Glued thin-webbed beams

STEP lecture B9
K.H. Solli
Norwegian Institute
of Wood Technology

Objectives

To introduce glued thin-webbed I-beams and box beams and to explain the background to the design method given in EC5.

Prerequisites

A4 Wood as a building material
A10 Wood-based panels - Plywood
A11 Wood-based panels - Fibre board, particle board and OSB
A17 Serviceability limit states - Deformations

Summary

The lecture starts with a general description of a thin-webbed beam. It then covers the necessary design controls based on EC5 and provides a brief theoretical background. A design example is given.

Introduction

A glued thin-webbed beam comprises three main parts as follows:

- flanges,
- web,
- and glued joints between flanges and web.

The flanges are often made of finger jointed structural timber, but they can also be made of other materials such as glued laminated timber or laminated veneer lumber (LVL). The main purpose of the flanges is to carry the stresses caused by bending moments and axial forces. Since the flanges normally have small dimensions it is important that the material has few and small defects.

The web (or webs) are made of different wood-based panel materials such as plywood, particleboard, fibreboard etc. The main purpose is to carry the stresses from shear forces. For long beams it may be necessary to have joints in the web. If the web joints are put in regions with low shear force they can be made as butt joints. If not it will be necessary to reinforce the web joint. It may also be necessary to reinforce the web at the supports. The reinforcement can be made with gusset plates of wood-based panels which are nailed or glued to the web. The reinforced web at joints and supports must be designed to accommodate the actual shear forces.

Production

Glued thin-webbed beams are normally produced in an industrial process. To achieve an adequate glued connection between the web and the flanges it is important that the temperature is correct (see STEP lecture A12). It is also important that the faces of the flanges have been planed and cleared just before gluing and that the moisture content in both the flange and web materials is under control.

Use of glued thin-webbed beams

Glued thin-webbed beams have a high load capacity and stiffness compared with their weight. This make them easy to handle. They can also be easily modified by hand tools. Such beams can be used principally in the same places as solid timber. For floor and roof constructions where it is difficult to obtain large enough sections of solid timber and where glued laminated timber might be too expensive (i.e. for a span of 5 - 8 *m*) glued thin-webbed beams often are used.

When the glued thin-webbed beams are used as members in floor, roof and wall constructions the depth of the beam might become quite big (300 - 500 *mm*). This makes it easy to accommodate different types of technical equipment. The depth will also give room for enough insulation material where this is required. In countries with cold winters the dimension of the studs are defined by the demand of insulation thickness. By using a glued thin-webbed profile it is possible to optimize the material consumed.

The use in service class 3 might be limited because of the web material's restrictions for use in this class.

Special aspects of production and transport

The stiffness about the z-axis is very low compared with the y-axis. This must be considered during the production and all transportation phases from factory to the building site. The web materials are in addition very sensitive to damage caused by transportation and handling.

The beams must be kept under dry conditions during the building period. If the moisture content in the web becomes too high, the risk of getting non-elastic deformations in the final construction is high.

Lateral stability

Flanges which carry compression stresses must be supported to prevent lateral deflection and buckling. When the beams are used in floor construction as simply supported beams the connection between the compression flange and the floor often will be sufficient to avoid lateral instability. Care must be taken where the compression stress changes from one flange to the other, as for example at the intermediate support of a continuous beam.

Effective values for the cross-section

It is a presumption in the calculations shown later that the web and flanges are glued together to form a structural unit. It is also assumed that the variation of strain over the depth of the beam is linear. Based on Hooke's law the stress at a certain point can be expressed by the product of the strain and the modulus of elasticity. A beam may be built up with materials which have different moduli of elasticity leading to different stresses at the same depth. Figure 1 shows an example of how the stresses might vary for such a beam profile subjected to a bending moment.

Since the moduli of elasticity are different over the cross-section, it is common practice to calculate so-called effective values for the cross-section. This can be done by regarding the whole profile as one homogeneous material with the same properties as the flange material. The contribution from the web must then be reduced in proportion to the ratio of the moduli of elasticity.

