Beam and post structures - Design

STEP lecture E16
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Objectives

To show the design of a small office building in Reutlingen, built in 1994. To present the calculation and detailing of the most important elements of the building, such as roof and floor beams, stringers and columns. To demonstrate the design of the connection between the stringer and column.

Introduction

The load bearing structure of the office building consists of single section beams, stringers and columns. The beams, stringers and the columns in the roof are designed with a level upper surface hence all beams are single-span girders. The beams are connected with T-shaped steel-plate castings to the stringers. Fastener joints are rod dowels in the beams and grooved nails in the stringers. The stringers and the columns are connected by slotted steel-plates and dowels. The horizontal force is transferred through the roof area and some of the walls, which are formed as wood-based diaphragms. The calculations allow for an additional storey to be added later. In the top of the columns screw sockets are included, to which the columns of the second storey can be fixed in the case of extension.

Design example

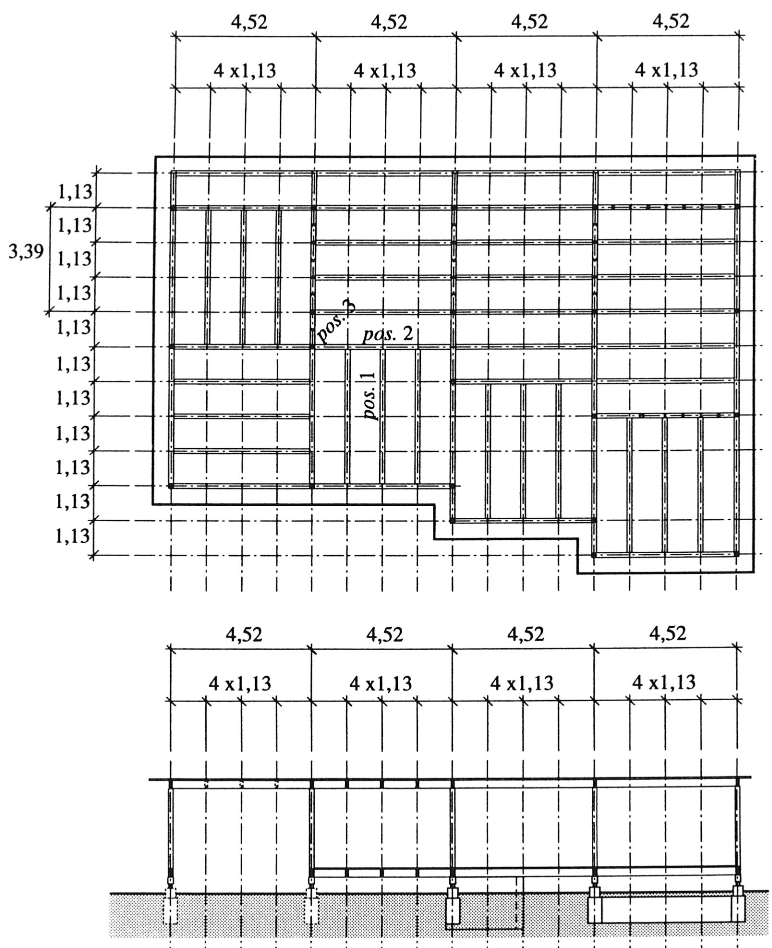


Figure 1 Ground plan and cross section.

Secondary beams (pos. 1 in Figure 1)
Single storey case; roof loading
Permanent actions

Roof felting	0,05 kN/m^2
Thermal insulation	0,12 kN/m^2
Timber boarding	0,18 kN/m^2
Self weight of the beams	0,20 kN/m^2
G_k =	0,55 kN/m^2

Variable actions

Snow	Q_k = 0,75 kN/m^2
Imposed load	2,00 kN/m^2

Wind pull with uplifting function not governing
Actions per beam spacing 1,13 m on centres

$$g_k = 0{,}55 \cdot 1{,}13 = 0{,}62\ kN/m$$
$$q_k = 0{,}75 \cdot 1{,}13 = 0{,}85\ kN/m$$

