Moment resisting connections

STEP lecture C16
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Objectives

To present the main types of joints used to transfer moments and forces. To examine the influence of the stiffness of moment-resisting joints on structural behaviour. To give the calculation methods associated with force distribution in the joint area.

Prerequisites

A4 Wood as a building material

Summary

Patterns of fasteners are described for several types of moment-resisting joints. Practical applications and general requirements are given for a splice joint and frame corner connections. The influence of joint stiffness on structural behaviour is indicated. After an examination of the force and stress distributions in the joint area, the calculation methods are presented. These methods define the forces acting on the fasteners and the members, based on the assumption of elastic behaviour. The design of a frame corner with the fasteners located in two concentric circular patterns complements the lecture.

Introduction

In timber structures, the joints are normally designed to transfer forces stressing the fasteners in the load direction. The analysis assumes the joints to be pinned because of the concentration of fasteners in a small area limiting the moment arm. However, the development of glued-laminated timber and other wood-based materials offers many structural possibilities. To fulfil the code requirements, or to optimise the construction work, designers increasingly use rotationally rigid joints. The type of joint and the jointing technologies will depend to a great extent on the layout of the structure and how the connected members work.

Moment-resisting joints can work according to three types of force diagram (Figure 1). Depending on the rotation centre C of the joint, the resulting moment of the fastener withdrawal (F_W) or lateral (F_L) loads balances the applied moment.

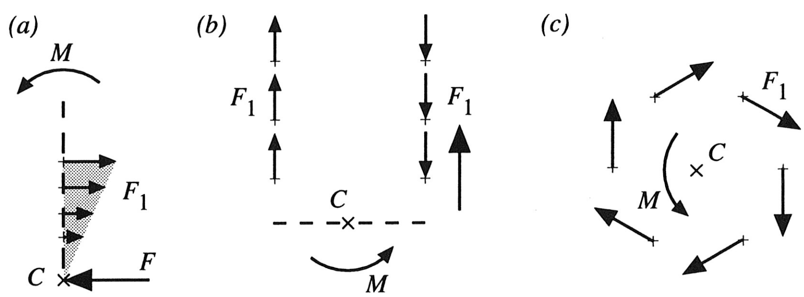

Figure 1   Force diagrams on the fasteners in moment-resisting joints.

Relating to the diagrams in Figure 1, Figure 2 presents some examples of moment-resisting joints such as: (a) a joint of handrail support to deck beam in a pedestrian bridge, (b) a splice joint in a continuous beam or arch, and (c) a knee joint in a frame. The diagrams (a) to (c), respectively, are associated with these joints.

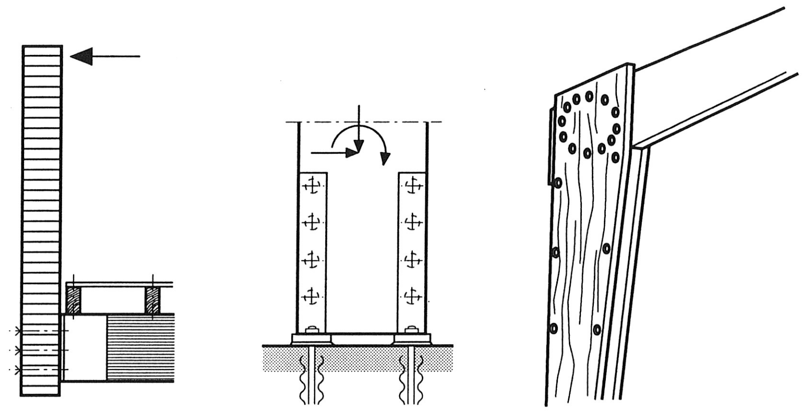

Figure 2 *Examples of moment-resisting joints.*

For the joint (2a), the action F and the bending moment ($F\ a$) induce withdrawal forces on the fasteners. In the example (2b), fastener forces balance the bending moment. Normal and shear forces are transferred to the foundation by steel hardware stressing the timber column in compression parallel and perpendicular to the grain. As large stresses perpendicular to the grain can occur, this technique is used for the transfer of small bending moments.