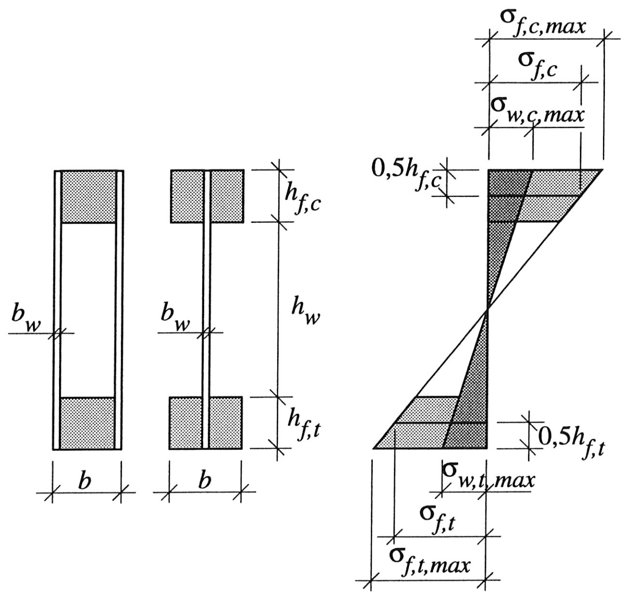

Figure 1 Example of stresses in glued I- and box beams.

Effective area:

$$A_{ef} = A_f + \left(\frac{E_w}{E_f}\right)\left(\frac{1+k_{def,f}}{1+k_{def,w}}\right)A_w \tag{1}$$

Effective second moment of area:

$$I_{ef} = I_f + \left(\frac{E_w}{E_f}\right)\left(\frac{1+k_{def,f}}{1+k_{def,w}}\right)I_w \tag{2}$$

Because the distribution of the stresses within a composite cross-section changes over time due to the different creep behaviour of the components, the stresses have to be calculated at instantaneous and at final deformation.

Control of the stresses in the flanges

In a beam profile carrying a bending moment most of the stresses in the flanges are caused by axial compressive or tensile stresses. The portion of pure bending will be quite small. For a symmetrical profile carrying only bending moment the stresses in the compression and tensile flanges will have the same absolute values. If the beam in addition to the bending moment also carries axial compression or tensile actions, the flange stresses shall be calculated as the sum of stresses from the moment and from the axial forces.

Flanges

The maximum stress in the extreme fibres of the compression flange is given by the following equation:

$$\sigma_{f,c,max,d} = \left(\frac{M_d}{I_{ef}}y_0\right) + \left(\frac{F_d}{A_{ef}}\right) \tag{3}$$

The axial stress at the centre of gravity of the compression flange is given by the following equation:

$$\sigma_{f,c,d} = \left(\frac{M_d}{I_{ef}}y_c\right) + \left(\frac{F_d}{A_{ef}}\right) \tag{4}$$

where

M_d is the design value of the bending moment,

y_0 is the distance between the neutral axis of the beam and the ultimate fibres of the flange. For symmetrical cross-sections $y_0 = h/2$ when h is the depth of the beam,

F_d is the axial load (in addition to the bending moment), can be compressive or tensile, and

y_c is the distance between the neutral axis of the beam and the centre of gravity in the compression flange.

EC5: Part 1-1: 5.3.1

When the actual stresses are calculated they must be compared with the design strength values of the flange:

$$\sigma_{f,c,max,d} \leq f_{m,d} \tag{5}$$

$$\sigma_{f,c,d} \leq k_c f_{c,0,d} \tag{6}$$

Where k_c is a factor which takes into account lateral instability. The factor k_c may be determined (conservatively, especially for box beams) according to EC5, 5.2.1 with

EC5: Part 1-1: 5.3.1(3)

$$\lambda = \frac{\sqrt{12}\, l_c}{b} \tag{7}$$

l_c is the distance between the sections where lateral deflection of the compression flange is prevented, and b is the thickness of the flange.

The stresses in the tensile flanges are calculated accordingly.

Control of axial stresses in the web

The main purpose of the web is to carry the stresses from shear forces but the web will also have to take some of the stresses caused by the bending moment and axial loads. Therefore the web capacity must also be controlled in accordance with these stresses. Since the strain variation is assumed to be linear over the depth, the web stresses can be expressed by the following general equation:

$$\sigma_w = \sigma_f \left(\frac{E_w}{E_f} \right) \tag{8}$$

When the equation is corrected in accordance to the actual load duration and service class, it can be expressed as:

$$\sigma_w = \sigma_f \left(\frac{E_w}{E_f} \right) \left(\frac{1 + k_{def,f}}{1 + k_{def,w}} \right) \tag{9}$$

As shown earlier the stresses in the flange are given by:

$$\sigma_f = \left(\frac{M}{I_{ef}} y_1 \right) + \left(\frac{F}{A_{ef}} \right) \tag{10}$$

where y_1 is the distance between the neutral axis of the beam and the point where the stress value is calculated.