Two storey case; floor beam
Permanent actions

Finishes	0,10 kN/m^2
Particle board flooring	0,20 kN/m^2
Concrete slabs, thickness 50 mm	1,10 kN/m^2
Boarding	0,18 kN/m^2
Self weight of the beams	0,20 kN/m^2
G_k =	1,78 kN/m^2

Variable actions

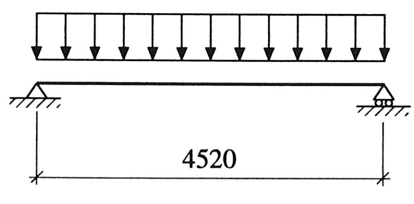
Imposed load	2,00 kN/m^2
Light partition walls	0,75 kN/m^2
Q_k =	2,75 kN/m^2

Actions per beam spacing = 1,13 m

$$g_k = 1{,}78 \cdot 1{,}13 = 2{,}01\ kN/m$$
$$q_k = 2{,}75 \cdot 1{,}13 = 3{,}11\ kN/m$$

Thus the flooring condition governs the beam design.

System



Internal forces and moments

$$A_{g,k} = B_{g,k} = V_{g,k} = 0{,}5 \cdot 2{,}01 \cdot 4{,}52 = 4{,}54\ kN$$
$$A_{q,k} = B_{q,k} = V_{q,k} = 0{,}5 \cdot 3{,}11 \cdot 4{,}52 = 7{,}03\ kN$$
$$M_{g,k} = 0{,}125 \cdot 2{,}01 \cdot 4{,}52^2 = 5{,}13\ kNm$$
$$M_{q,k} = 0{,}125 \cdot 3{,}11 \cdot 4{,}52^2 = 7{,}94\ kNm$$

Design value of the internal forces
for ultimate limit state $1{,}35\ g_k + 1{,}5\ q_k$
for serviceability limit state $1{,}0\ g_k + 1{,}0\ q_k$

$A_d = B_d = V_d$	$= 1{,}35 \cdot 4{,}54 + 1{,}5 \cdot 7{,}03$	$= 16{,}7\ kN$
M_d	$= 1{,}35 \cdot 5{,}13 + 1{,}5 \cdot 7{,}94$	$= 18{,}8\ kNm$

chosen GL24 $b \times h = 120 \times 260\ mm$
service class 1
load duration class: medium

$A = 31{,}2 \cdot 10^3\ mm^2$	$W = 1{,}35 \cdot 10^6\ mm^3$	$I = 176 \cdot 10^6\ mm^4$
$E = 10800\ N/mm^2$		
$k_{mod} = 0{,}8$	$k_{def,g} = 0{,}6$ (permanent)	$k_{def,q} = 0{,}25$ (medium)

Bending

$$\sigma_d = \frac{18{,}8 \cdot 10^6}{1{,}35 \cdot 10^6} = 14{,}0 < f_{m,d} = \frac{0{,}8 \cdot 24}{1{,}3} = 14{,}8\ N/mm^2$$

Shear

$$\tau_d = 1{,}5\ \frac{16{,}7 \cdot 10^3}{31{,}2 \cdot 10^3} = 0{,}80 < f_{v,d} = \frac{0{,}8 \cdot 2{,}1}{1{,}3} = 1{,}3\ N/mm^2$$

Deflection

$$u_{g,inst} = \frac{5\ g_d\ l^4}{384\ EI} = \frac{5 \cdot 2{,}01 \cdot 4520^4}{384 \cdot 10800 \cdot 176 \cdot 10^6} = 5{,}6\ mm$$

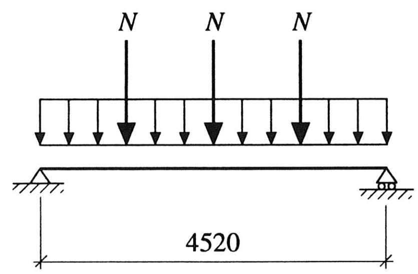
$$u_{q,inst} = \frac{5\ q_d\ l^4}{384\ EI} = \frac{5 \cdot 3{,}11 \cdot 4520^4}{384 \cdot 10800 \cdot 176 \cdot 10^6} = 8{,}7\ mm < \frac{l}{300} = 15{,}1\ mm$$

