Carpentry joints

STEP lecture C12
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Objective

To describe design procedures for carpentry joints.

Prerequisites

A6 Strength grading
A7 Solid timber - Strength classes
B2 Tension and compression

Summary

Supported by drawings and figures, the most frequently used carpentry joints are presented. By explaining the deformation and load-carrying behaviour of such joints, the possible field of application is described. Special execution rules and recommendations are given.

Design rules for ultimate limit state as well as serviceability limit state are evaluated, and their application is demonstrated by typical examples.

Introduction

An ancient timber structure usually consisting of single timber members is only efficient if the individual parts are formed into a reasonable construction. Joints transfer the inner forces caused by external actions from one member to another. Two or more members of the construction are assembled at nodes. In many cases the forces will be passed on by contact of the joint areas or by friction. Some carpentry joints are completed by fasteners made of iron or wood in order to ensure a correct fit of the connection or to allow the transmission of additional forces. Although there are a lot of forms of carpentry joints, it is possible to reduce the multitude of joints to some basic types. Some typical basic carpentry joints, such as half-lap joints, framed joints, tenon joints and cogging joints are shown in Figures 1 and 2. These joints are either used to lengthen single members parallel to grain or to join elements that meet each other at an angle. In the following sections the deformation and load-carrying behaviour of framed and tenon joints is explained.

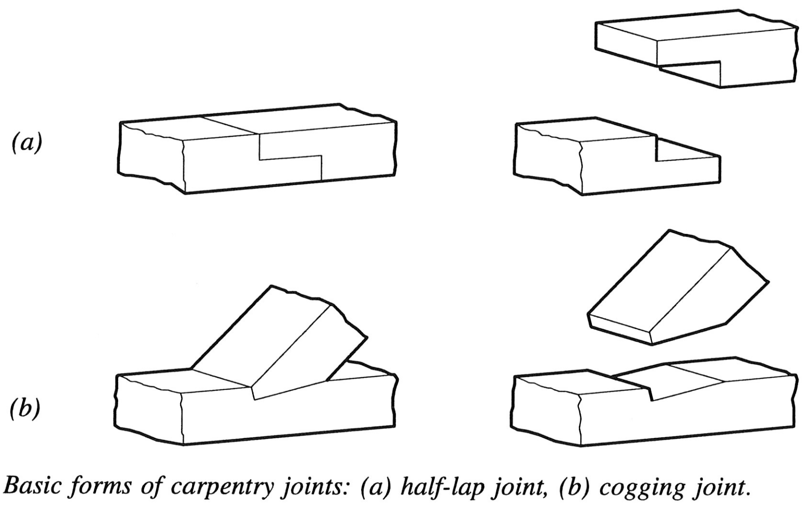

Figure 1 Basic forms of carpentry joints: (a) half-lap joint, (b) cogging joint.

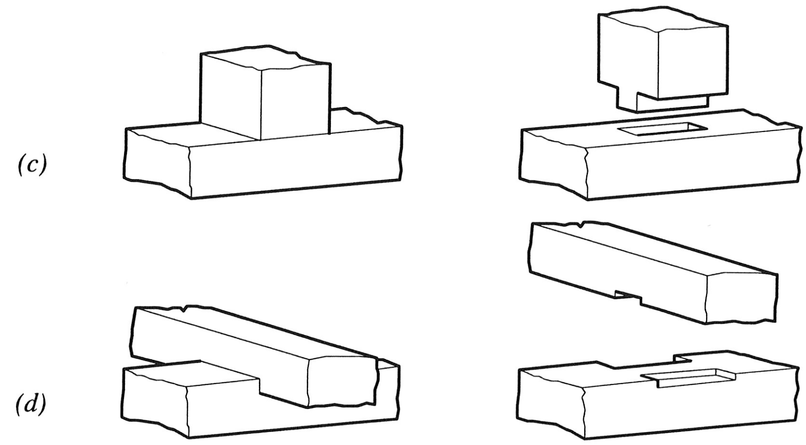

Figure 2 Basic forms of carpentry joints: (c) framed joint, (d) tenon joint.

Framed joints

Framed joints are used to transmit compression forces from one member inclined to another at a given angle. The compression force of the strut is transmitted by contact using the frontal area of the joint. The chord is loaded in shear. In the past an additional tenon was used to keep the joint in the right position. Today this is mostly brought about by nails, sometimes also by screws, bolts or laterally nailed cover plates. Framed joints can be formed with a notch in the front area or in the rear of the strut. Combinations of both approaches are also possible (see Figure 3).