In the last joint (2c), fasteners are designed to carry forces and bending moments given by the structural analysis. The following sections deal with this kind of moment-resisting joint.

The trend now is to install joints working in accordance with diagram (c) as the traditional knee connection in frames. For this type of connection, the designer must be aware of the possible additional stresses perpendicular to the grain. They are induced by swelling or shrinkage across the restrained cross-section. Either, these stresses have to be taken into account in the calculations, or the height of the restrained area should be limited. Apart from the possibility of splitting, this type of joint offers many advantages. For long-span structures, they are an effective way of overcoming transport limitations and/or of using economical timber sizes. Although curved frames or arches work more efficiently, timber frames are often designed with tapered members connected by moment-resisting joints. This design maximises internal space in the building.

For the joint calculations, the global analysis of the structure and the local analysis of the connection should both be considered. The stiffness of the joints can affect the structural distribution of forces and deformations. The examination of the fastener forces and timber stresses allows the derivation of the design rules.

Structural influence of moment-resisting joints

Current design calculations assume the connections to be either pinned or fixed. As embedding deformations in the timber produce large joint deformations, the modelling assumption should consider the joint stiffness. It affects the deformation of the structures and the distribution of the forces in the case of indeterminate structures (Leijten, 1988; Komatsu, 1992). To produce accurate designs, the joint can conveniently be classified by considering the coefficient β_r:

$$K_r = \beta_r \frac{EI}{L} \tag{1}$$

where K_r is the rotational stiffness of the joint as defined in the following section and EI is the bending stiffness of the connected member with a span L.

The classification of the joint as fixed, semi-rigid or pinned is examined in the example of a two-hinged frame (Figure 3a). Neglecting the deformations induced by normal and shear forces, the moment in the joint is given by:

$$M_j = \frac{q\,L^2}{8} \cdot \frac{1}{1{,}5 + \alpha \frac{H}{L} + \frac{3}{\beta_r}} \tag{2}$$

where q is the uniformly distributed load on the rafter.

The joint efficiency is measured by the ratio R_M, which relates the moment M_j to the moment of a rigid joint corresponding to $K_r = \infty$. For different layouts of the frame, Figures 3b and 3c present the influence of the joint stiffness on the ratio R_M.

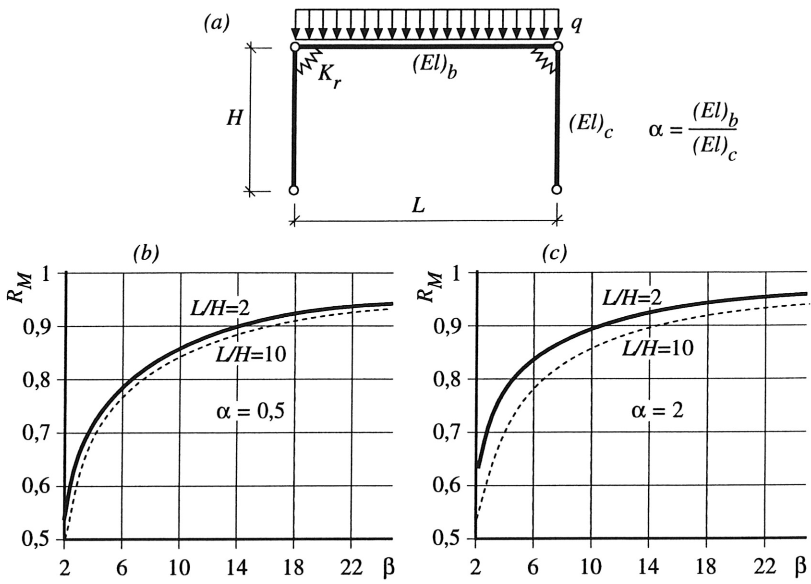

Figure 3 Geometry of the frame(a) and variations in the joint efficiency (b,c).