$u_{g,fin}$	$= 5{,}6\ (1 + 0{,}6)$	$= 9{,}0\ mm$
$u_{q,fin}$	$= 8{,}7\ (1 + 0{,}25)$	$= 10{,}9\ mm < l / 200 = 22{,}6\ mm$
u_{fin}	$= 9{,}0 + 10{,}9$	$= 19{,}9\ mm < l / 200 = 22{,}6\ mm$

Main beam (pos. 2 in Figure 1)

Permanent actions		
Self weight of the stringer		$= 0{,}40\ kN/m$
Floor load $1{,}78 \cdot 1{,}13 / 2$		$= 1{,}01\ kN/m$
	G_k	$= 1{,}41\ kN/m$
Reaction from floor beams	N_g	$= 4{,}54\ kN$
Variable actions		
from floor $2{,}75 \cdot 1{,}13 / 2$	Q_k	$= 1{,}55\ kN/m$
Reaction from floor beams	N_q	$= 7{,}03\ kN$

System



Internal forces and moments

$A_{g,k} = B_{g,k} = V_{g,k}$ $= 0,5 \cdot 1,41 \cdot 4,52 + 1,5 \cdot 4,54$ $= 10,0\ kN$

$A_{q,k} = B_{q,k} = V_{q,k}$ $= 0,5 \cdot 1,55 \cdot 4,52 + 1,5 \cdot 7,03$ $= 14,1\ kN$

$M_{g,k}$ $= 0,125 \cdot 1,41 \cdot 4,52^2 + 0,5 \cdot 4,54 \cdot 4,52$ $= 13,9\ kNm$

$M_{q,k}$ $= 0,125 \cdot 1,55 \cdot 4,52^2 + 0,5 \cdot 7,03 \cdot 4,52$ $= 19,9\ kNm$

Design value of the internal forces

for ultimate limit state $1,35\ g_k + 1,5\ q_k$

for serviceability limit state $1,0\ g_k + 1,0\ q_k$

$A_d = B_d = V_d$ $= 1,35 \cdot 10,0 + 1,5 \cdot 14,05$ $= 34,6\ kN$

M_d $= 1,35 \cdot 13,86 + 1,5 \cdot 19,85$ $= 48,5\ kNm$

chosen GL24 $b \times h = 160 \times 360\ mm$

service class 1

load duration class: medium

$A = 576 \cdot 10^3\ mm^2$ $W = 3,46 \cdot 10^6\ mm^3$ $I = 622 \cdot 10^6\ mm^4$

$E = 10800\ N/mm^2$

$k_{mod} = 0,8$ $k_{def,g} = 0,6$ (permanent) $k_{def,q} = 0,25$ (medium)

Bending

$$\sigma_d = \frac{48,5 \cdot 10^6}{3,46 \cdot 10^6} = 14,0 < f_{m,d} = \frac{0,8 \cdot 24}{1,3} = 14,8\ N/mm^2$$

Shear

$$\tau_d = 1,5\ \frac{34,6 \cdot 10^3}{57,6 \cdot 10^3} = 0,90 < f_{v,d} = \frac{0,8 \cdot 2,1}{1,3} = 1,3\ N/mm^2$$

Deflection

$$u_{g,inst} = \frac{5\ g_d\ l^4}{384\ EI} + \frac{N_{g,d}\ l^3}{2 \cdot 10,11\ EI}$$

$$u_{g,inst} = \frac{5 \cdot 1,41 \cdot 4520^4}{384 \cdot 10800 \cdot 622 \cdot 10^6} + \frac{4540 \cdot 4520^3}{2 \cdot 10,11 \cdot 10800 \cdot 622 \cdot 10^6} = 4,2\ mm$$

$$u_{q,inst} = \frac{5\ q_d\ l^4}{384\ EI} + \frac{N_{q,d}\ l^3}{2 \cdot 10,11\ EI}$$

$$u_{g,inst} = \frac{5 \cdot 1,55 \cdot 4520^4}{384 \cdot 10800 \cdot 622 \cdot 10^6} + \frac{7030 \cdot 4520^3}{2 \cdot 10,11 \cdot 10800 \cdot 622 \cdot 10^6} = 5,9\ mm$$