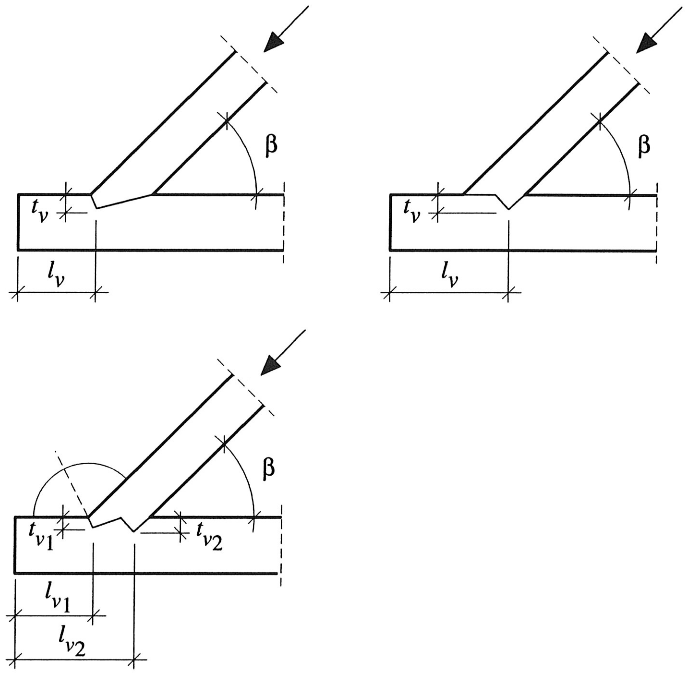

Figure 3 Framed joints with notch in the front area (top left), in the rear area (top right) and in combination (bottom).

When designing a framed joint it is necessary to prove the load-carrying capacity of the available areas of the joint. Thus, if the strut has a slope between 30° and 60°, only the frontal area of the joint is taken into account. The size of the frontal area can be calculated from the effective width b_{ef} and the cutting depth t_v in the chord.

The resulting compression stresses at an angle to the grain should satisfy the following condition:

$$\sigma_{c,\alpha,d} \leq \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{f_{c,90,d}} \sin^2 \alpha + \cos^2 \alpha} \tag{1}$$

For optimising the joint it is recommended that the angle of the frontal area is set to half of the angle between strut and chord. Thus, the angle α between force and grain is the lowest possible for both the chord and the strut ($\alpha = \beta/2$).

In this case the compression stress in the front area of the joint is:

$$\sigma_{c,\alpha,d} = \frac{F_d \cos^2(\beta/2)}{b_{ef}\, t_v} \tag{2}$$

If the notched area is at the rear of the strut the cut is made perpendicular to the longitudinal axis of the strut. In this case, the angle between force and grain is the same as between strut and chord. Then the compression stress is:

$$\sigma_{c,\alpha,d} = \frac{F_d \cos\beta}{b_{ef}\, t_v} \tag{3}$$

Using double framed joints, it is possible to transmit the sum of the two single framed joints as described before. In this case it is important that the frontal area as well as the rear area of the strut fit perfectly into the corresponding parts of the chord.

Assuming a uniformly distributed compression stress in the strut, the force is transmitted into the chord by shear stresses. The average shear stress in the chord is:

$$\tau_d = \frac{F_d \cos\beta}{b_{ef}\, l_v} \tag{4}$$

In double framed joints the shear areas should not coincide. Therefore, it is recommended that the following condition is satisfied:

$$t_{v1} \leq \begin{cases} t_{v2} - 10\ mm \\ 0{,}8\ t_{v2} \end{cases} \tag{5}$$

When determining the required length l_{v2} in the chord, the total horizontal component of the compression force of the strut should be taken into account.

When designing the strut, any eccentricities from the joint configuration may cause additional bending stresses in the strut. In the tension chord the reduced cross-sectional area must be considered.

Design example

Joint of a compression member with a rectangular cross section b x h = 140 x 140 *mm*, slope $\beta = 45°$, with a chord b x h = 140 x 180 *mm*. Cutting depth $t_v = 45\ mm$, shear length in the chord $l_v = 250\ mm$. Timber of strength class C24 according to prEN 338 "Structural timber. Strength classes" .