A substantial decrease in the moment in the joint occurs when the stiffness coefficient β_r is lower than 6. Considering this variation, a joint may be considered as fixed when $R_M \geq 0{,}85$, which requires a β_r-value from 8 to 12. In the opposite extreme, a pinned joint is assumed if $R_M \leq 0{,}20$ relating to a mean value $\beta_r = 0{,}5$. In all other cases, the structure shall be designed as a structure with semi-rigid joints. Considering second order effects, this classification is related to braced structures, i.e. those prevented from swaying. For unbraced structures, EC3 specifies a minimum value of 25 for β_r when assuming fixed joints.

Furthermore, the assumption of semi-rigid joints gives the opportunity for a moment redistribution in the timber structures. Related to the aspect ratio L/H of the frame and the coefficient β_r, the relative values of the moments in the joint and at mid-span are given in Figure 4a. The upper curve is associated with the fully-rigid frame.

Thus, the designer can choose a situation where the moments are equal in the joint and at mid-span. In the example, the joint shall be designed to give a stiffness coefficient respectively equal to 8 and 12 for the aspect ratio L/H of 4 and 8. Figure 4b shows that, simultaneously, the maximum deflection increases slightly. As the joints alone no longer dictate the sizes of the members, the use of the timber can be improved.

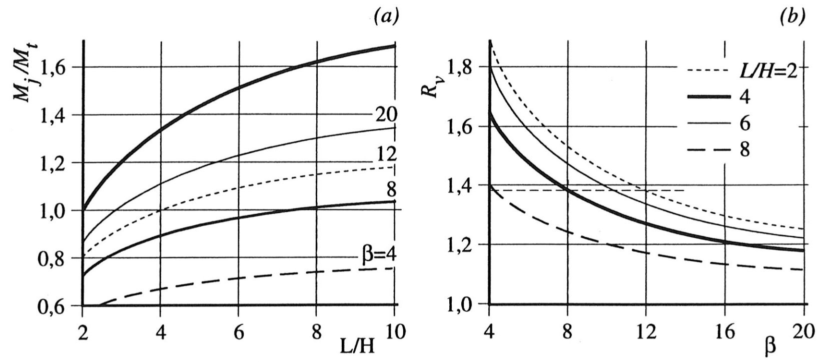

Figure 4 *Influences of the joint stiffness on (a) - the ratio between the moment in the joint (M_j) and at mid-span (M_t), and (b) - the ratio (R_u) between deflection at mid-span in semi-rigid and rigid frame.*

Local behaviour of the joints

To derive design equations, the mechanical behaviour of the joint is first examined when resisting a moment. To counteract the applied moment, the fasteners are loaded at a varying angle to the grain. Considering the orthotropic behaviour of the timber, the load on the fasteners depends on the slip modulus in the direction of load (Ohashi and Sakamoto, 1989). In addition, the layout of the joint should be taken into account. Two main types of joints are distinguished:

- splice joints where the timber members are parallel (Figure 5a),
- cross-grained timber to timber joints (Figure5b).

To achieve an elastic analysis, the members are assumed to be rigid since they are stiffer and stronger than the joint. Therefore, the joint rotation results from the rotational displacement ω of the fasteners (Figure 5c). Defining the rotation centre C as the geometrical centre of the joint, the equilibrium condition is given by:

$$M = \sum_{j=1}^{n} F_{M,j}\, r_j \tag{3}$$

where $F_{M,j}$ is the load on the fastener j, and r_j its distance from the rotation centre.