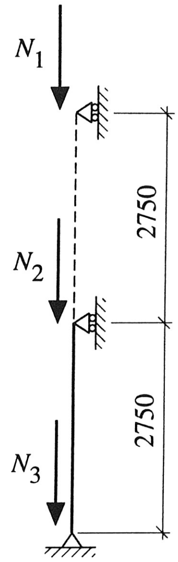
$u_{g,inst} = 5,9\ mm < l\ /\ 300$ $= 15,1\ mm$

$u_{g,fin} = 4,2\ (1 + 0,6)$ $= 6,7\ mm$

$u_{q,fin} = 5,9\ (1 + 0,25)$ $= 7,4\ mm < l\ /\ 200 = 22,6\ mm$

$u_{fin} = 6,7 + 7,4$ $= 14,1\ mm < l\ /\ 200 = 22,6\ mm$

Column, (pos. 3 in Figure 1)



Permanent actions
$N_{1,g} = 0{,}55 \cdot 4{,}52^2 \qquad = 11{,}2\ kN$
$N_{2,g} = N_{3,g} = 1{,}78 \cdot 4{,}52^2 \qquad = 36{,}4\ kN$
$N_{1,g} + N_{2,g} = 11{,}2 + 36{,}4 \qquad = 47{,}6\ kN$

the self weight of the column may be disregarded.

Variable actions
$N_{1,q} = 0{,}75 \cdot 4{,}52^2 \qquad = 15{,}3\ kN$
$N_{2,q} = N_{3,q} = 2{,}75 \cdot 4{,}52^2 \qquad = 56{,}2\ kN$

the eccentricity of the load N_2 will be disregarded.

Load combinations

Value of combinations	snow	$\psi_{0,1}$	= 0,7
	imposed load	$\psi_{0,2}$	= 0,7

Design value of internal forces and moments
$\Sigma\gamma_{G,j}G_{k,j}$ "+" $\gamma_{Q,1}Q_{k,1}$ "+" $\Sigma\gamma_{Q,i}\psi_{0,i}Q_{k,i}$

Buckling in the middle of the column

Combination 1 (permanent load + imposed load + snow)
$N_d \quad = 1{,}35 \cdot 47{,}6 + 1{,}5 \cdot 56{,}2 + 1{,}5 \cdot 0{,}7 \cdot 15{,}3 \qquad = 165\ kN$ short
Combination 2 (permanent load + snow + imposed load)
$N_d \quad = 1{,}35 \cdot 47{,}6 + 1{,}5 \cdot 15{,}3 + 1{,}5 \cdot 0{,}7 \cdot 56{,}2 \qquad = 146\ kN$ short
Combination 3 (permanent load + imposed load)
$N_d \quad = 1{,}35 \cdot 47{,}6 + 1{,}5 \cdot 56{,}2 \qquad = 149\ kN$ medium

Combination 3 is governing

chosen GL24 b x h = 160 x 160 mm
service class 1
load duration class: medium

$A \quad = 25{,}6 \cdot 10^3\ mm^2$

$E_{0,g,05} \quad = 8800\ N/mm^2$

$\lambda = l_{ef} / i \quad = 2750 / 46{,}2 = 60$

$$\sigma_{c,crit} = \pi^2 \frac{E_{0,g,05}}{\lambda^2} = \pi^2 \frac{8800}{60^2} = 24{,}1\ N/mm^2$$

$$\lambda_{rel} = \sqrt{\frac{f_{c,0,k}}{\sigma_{c,crit}}} = \sqrt{\frac{24}{24{,}1}} = 0{,}998$$

$$k = 0{,}5\ (1 + \beta_c\ (\lambda_{rel} - 0{,}5) + \lambda_{rel}^2)$$

$$k = 0{,}5\ (1 + 0{,}1(0{,}998 - 0{,}5) + 0{,}998^2) = 1{,}02$$

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}} = \frac{1}{1{,}02 + \sqrt{1{,}02^2 - 0{,}998^2}} = 0{,}81$$

$$k_c\ f_{c,0,d} = 0{,}81 \cdot 16{,}0 = 13{,}0\ N/mm^2$$

$$\frac{\sigma_{c,0,d}}{k_c\ f_{c,0,d}} = \frac{149 \cdot 10^3}{13 \cdot 25{,}6 \cdot 10^3} = 0{,}45 < 1$$