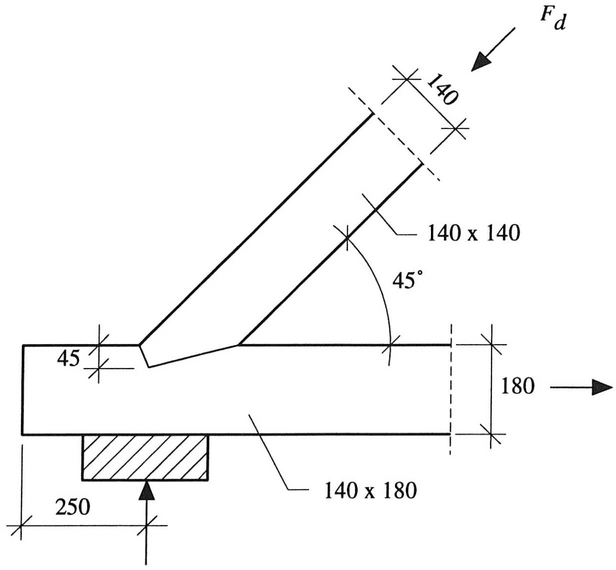

Figure 4 Design example of a framed joint.

Governing design value of permanent and medium-term load:
$F_d = 63\ kN$
$\gamma_M = 1{,}3$
Service class 1: $k_{mod} = 0{,}8$

Characteristic material properties:
The characteristic values of compression and shear strengths are taken from prEN 338 "Structural timber. Strength classes".

$f_{c,0,k} = 21\ N/mm^2$ $f_{c,90,k} = 5{,}7\ N/mm^2$ $f_{v,k} = 2{,}4\ N/mm^2$

The design values of the compression and shear strengths are:

$$f_{c,0,d} = \frac{k_{mod}\, f_{c,0,k}}{\gamma_M} = \frac{0{,}8 \cdot 21}{1{,}3} = 12{,}9\ N/mm^2$$

$$f_{c,90,d} = \frac{k_{mod}\, f_{c,90,k}}{\gamma_M} = \frac{0{,}8 \cdot 5{,}7}{1{,}3} = 3{,}5\ N/mm^2$$

With an angle $\alpha = \beta/2 = 22{,}5°$ between the direction of the force and the grain of chord and strut, the design compression strength shall satisfy the following condition:

$$\sigma_{c,\alpha,d} \le \frac{f_{c,0,d}}{\dfrac{f_{c,0,d}}{f_{c,90,d}} \sin^2\alpha + \cos^2\alpha} = 9{,}3\ N/mm^2$$

$$f_{v,d} = \frac{k_{mod}\, f_{v,k}}{\gamma_M} = \frac{0{,}8 \cdot 2{,}4}{1{,}3} = 1{,}48\ N/mm^2$$

Verification of failure condition:

$$\sigma_{c,\alpha,d} = \frac{F_d \cos^2(\beta/2)}{b_{ef}\, t_v} = \frac{63000 \cdot \cos^2 22{,}5^\circ}{140 \cdot 45} = 8{,}5\ N/mm^2 < 9{,}3\ N/mm^2$$

$$\tau_d = \frac{F_d \cos\beta}{b_{ef}\, l_v} = \frac{63000 \cdot \cos 45^\circ}{140 \cdot 250} = 1{,}27\ N/mm^2 < 1{,}48\ N/mm^2$$

Tenon joints

In carpentry tenon joints are used for joining members transmitting transverse forces in ceilings, walls and roof constructions. Today, due to economical reasons, tenon joints are only used if they are produced by machines. Basically there are joints with a central tenon or ones with a tenon at the bottom edge of a member. Joints with a central tenon are normally used for joining members of the same depth, whereas joints with a bottom tenon are used to connect members with different depths, e.g. girders.

The depth h_e of hand made tenons is usually one third of the beam depth h. In modern constructions the tenon depth depends on the size of the processing machines. The tenon lengths vary from 40 to 60 *mm*. If the tenon joint is additionally fastened by a peg, greater lengths can be realised.

Mortises should only be arranged in the centre or in the compressive area of a beam. For designing the beam the reduced cross-sectional area shall be taken into account.

The design of tenon joints can be carried out in line with end-notched beams. Therefore, the following condition should be satisfied:

$$\tau_d = \frac{1{,}5\ V_d}{b\ h_e} \le k_v\, f_{v,d} \qquad (6)$$

where h_e is the tenon depth. The factor k_v is a reducing factor taking into account the geometry of the tenon joint, such as the beam depth h, the tenon depth h_e and the distance x of the shear load from the tenon corner.