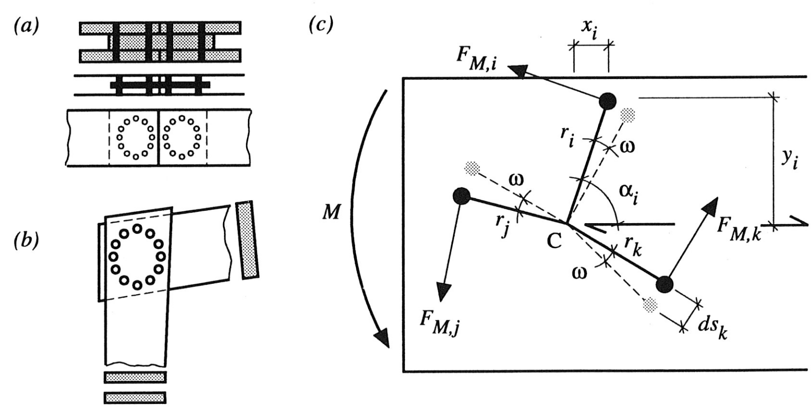

Figure 5 *Moment-resisting joints. (a) joint with parallel members, (b) cross-grained*

members, (c) geometrical definitions and forces on fasteners.

As the fasteners behave linearly, the following relationships apply:

$$ds_k = \frac{F_{M,k}}{K_{\alpha_k}} \quad and \quad \omega = \frac{ds_k}{r_k} = \frac{F_{M,k}}{K_{\alpha_k} r_k} = \frac{F_{M,j}}{K_{\alpha_j} r_j} = \frac{F_{M,i}}{K_{\alpha_i} r_i} \tag{4}$$

where $K_{\alpha k}$ is the slip modulus in the force direction ($\alpha_k + \pi/2$).

From Equations (3) and (4), the load on the fastener i is expressed as follows:

$$F_{M,i} = \frac{K_{\alpha_i} r_i}{K_r} M \tag{5}$$

with the rotational stiffness K_r :

$$K_r = \sum_{j=1}^{n} K_{\alpha_j} r_j^2 \tag{6}$$

For parallel members, the slip modulus is determined from the Hankinson formula (Equation 7). For cross-grained members, the compatibility of the embedding deformations requires a modified slip modulus to be considered. If the members are perpendicular to each other, this slip modulus is independent of the fastener position (Equation 8).

$$K_{\alpha_i} = \frac{K_{0,i} K_{90,i}}{K_{0,i} \cos^2\alpha_i + K_{90,i} \sin^2\alpha_i} \tag{7}$$

$$K_{\alpha_i} = \frac{2 K_{0,i} K_{90,i}}{K_{0,i} + K_{90,i}} \tag{8}$$

In addition to the load distribution on the fasteners, the timber stresses have to be considered in the jointed area. Figure 6 gives the stress distribution in shear and in tension perpendicular to the grain resulting from a linear orthotropic model.

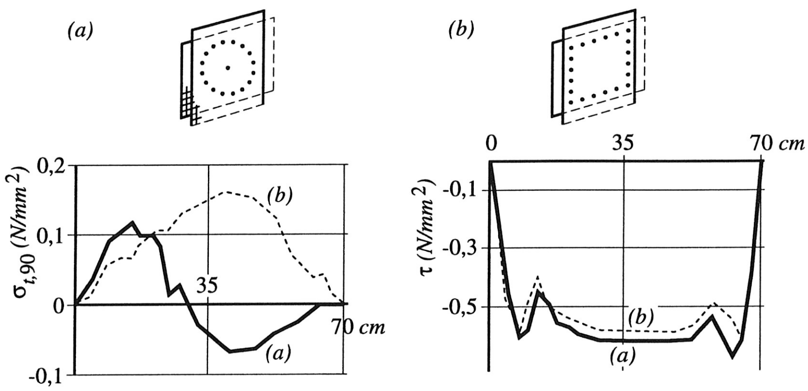

Figure 6 *Stresses in moment resisting joint. (a) fastener patterns, (b) shear stresses on the middle plane of joints, and (c) stresses in tension perpendicular to the grain 100 mm from the end (Racher and Gallimard, 1991).*

Figure 6 shows that a rectangular pattern leads to the most dangerous combination of shear and tension perpendicular to the grain. Locating the fasteners along the edges of the members results in higher stresses perpendicular to the grain near to the end (Boult, 1988). For such patterns, the risk of splitting can be reduced by not placing fasteners in the joint corner, by using small diameter fasteners, or by gluing on some form of reinforcement.