Check axial load at base (no buckling):

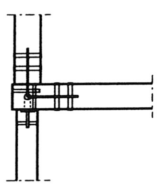
Combination 3 (permanent load + imposed load)

$N_d \quad = 1{,}35 \cdot 47{,}6 + 1{,}5 \cdot 2 \cdot 56{,}2 \qquad = 233\ kN$ medium

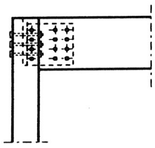
$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{233 \cdot 10^3 \cdot 1{,}3}{25{,}6 \cdot 10^3 \cdot 0{,}8 \cdot 26{,}0} = 0{,}57 < 1$$

The columns loaded by wind bracing loads will not be treated here.

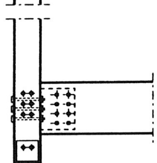
Joint stringer-column



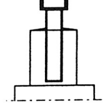
Plan view of the upper connection



Elevation showing upper connection

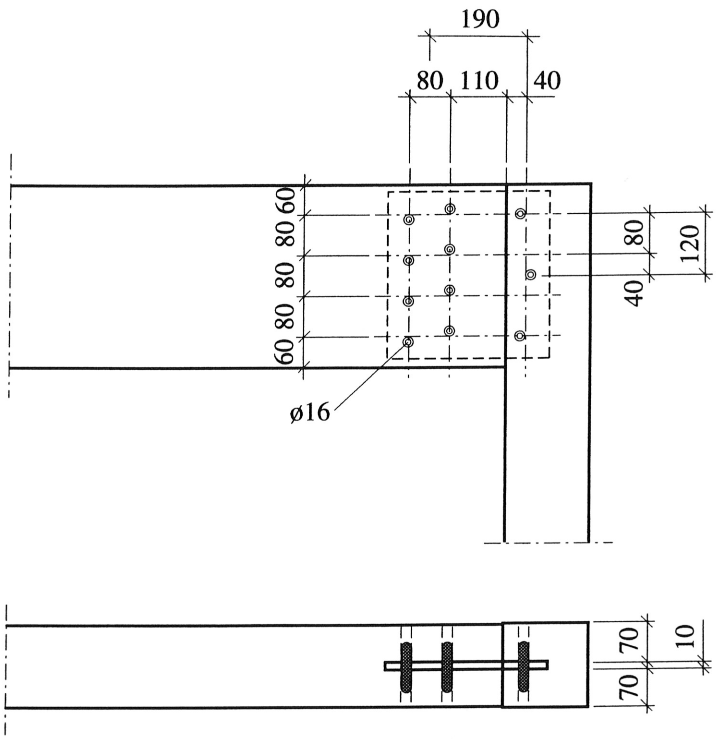


Elevation showing lower connection



Column base

The following calculations show the design of the main beam to column connection.
$V_d = 34{,}6\ kN$ (support reaction of the stringer)