For joints with a tenon at the bottom edge of the member, k_v=1. For joints with a central tenon:

$$k_v = \min \begin{cases} 1 \\ \dfrac{5}{\sqrt{h}\left(\sqrt{\dfrac{h_e}{h}\left(1-\dfrac{h_e}{h}\right)} + 0{,}8\,\dfrac{x}{h}\sqrt{\dfrac{h}{h_e} - \left(\dfrac{h_e}{h}\right)^2}\right)} \end{cases} \qquad (7)$$

Furthermore it shall be proved that the design compression stresses perpendicular to the grain do not exceed the design compression strength.

$$\sigma_{c,90,d} \le k_{c,90}\, f_{c,90,d} \qquad (8)$$

For the most common tenon joints it may be assumed that $k_{c,90}$ = 1.

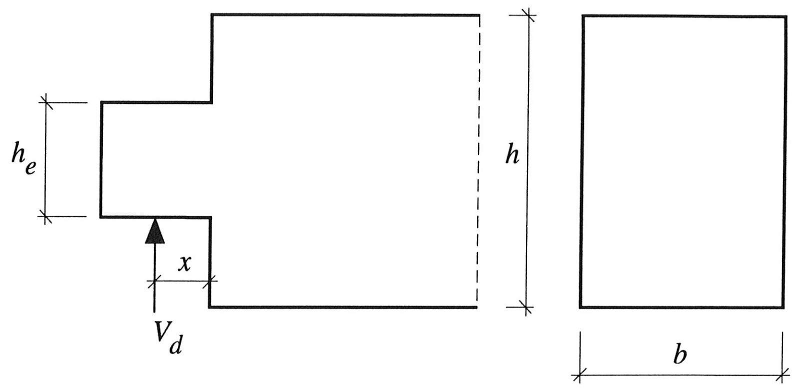

Figure 5 Joint with a central tenon.

Design example

Joint of a girder with a rectangular cross section b x h = 100 x 180 *mm*, with central tenon (tenon depth h_e = *60 mm*). Timber of strength class C24 according to prEN 338.

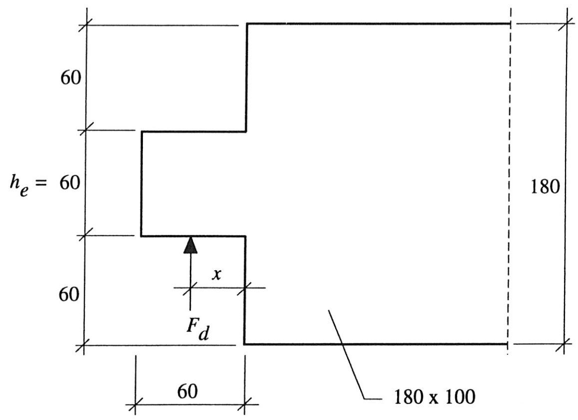

Figure 6 Design example of a joint with a central tenon.

Governing design value of permanent and medium-term load:
$V_d = 3$ *kN*

Service class 1: $k_{mod} = 0{,}8$

Characteristic material properties:

The characteristic values of compression and shear strengths are taken from prEN 338 "Structural timber - Strength classes".

$f_{v,k} = 2{,}4$ *N/mm²* $f_{c,90,k} = 5{,}7$ *N/mm²*

The design values of the shear and compression strengths are:

$$f_{v,d} = \frac{k_{mod}\, f_{v,k}}{\gamma_M} = \frac{0{,}8 \cdot 2{,}4}{1{,}3} = 1{,}48\ N/mm^2$$

$$f_{c,90,d} = \frac{k_{mod}\, f_{c,90,k}}{\gamma_M} = \frac{0{,}8 \cdot 5{,}7}{1{,}3} = 3{,}5\ N/mm^2$$

Assumption: $x = 30\ mm$
$h/h_e = 3$
$x/h = 1/6$

Then,

$$k_v = \frac{5}{\sqrt{180}\left(\sqrt{\frac{1}{3}\,(1 - \frac{1}{3})} + 0{,}8\ \frac{1}{6}\sqrt{3 - \frac{1}{9}}\right)} = 0{,}534$$

$$k_v\, f_{v,d} = 0{,}534 \cdot 1{,}48 = 0{,}79\ N/mm^2$$

$$k_{c,90}\, f_{c,90,d} = 1 \cdot 3{,}5 = 3{,}5\ N/mm^2$$

Verification of failure condition:

$$\tau_d = \frac{1{,}5\ V_d}{b\ h_e} = \frac{1{,}5 \cdot 3 \cdot 10^3}{100 \cdot 60} = 0{,}75\ N/mm^2 < 0{,}79 N/mm^2$$

$$\sigma_{c,90,d} = \frac{V_d}{b\ l} = \frac{3 \cdot 10^3}{100 \cdot 60} = 0{,}5\ N/mm^2 < 3{,}5 N/mm^2$$