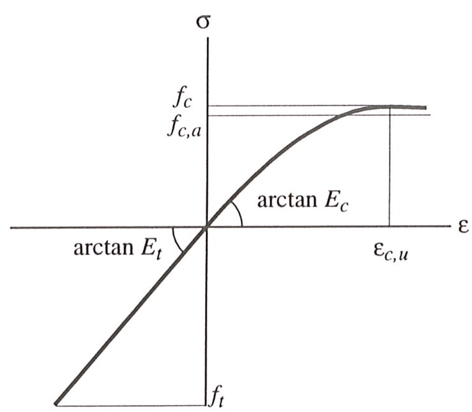

Figure 2 *Stress-strain curve of timber according to Glos (1978). $f_{c,a}$ is the asymptotic value of compression strength and $\varepsilon_{c,u}$ is the compressive strain at failure.*

EC5: Part 1-1: 7.2

The most important geometric imperfections of timber compression members are initial curvature, inclination of the member axis and deviations of cross-sectional dimensions from the nominal values. Deviation from straightness is limited to 1/500 of the length for glued laminated members and to 1/300 of the length for structural timber. Deviations in cross-sectional dimensions from target sizes are limited by values for tolerance class 1 in prEN 336 "Structural timber. Coniferous and poplar - timber sizes - permissible deviations" for structural timber and by prEN 390 "Glued laminated timber. Sizes. Permissible deviations" for glued laminated timber.

Material imperfections include growth characteristics and other factors which influence the stress-strain behaviour of timber. Generally, the stress-strain curve is linear elastic until failure, for timber subjected to tensile stresses, and non-linear with considerable plastic deformations, under compression stresses (see Figure 2). The shape of the stress-strain curve of European softwoods depends mainly on the following properties (Glos, 1978): density, knot size (knot area ratio), content of compression wood and moisture content. Glos (1978) derived relationships between these properties and the shape of the stress-strain curve for both sawn timber and laminations for glued laminated timber. From a knowledge of the density, the knot size, the content of compression wood and the moisture content it is possible to calculate the stress-strain curve for a piece of timber.

Background to the buckling curves of Eurocode 5

Buckling curves generally describe the influence of slenderness on the characteristic load-bearing capacity of two-hinged columns. Each value on a buckling curve consequently represents the characteristic load-bearing capacity of columns with the corresponding slenderness ratio. The slenderness ratio is defined as the largest ratio of the unbraced length to the radius of gyration. There are several possibilities for deriving characteristic column strength values. One possibility is to determine characteristic values R_k from tests. However, because of the vast amount of necessary tests, this procedure is too expensive to be justified.

EC5: Part 1-1: 2.2.3.2

To derive the buckling curves for EC5 a different method was chosen (Blass, 1986; Blass, 1987 and Blass, 1988). This method is based on the simulation of tests by computer. Here, columns are modelled by assigning them material properties and geometric imperfections based on observations of real columns. This means that strength and stiffness values as well as initial curvature or deviations from target sizes are chosen randomly for a certain column. Of course, the assigned properties have to be realistic and the correlation between the different properties has to be taken into account during the simulation process. Like a real column, a simulated column is then characterised by a set of properties determining its load-carrying capacity.

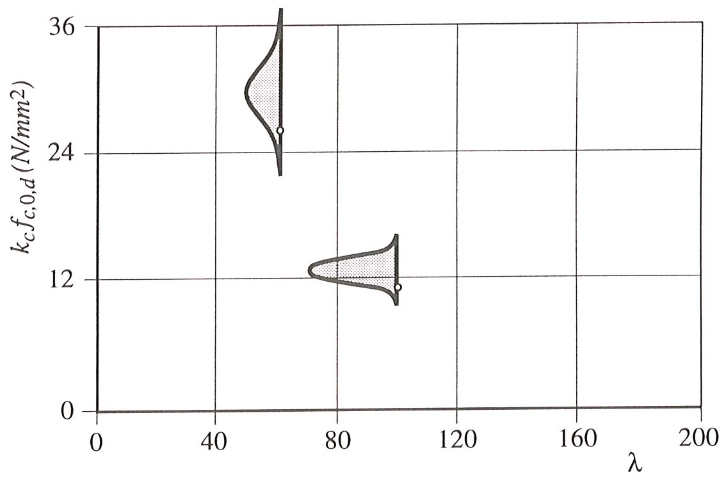

Figure 3 *Distribution of buckling strength and characteristic values for two slenderness ratios.*