To calculate the maximum shear stress, the fastener forces are projected on the *y*-direction, considering half the joint. Using the previous notation (Figure 5), the shear force V_M due to the fasteners is given by:

$$V_M = M \sum_{\alpha_i=-\pi/2}^{\pi/2} \frac{K_{u,\alpha_i}}{K_{r,u,d}} x_i \quad (9)$$

Design calculations

The design has to refer to the value:

- $K_{r,ser,d}$ for the calculation of structural deformations,
- $K_{r,u,d}$ to check the load-carrying capacity of the joint and the second-order effects (see STEP lecture B7).

Rotational stiffness

For dowel-type fasteners, the rotational stiffness is calculated using the slip modulus K_{ser} specified in EC5. This low value can be assumed as an average for both the directions parallel and perpendicular to the grain. If the joint is made with one type of fastener, Equation (6) gives the design rotational stiffnesses as:

$$K_{r,ser,d} = K_{ser} \sum_{j=1}^{n} r_j^2 \quad and \quad K_{r,u,d} = \frac{2}{3} K_{r,ser,d} \quad (10)$$

Considering the connections shown in Figure 7, the rotational stiffness at the serviceability limit state is (Kessel, 1991):

$$K_{r,ser,d} = K_{ser} (n_1 r_1^2 + n_2 r_2^2) \quad joint\ type\ A \quad (11)$$

$$K_{r,ser,d} = K_{ser} (\mu_x e_x^2 + \mu_y e_y^2) \quad joint\ type\ B \quad (12)$$

where:

$$\mu_x = 4 m_y \sum_{i=1}^{m_x} (i - 0{,}5)^2 \qquad \mu_y = 4 m_x \sum_{j=1}^{m_y} (j - 0{,}5)^2$$

$$m_x = \text{mod}\left[\frac{n_x + 1}{2}\right] \qquad m_y = \text{mod}\left[\frac{n_y + 1}{2}\right]$$

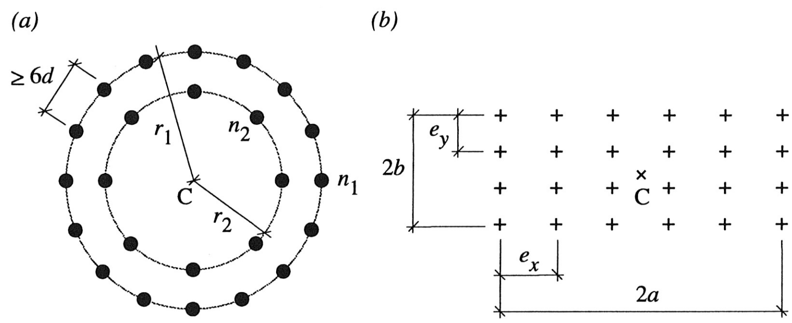

Figure 7 Geometry of common patterns for moment-resisting joints. (a) joint type A, (b) joint type B.