Compression side of the web

The maximum stress in the compression zone of the web can be calculated as:

$$\sigma_{w,c,\max,d} = \left[\left(\frac{M_d}{I_{ef}} y_{w,c}\right) + \left(\frac{F}{A_{ef}}\right)\right]\left(\frac{E_w}{E_f}\right)\left(\frac{1 + k_{def,f}}{1 + k_{def,w}}\right) \qquad (11)$$

where $y_{w,c}$ is the distance between the neutral axis of the beam and the compression edge of the web.

This stress shall satisfy the following condition:

EC5: Part 1-1: 5.3.1e

$$\sigma_{w,c,\max,d} \le f_{c,w,d} \qquad (12)$$

Tensile side of the web

The maximum stress in the tensile zone of the web can be calculated as:

$$\sigma_{w,t,\max,d} = \left[\left(\frac{M_d}{I_{ef}} y_{w,t}\right) + \left(\frac{F}{A_{ef}}\right)\right]\left(\frac{E_w}{E_f}\right)\left(\frac{1 + k_{def,f}}{1 + k_{def,w}}\right) \qquad (13)$$

where $y_{w,t}$ is the distance between the neutral axis of the beam and the tensile edge of the web.

This stress shall satisfy the following condition:

EC5: Part 1-1: 5.3.1f

$$\sigma_{w,t,\max,d} \le f_{t,w,d} \qquad (14)$$

$f_{c,w,d}$ and $f_{t,w,d}$ are the compressive and tensile bending strengths of the web. Unless other values are given, the design compressive and tensile strength of the web should be taken as the in-plane design compressive and tensile strengths.

Shear stresses in the web

EC5: Part 1-1: 5.3.1

Unless a detailed buckling analysis is made it should be verified that:

$$h_w \le 70\, b_w \qquad (15)$$

$$V_d \le f_{v,0,d}\, n\, b_w\, h_w \left(1 + 0{,}5\left(\frac{h_{f,t} + h_{f,c}}{h_w}\right)\right) \quad \text{for } h_w \le 35\, b_w \qquad (16)$$

$$V_d \le 35 f_{v,0,d}\, n\, b_w^2 \left(1 + 0{,}5\left(\frac{h_{f,t} + h_{f,c}}{h_w}\right)\right) \quad \text{for } 35\, b_w \le h_w \le 70\, b_w \qquad (17)$$

where

V_d is the design value of the shear force in the actual section,
$f_{v,0,d}$ is the design panel shear strength,
n is the number of webs.

Shear stresses in the glued joint between the flanges and the web

As previously mentioned it is advantageous that the capacity of the glue-line is higher than the corresponding capacities of the flange and web material. Normally the weakest link in this joint will be the rolling shear strength of the web, $f_{v,90}$. It is assumed that the design shear stress ($\tau_{mean,d}$) at the actual section is uniformly distributed.

$\tau_{mean,d}$ can be expressed by the following equation:

$$\tau_{mean,d} = \frac{V_d S_f}{I_{ef}\, l_g} \qquad (18)$$

where S_f is the first moment of plane area for a flange, calculated from the neutral axis of the beam cross-section and l_g is the total length of the glue-line in the same flange.

EC5: Part 1-1: 5.3.1

The calculated shear stress shall satisfy the following condition:

$$\tau_{mean,d} \le f_{v,90,w,d} \qquad for\ h_f \le 4\,b_w \tag{19}$$

$$\tau_{mean,d} \le f_{v,90,w,d} \left(\frac{4\,b_w}{h_f}\right)^{0,8} \qquad for\ h_f > 4\,b_w \tag{20}$$

Example

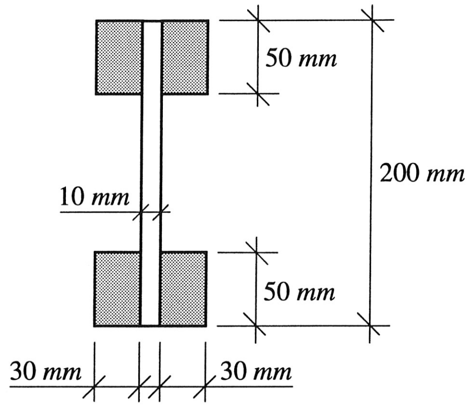

Figure 2 Cross-section of the I-beam used in the example.