Simulating a large number of columns of the same slenderness ratio and strength class, and subsequently calculating their ultimate loads, results in a distribution of ultimate load values. The variation in the resulting column strength values is determined by the variation in strength and stiffness properties as well as the geometric imperfections. From the distribution of ultimate load values, the 5-percentile as the characteristic value is determined. This characteristic value then represents one point on the buckling curve (see Figure 3).

Such simulations and ultimate load calculations may be performed for a range of slenderness ratios, resulting in a series of characteristic load-carrying capacities, or buckling strengths. Characteristic buckling strengths for a range of slenderness ratios, obtained by such simulations, are shown in Figure 4. Since a diagram is more difficult to manipulate mathematically than an equation, approximate curves have been fitted to a series of buckling simulations like those shown in Figure 4. The form of the equations corresponds to those used in Eurocode 3 for the design of steel columns. Figure 5 shows an example of a series of characteristic buckling strengths determined by simulation, together with the corresponding fitted curve.

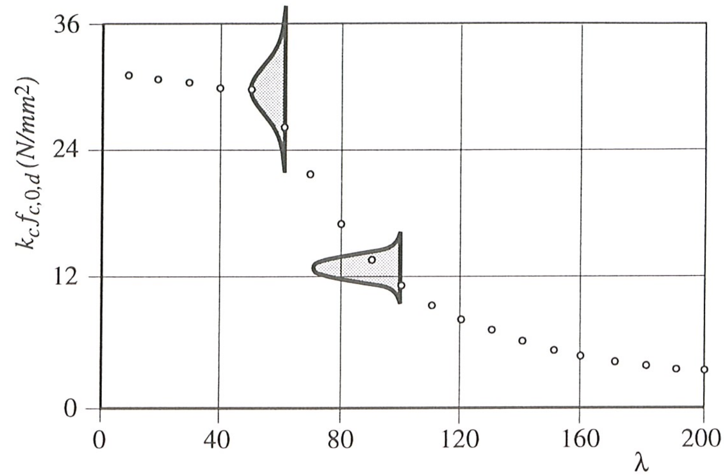

Figure 4 Characteristic buckling strength values for different slenderness ratios.

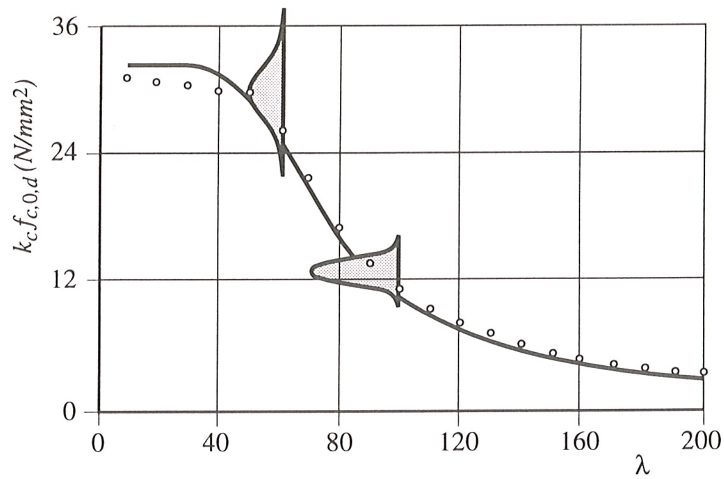

Figure 5 Approximate curve fitted to the characteristic buckling strength values.