Load-carrying capacity

The moment induces a load F_M perpendicular to the polar radius of the fastener. The maximum value is calculated as:

$$F_M = \frac{K_{\alpha_u} r_i}{K_{r,u,d}} M_{u,d} = \frac{r_1}{n_1 r_1^2 + n_2 r_2^2} M_{u,d} \quad joint\ type\ A \quad (13)$$

$$= \frac{\sqrt{a^2 + b^2}}{(\mu_x e_x^2 + \mu_y e_y^2)} M_{u,d} \quad \textit{joint type B} \tag{14}$$

In addition, shear and normal forces (V and N) are assumed to be uniformly distributed on the fasteners:

$$F_V = \frac{V_{u,d}}{n} \qquad F_N = \frac{N_{u,d}}{n} \tag{15}$$

The total load is calculated by the vectorial summation of F_M, F_N and F_V. Depending on the ratio N/V, the maximum load is obtained for:

- the fastener located at an angle α where $\tan \alpha = N/V$ in a circular pattern,
- the furthest fastener in a rectangular pattern.

To check the load-carrying capacity of the joint, the variation of the embedding strength $f_{h,\alpha,d}$ with the angle α to the grain has to be considered (Heimeshoff, 1977). The joint must be designed for the largest value of the relative fastener load S_f defined as the ratio of load to strength for a fastener located at an angle α. According to EC5 rules, Figures 8a and 8b show the variation of S_f for dowel-type fasteners (d=24 *mm*) in a circular pattern. This variation depends on the fastener slenderness $\lambda_f = M_{y,d} / (f_{h,0,d}\, t_1^2\, d)$. As illustrated by Figure 8, the critical fastener in a circular pattern is located close to the longitudinal axis of the connected members. In a rectangular or trapezoidal pattern, the critical one can be the same or the furthest fastener depending on the joint components and the geometrical ratio *a/b*.

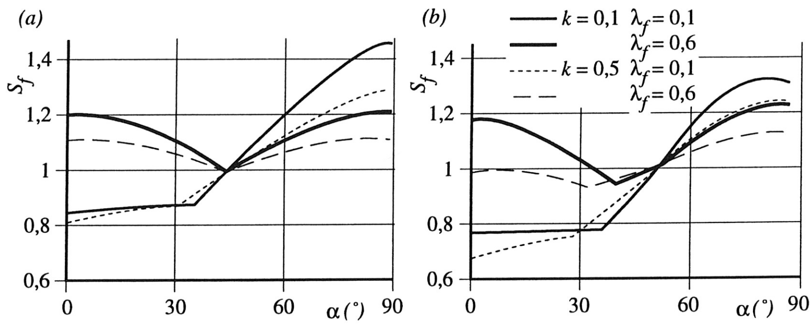

Figure 8 *Influence of the fastener location on the relative capacity S_f (b) for circular pattern and different ratio $k = F_N / F_M = F_V / F_M$. (a) connected members at an angle of 90° and (b) 110°.*

With forces defined by Equations (13) to (15), moment resisting joints are designed considering the load on the fastener located on the longitudinal axis:

$$F_{1,d} = \sqrt{(F_M + F_V)^2 + F_N^2} \tag{16}$$

and acting at an angle to the grain :

$$\alpha_1 = \arctan\left[\frac{F_M + F_V}{F_N}\right] \tag{17}$$

In the case of rectangular or trapezoidal pattern (type B), the furthest fastener should also be checked for the load:

$$F_{2,d} = \sqrt{\left(F_V + \frac{a}{\sqrt{a^2 + b^2}} F_M\right)^2 + \left(F_N + \frac{b}{\sqrt{a^2 + b^2}} F_M\right)^2} \tag{18}$$

at an angle:

$$\alpha_2 = \arctan\left[\frac{a\ F_M + \sqrt{a^2 + b^2}\ F_V}{b\ F_M + \sqrt{a^2 + b^2}\ F_N}\right] \tag{19}$$