Figure 2 shows the actual cross-section.

Service class 2

M_d = 5,0 kNm $\quad$ $F_{c,d}$ = 18 kN $\quad$ V_d = 4,8 kN

The actions are assumed to consist of 25% permanent load and 75% medium-term load.

Flanges: strength class C30 according to prEN 338.

$f_{m,d}$ = 18,5 N/mm^2 $\quad$ $f_{c,0,d}$ = 14,2 N/mm^2 $\quad$ k_c = 0,95

$f_{t,0,d}$ = 11,1 N/mm^2 $\quad$ E_{mean} = 12000 N/mm^2

Medium-term: k_{def} = 0,25 $\quad$ k_{mod} = 0,80

Permanent: k_{def} = 0,80

Web: particleboard according to EN 312-6, t = 10 mm.

$f_{c,w,d}$ = 5,97 N/mm^2 $\quad$ $f_{t,w,d}$ = 4,44 N/mm^2 $\quad$ $f_{v,0,d}$ = 3,30 N/mm^2

$f_{v,90,d}$ = 0,804 N/mm^2 $\quad$ E_{mean} = 2475 N/mm^2

Medium-term: k_{def} = 0,75 $\quad$ k_{mod} = 0,55

Permanent: k_{def} = 2,25

Since the particleboard web shows larger creep deflections than the solid timber flanges, the normal stresses in the flanges will increase and in the web decrease in time. Consequently, the normal stresses in the web are calculated at instantaneous and in the flanges at final deformation.

For control at instantaneous deformation

$$A_{ef} = \left[(4 \cdot 50 \cdot 30) + \frac{(1+0{,}0)\,2475}{(1+0{,}0)\,12000}(10 \cdot 200)\right] mm^2 = 6{,}41 \cdot 10^3\, mm^2$$

$$I_f = 4\left[\left(\frac{1}{12} 30 \cdot 50^3\right) + (30 \cdot 50 \cdot 75^2)\right] mm^4 = 35{,}0 \cdot 10^6\, mm^4$$

$$I_w = \left(\frac{1}{12} 10 \cdot 200^3\right) mm^4 = 6{,}67 \cdot 10^6\, mm^4$$

$$I_{ef} = \left[35 + \frac{(1+0{,}0)\,2475}{(1+0{,}0)\,12000} 6{,}67\right] 10^6\, mm^4 = 36{,}4 \cdot 10^6\, mm^4$$

For control at final deformation

$$A_{ef} = \left[(4 \cdot 50 \cdot 30) + \frac{(1+(0{,}80 \cdot 0{,}25 + 0{,}25 \cdot 0{,}75))\,2475}{(1+(2{,}25 \cdot 0{,}25 + 0{,}75 \cdot 0{,}75))\,12000}(10 \cdot 200)\right] mm^2 = 6{,}27 \cdot 10^3\, mm$$

$$I_{ef} = \left[35 + \frac{(1+(0{,}80 \cdot 0{,}25 + 0{,}25 \cdot 0{,}75))2475}{(1+(2{,}25 \cdot 0{,}25 + 0{,}75 \cdot 0{,}75))\,12000} 6{,}67\right] 10^6\, mm^4 = 35{,}9 \cdot 10^6\, mm^4$$

Control of compressive flange at final deformation

$$\sigma_{f,c,\max,d} = \left[\left(\frac{5{,}0 \cdot 10^6}{35{,}9 \cdot 10^6} 100\right) + \left(\frac{18 \cdot 10^3}{6{,}27 \cdot 10^3}\right)\right] N/mm^2 = 16{,}8\, N/mm^2$$

$$\sigma_{f,c,\max,d} < f_{m,d} = 18{,}5\, N/mm^2$$

$$\sigma_{f,c,d} = \left[\left(\frac{5{,}0 \cdot 10^6}{35{,}9 \cdot 10^6} 75\right) + \left(\frac{18 \cdot 10^3}{6{,}27 \cdot 10^3}\right)\right] N/mm^2 = 13{,}3\, N/mm^2$$

$$\sigma_{f,c,d} < k_c\, f_{c,0,d} = 0{,}95 \cdot 14{,}2\, N/mm^2 = 13{,}5\, N/mm^2$$