The calculation of the ultimate loads of the simulated columns is based on a second order plastic analysis, using the plastic deformation potential of timber subjected to compressive stresses. This method - although requiring a comparatively long calculation time, caused by the necessary iteration procedures - leads to higher ultimate loads compared with results based on an elastic solution, where the ultimate load is defined as reaching the material strength in the most stressed fibre in the critical cross-section. The plastic approach results in an increase in the performance under combined axial compression forces and bending moments.

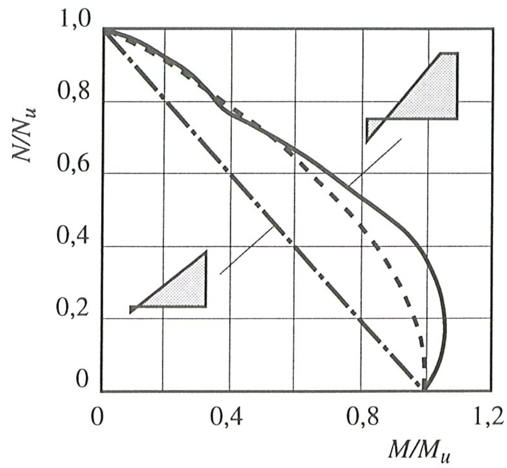

Figure 6 Bending moment-axial force interaction curves.

EC5: Part 1-1: 5.1.10

Figure 6 shows a bending moment-axial force interaction diagram of a rectangular cross-section, where the linear interaction represents elastic behaviour and the solid curve the characteristic strength of the cross-section when the plastic behaviour of the timber is considered. The dashed line shows the EC5 design rule for combined uni-axial bending and axial compression when no instability condition is to be considered or when the internal forces and moments have been determined using a second order analysis.

For members under combined compression and bending, which are able to deflect sideways, the interaction curve changes from the shape shown in Figure 6 for very stocky members into a nearly linear interaction for members with a high slenderness ratio. For the design of stocky members, the dashed line is valid for slenderness ratios λ up to about 30 (corresponding to λ_{rel} = 0,5) and a simple linear interaction was chosen for all members with a slenderness ratio exceeding this threshold value.

Buckling curves

EC5: Part 1-1: 5.2.1

In the following, the buckling curves of EC5 are presented.

The relative slenderness ratios are defined by:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{c,crit,y}}} \tag{1}$$

and

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{c,crit,z}}} \tag{2}$$

where

$$\sigma_{c,crit,y} = \pi^2 \frac{E_{0,05}}{\lambda_y^2} \tag{3}$$

$$\sigma_{c,crit,z} = \pi^2 \frac{E_{0,05}}{\lambda_z^2} \tag{4}$$

λ_y and $\lambda_{rel,y}$ correspond to bending about the y-axis (deflection in the z-direction), λ_z and $\lambda_{rel,z}$ correspond to bending about the z-axis (deflection in the y-direction).

For both $\lambda_{rel,y} \leq 0,5$ and $\lambda_{rel,z} \leq 0,5$ the stresses should satisfy the following conditions:

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (5)$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (6)$$

where $\sigma_{c,0,d}$ is the design compressive stress and $f_{c,0,d}$ is the design compressive strength. $\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the respective design bending stresses and $f_{m,y,d}$ and $f_{m,z,d}$ the design bending strengths. k_m is 0,7 for rectangular sections (see STEP lecture B3) and 1,0 for other cross-sections.

In all other cases the stresses should satisfy the following conditions:

$$\frac{\sigma_{c,0,d}}{k_{c,z}\, f_{c,0,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \qquad (7)$$

$$\frac{\sigma_{c,0,d}}{k_{c,y}\, f_{c,0,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \qquad (8)$$

where the symbols are defined as follows:

σ_m bending stress due to any lateral loads

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} \qquad \text{(similarly for } k_{c,z}\text{)} \quad (9)$$

$$k_y = 0,5\ (1 + \beta_c\ (\lambda_{rel,y} - 0,5) + \lambda_{rel,y}^2) \qquad \text{(similarly for } k_z\text{)} \quad (10)$$

β_c is a factor for members within the straightness limits mentioned above:

- for solid timber: $\beta_c = 0,2$
- for glued laminated timber: $\beta_c = 0,1$

The difference between solid timber and glued laminated timber is mainly caused by the smaller initial curvature of glued laminated timber members and their smaller deviations from target sizes. Moreover, the mean value, as well as the variation of the moisture content, is lower in glued laminated timber columns compared with solid timber columns. A higher moisture content causes a decrease in compression strength of the timber and consequently a decrease in column strength for low and medium slenderness ratios whereas the modulus of elasticity, which mainly determines the load-bearing capacity of slender columns, is hardly affected by a change in moisture content.