As previously shown, the shear strength of the members has to be checked in the joint area. Considering Equation (9) and the equilibrium conditions, the design shear force is given by:

$$F_{V,d} = V_M - \frac{V_{u,d}}{2} = \left[\frac{M_{u,d}}{\pi}\ \frac{n_1 r_1 + n_2 r_2}{n_1 r_1^2 + n_2 r_2^2}\right] - \frac{V_{u,d}}{2} \qquad \textit{joint type A} \tag{20}$$

$$= M_{u,d}\ \frac{2\mu_y e_x}{\mu_x e_x^2 + \mu_y e_y^2}\ \sum_{i=1}^{\mu_y}\left(i - \frac{1}{2}\right) - \frac{V_{u,d}}{2} \qquad \textit{joint type B}$$

Specific rules

As load direction varies with fastener location, the load-carrying capacities per fastener are not reduced with the number of fasteners used. Furthermore, placement of fasteners for type A and B joints should be in accordance with modified distances (Table 1).

	Bolt, Dowel	Ring, Shear-plate	Toothed-plate
Loaded end	7 d	2 d_c	1,5 d_c
Edge distance	4 d	d_c	d_c
Spacings:			
on a circular or rectangular pattern	6 d	2 d_c	1,5 d_c
between patterns	5 d	1,5 d_c	1,5 d_c

Table 1 Moment resisting joint: placement of fasteners (diameter d or d_c).

Design example

A three-hinged frame (Figure 9) is designed with glued-laminated members of strength class GL24. The calculation of the knee joint of the frame is to be considered. Related to the short-term load duration class, the critical load combination gives the forces in the column (see STEP lecture A3) to be used for the design:

$M_{u,d} = 622 \cdot 10^3\ Nm \qquad V_{u,d} = 138 \cdot 10^3\ N \qquad N_{u,d} = 166 \cdot 10^3\ N$

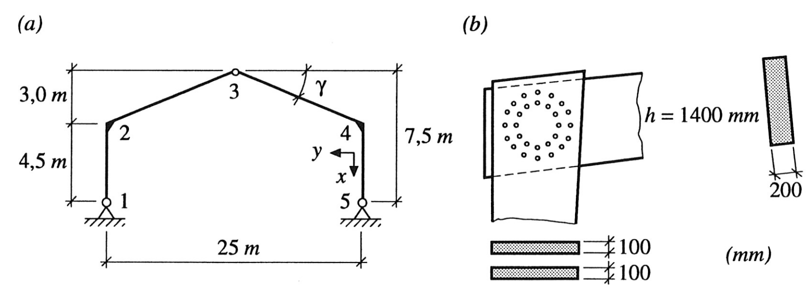

Figure 9 Geometry of the frame (a) and layout of the frame corner (b).

The characteristic timber properties are: $f_{v,k} = 2{,}82\ N/mm^2 \qquad \rho_k = 380\ kg/m^3$

Loads on the fasteners and timber

Considering the thickness of members ($t_2 = 2\ t_1 = 200\ mm$), the designer chooses slender dowels with a diameter of 24 *mm* increasing the joint ductility.

With distances given in Table 1, the radii are:

$$r_1 = 0{,}5\,h - 4\,d = 600\ mm$$
$$r_2 = r_1 - 5\,d = 480\ mm$$

Maximum number of dowels per circle are:

$$n_1 \leq \frac{2\pi r_1}{6\ d} = 26 \qquad n_2 \leq \frac{2\pi r_2}{6\ d} = 20$$

The load induced by the moment has the value:

$$F_M = \frac{r_1}{n_1\ r_1^2 + n_2\ r_2^2}\ M_{u,d} = \frac{600}{26 \cdot 600^2 + 20 \cdot 480^2}\ 622 \cdot 10^6 = 26{,}7 \cdot 10^3\ N$$

Loads due to shear and normal forces in the column are:

$$F_{V,c} = \frac{V_{u,d}}{n_1 + n_2} = \frac{138 \cdot 10^3}{46} = 3 \cdot 10^3\ N$$

$$F_{N,c} = \frac{N_{u,d}}{n_1 + n_2} = \frac{166 \cdot 10^3}{46} = 3{,}61 \cdot 10^3\ N$$

and in the rafter: $F_{V,r} = 2{,}81 \cdot 10^3\ N \qquad F_{N,r} = 3{,}76 \cdot 10^3\ N$