Control of tensile flange at final deformation

$$\sigma_{f,t,\max,d} = \left[\left(\frac{5{,}0 \cdot 10^6}{35{,}9 \cdot 10^6} 100\right) - \left(\frac{18 \cdot 10^3}{6{,}27 \cdot 10^3}\right)\right] N/mm^2 = 11{,}1\, N/mm^2$$

$$\sigma_{f,t,\max,d} < f_{m,d} = 18{,}5\, N/mm^2$$

$$\sigma_{f,t,d} = \left[\left(\frac{5{,}0 \cdot 10^6}{35{,}9 \cdot 10^6} 75\right) - \left(\frac{18 \cdot 10^3}{6{,}27 \cdot 10^3}\right)\right] N/mm^2 = 7{,}57\, N/mm^2$$

$$\sigma_{f,t,d} < f_{t,0,d} = 11{,}1\, N/mm^2$$

Control of compressive stress in the web at instantaneous deformation

$$\sigma_{w,c,d} = \left[\left(\frac{5{,}0 \cdot 10^6}{36{,}4 \cdot 10^6} 100\right) + \left(\frac{18 \cdot 10^3}{6{,}41 \cdot 10^3}\right)\right]\left(\frac{(1+0{,}0)\,2475}{(1+0{,}0)\,12000}\right) N/mm^2 = 3{,}41\, N/mm^2$$

$$\sigma_{w,c,d} < f_{c,w,d} = 5{,}97\, N/mm^2$$

Control of tensile stress in the web at instantaneous deformation

$$\sigma_{w,t,d} = \left[\left(\frac{5,0 \cdot 10^6}{36,4 \cdot 10^6} 100\right) - \left(\frac{18 \cdot 10^3}{6,41 \cdot 10^3}\right)\right]\left(\frac{(1+0,0)\,2475}{(1+0,0)\,12000}\right) N/mm^2 = 2,25\,N/mm^2$$

$$\sigma_{w,t,d} < f_{t,w,d} = 4,44\,N/mm^2$$

Control of the shear stress in the web

$$h_w = 100\,mm < 70\,b_w = 700\,mm$$

$$h_w < 35\,b_w = 350\,mm$$

$$V_d = 4,8\,kN \le 10 \cdot 100\left[1 + 0,5\left(\frac{50+50}{100}\right)\right] 3,30 = 4,95\,kN$$

Control of the shear stress in the glued joint between flanges and web at final deformation

Since the normal stresses in the flanges will increase in time, the shear stresses in the glueline between web and flanges will also increase. Consequently, the shear stresses in the glueline are calculated at final deformation.

$$S_f = 2\,(50 \cdot 30 \cdot 75)\,mm^3 = 225 \cdot 10^3\,mm^3$$

$$l_g = 2 \cdot 50\,mm = 100\,mm$$

$$\tau_{mean,d} = \frac{4,8 \cdot 10^3 \cdot 225 \cdot 10^3}{35,9 \cdot 10^6 \cdot 100} N/mm^2 = 0,301\,N/mm^2$$

$$h_f = 50\,mm > 4\,b_w = 40\,mm$$

$$0,804\left(\frac{4 \cdot 10}{50}\right)^{0,8} N/mm^2 = 0,673\,N/mm^2 > \tau_{mean,d} = 0,301\,N/mm^2$$

Calculation of deflections

Deflections of glued thin-webbed beams are calculated according to the same principles as given for solid timber. Nevertheless it is important to remember that the shear deflection in this case also has to be considered.

The deflection from a given load is then expressed by:

$$u = \left(\frac{A}{E_f I_{ef}}\right)(1 + k_{def,f}) + \left(\frac{B}{G_w A_w}\right)(1 + k_{def,w})$$

where A and B denote factors given by the type of load and the structural system. Unless a more detailed analysis is made, the shear deflection can be based on the real area of the web (A_w).