Design example

Timber column with square cross-section 200 x 200 *mm*, buckling length l = 4,0 *m*. Strength class C24 according to prEN 338 "Structural timber. Strength classes".

Design values of permanent and variable load for the governing load case:
permanent load: $G_d = 162\ kN$ (axial load, permanent)
variable load: $Q_d = 5{,}25\ kN/m$ (line load, short-term)

EC5: Part 1-1: 3.1.7

Service class 1: $k_{mod} = 0{,}9$

Design compressive stress:

$$\sigma_{c,0,d} = \frac{N_d}{A} = \frac{162 \cdot 10^3}{40 \cdot 10^3} = 4{,}05\ N/mm^2$$

Design bending stress:

$$\sigma_{m,d} = \frac{Q_d\ l^2}{8\ W} = \frac{10{,}5 \cdot 10^6}{1{,}33 \cdot 10^6} = 7{,}88\ N/mm^2$$

Characteristic material properties:

prEN 338: 1991

The characteristic values of bending and compression strength as well as the modulus of elasticity are taken from prEN 338 "Structural timber - Strength classes". For the modulus of elasticity, the 5-percentile value is used in the design since an ultimate limit state is considered.

$f_{m,k} = 24\ N/mm^2$ $f_{c,0,k} = 21\ N/mm^2$ $E_{0,05} = 7400\ N/mm^2$

EC5: Part 1-1: 2.2.3.2

The design values of the bending and compression strength are:

$$f_{m,d} = \frac{k_{mod}\ f_{m,k}}{\gamma_M} = \frac{0{,}9 \cdot 24}{1{,}3} = 16{,}6\ N/mm^2$$

$$f_{c,0,d} = \frac{k_{mod}\ f_{c,0,k}}{\gamma_M} = \frac{0{,}9 \cdot 21}{1{,}3} = 14{,}5\ N/mm^2$$

$\beta_c = 0{,}2$ (solid timber)

The design value of the member buckling resistance is calculated using the buckling curves for solid timber:

$$\sigma_{c,crit} = \pi^2 \frac{E_{0,05}}{\lambda_y^2} = \pi^2 \frac{7400}{69^2} = 15{,}3\ N/mm^2$$

$$\lambda_{rel} = \sqrt{\frac{f_{c,0,k}}{\sigma_{c,crit}}} = \sqrt{\frac{21}{15{,}3}} = 1{,}17$$

$$k = 0{,}5\ (1 + \beta_c\ (\lambda_{rel} - 0{,}5) + \lambda_{rel}^2) = 1{,}25$$

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}} = \frac{1}{1{,}25 + \sqrt{1{,}25^2 - 1{,}17^2}} = 0{,}59$$

$$k_c\ f_{c,0,d} = 0{,}59 \cdot 14{,}5 = 8{,}56\ N/mm^2$$

EC5: Part 1-1: 2.3.2.1b

Verification of failure condition:

$$\frac{4,05}{8,56} + \frac{7,88}{16,6} = 0,95 < 1$$

Concluding summary

- Timber columns that are not adequately restrained along their length are subject to flexural buckling.

- Buckling length, slenderness ratio, compression strength and modulus of elasticity as well as geometric and structural imperfections are the primary influences on buckling resistance.

- The buckling curves of EC5 are based on a second order analysis where the plastic behaviour of timber under compression stress was taken into account.

- The design of columns with $\lambda > 30$ and subjected to bending stresses due to lateral loads and eccentric axial load is based on a linear interaction of buckling strength and bending strength.

References

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