Considering the critical fastener located respectively on the longitudinal axis of column or rafter, the design load is:

$$F_{d,c} = \sqrt{(F_M + F_{V,c})^2 + F_{N,c}^2} = 29{,}9 \cdot 10^3\ N \qquad F_{d,r} = 29{,}8 \cdot 10^3\ N$$

The maximum shear force stressing the timber in the joint area has the value:

$$F_{V,d} = V_M - \frac{V_{u,d}}{2}$$

with:

$$V_M = \left[\frac{M_{u,d}}{\pi}\ \frac{n_1 r_1 + n_2 r_2}{n_1 r_1^2 + n_2 r_2^2}\right] = \left[\frac{622 \cdot 10^3}{\pi}\ \frac{26 \cdot 600 + 20 \cdot 480}{26 \cdot 600^2 + 20 \cdot 480^2}\right] = 357 \cdot 10^3\ N$$

and gives: $F_{V,d} = 288 \cdot 10^3\ N$

Fastener capacity

In the direction parallel to the grain, the embedding strength has the following design value:

$$f_{h,0,d} = \frac{0{,}9}{1{,}3}\ 0{,}082\ (1 - 0{,}01 \cdot 24)\ 380 = 17{,}7\ N/mm^2$$

The coefficient k_{90} is equal to 1,71. Then, the load-carrying capacity (see STEP lecture C6) is calculated considering for the fastener on column axis:

- angles between load and grain:

$$\alpha_1 = \arctan\left[(F_M + F_{V,c})/F_{N,c}\right] = 83{,}1°$$

$$\alpha_2 = \alpha_1 + \pi/2 - 13{,}5 = 6{,}6°$$

- the embedding strengths: $f_{h,1,d} = 9{,}65\ N/mm^2 \quad f_{h,2,d} = 16{,}2\ N/mm^2$
- the ratio β: $\beta = [f_{h,2,d} / f_{h,1,d}] = 1{,}68$

The calculations result in the minimum strength per shear plane defined by:
$R_d = \min[23{,}1\ ;\ 38{,}9\ ;\ 15{,}6\ ;\ 20{,}6] \cdot 10^3 = 15{,}6 \cdot 10^3\ N$

For the fastener on the rafter axis, calculations result in:

- angles between load and grain:
  $\alpha_1 = 20{,}8° \quad \alpha_2 = 82{,}7°$
- embedding strengths: $f_{h,1,d} = 15{,}1\ N/mm^2 \quad f_{h,2,d} = 9{,}65\ N/mm^2$
- ratio β: $\beta = [f_{h,2,d} / f_{h,1,d}] = 0{,}64$
- and design strength: $R_d = \min[36{,}3 ; 23{,}1 ; 17{,}5 ; 20{,}3] \cdot 10^3 = 17{,}5 \cdot 10^3\ N$

For the chosen patterns, the load-carrying condition is checked:

on column: $R_{j,d} = 2 \cdot 15{,}6 \cdot 10^3 = 31{,}1 \cdot 10^3\ N > F_{d,c} = 29{,}9 \cdot 10^3\ N$

and on rafter: $R_{j,d} = 2 \cdot 17{,}5 \cdot 10^3 = 34{,}9 \cdot 10^3\ N > F_{d,c} = 29{,}8 \cdot 10^3\ N$

Timber shear

In the joint area, the strength of the timber is checked for the calculated force $F_{V,d}$:

$$\tau_V = \frac{3\ F_{V,d}}{2\ b\ h} = \frac{3\ 288 \cdot 10^3}{2\ 200\ 1400} = 1{,}54\ N/mm^2 < f_{V,d} = \frac{0{,}9}{1{,}3}\ 2{,}8 = 1{,}94\ N/mm^2$$

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