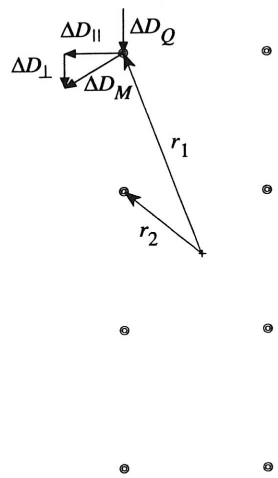
Maximum force on dowel in the column:
$D_l \quad = 34{,}6\ /\ 3 \qquad = 11{,}5\ kN$
Moment
$M \quad = 34{,}6 \cdot 0{,}19 \quad = 6{,}57\ kNm$

assumed: hinge in the column
axial force on dowel in the stringer:
$\Delta D_V \quad = 34{,}6\ /\ 8 \qquad = 4{,}32\ kN$

$$r_1 = \sqrt{0{,}12^2 + 0{,}04^2} = 0{,}126\ m$$

$$r_2 = \sqrt{0{,}04^2 + 0{,}04^2} = 0{,}056\ m$$

$$\Delta D_M = 6{,}57\ \frac{0{,}126}{4 \cdot 0{,}126^2 + 4 \cdot 0{,}056^2} = 10{,}9\ kN$$



Resultant force on dowel in main beam:
$\alpha \quad = \arctan\ (4\ /\ 12) \qquad = 18{,}4°$
$\Delta D_\perp \quad = 10{,}9\ \sin 18{,}4° \qquad = 3{,}44\ kN$
$\Delta D_l \quad = 10{,}9\ \cos 18{,}4° \qquad = 10{,}3\ kN$
$D_\perp \quad = 3{,}44 + 4{,}32 \qquad = 7{,}76\ kN$
$D_l \quad = \qquad = 10{,}3\ kN$
$\beta \quad = \arctan\ (7{,}76\ /\ 10{,}3) \quad = 37{,}0°$

$$D = \sqrt{7{,}76^2 + 10{,}3^2} = 12{,}9\ kN$$

Design value of load-carrying capacity
$k_{mod} \quad = 0{,}8$
$\gamma_M \quad = 1{,}3$ for timber

γ_M = 1,1 for steel used in joints
ρ_k = 410 kg/m^3
k_{90} = 1,35 + 0,015 d = 1,35 + 0,015 · 16 = 1,59

Embedding strength

$$f_{h,0,d} = 0{,}082\ (1 - 0{,}01 \cdot 16) \cdot 410 \cdot \frac{0{,}8}{1{,}3} = 17{,}4\ N/mm^2$$

$$f_{h,37,d} = \frac{17{,}4}{1{,}59\ \sin^2 37^\circ + \cos^2 37^\circ} = 14{,}3\ N/mm^2$$

EC5: Part 1-1: 6.2.1 (3)

Yield moment (γ_M = 1,1):

EC5: Part 1-1: 6.5.1.2 (2)

$$M_{y,d} = 0{,}8 \cdot 360 \cdot \frac{16^3}{6} \cdot \frac{1}{1{,}1} = 179 \cdot 10^3\ Nmm$$

Design value of the load-carrying capacity
β = 0°

$$R_d = \min \begin{cases} 17{,}4 \cdot 70 \cdot 16 \cdot 10^{-3} = 19{,}5\ kN \\ 1{,}1 \cdot 17{,}4 \cdot 70 \cdot 16 \left(\sqrt{2 + \dfrac{4 \cdot 179000}{17{,}4 \cdot 16 \cdot 70^2}} - 1 \right) 10^{-3} = 12{,}6\ kN \\ 1{,}5 \cdot \sqrt{2 \cdot 179000 \cdot 17{,}4 \cdot 16} \cdot 10^{-3} = 15{,}0\ kN \end{cases}$$

$R_{0,d}$ = 2 · 12,6 = 25,2 kN > 11,5 kN

β = 37,0°

$$R_d = \min \begin{cases} 14{,}3 \cdot 70 \cdot 16 \cdot 10^{-3} = 16{,}0\ kN \\ 1{,}1 \cdot 14{,}3 \cdot 70 \cdot 16 \left(\sqrt{2 + \dfrac{4 \cdot 179000}{14{,}3 \cdot 16 \cdot 70^2}} - 1 \right) 10^{-3} = 11{,}0\ kN \\ 1{,}5 \cdot \sqrt{2 \cdot 179000 \cdot 14{,}3 \cdot 16} \cdot 10^{-3} = 13{,}6\ kN \end{cases}$$

$R_{37,d}$ = 2 · 11,0 = 22,0 kN > 12,9 kN

Concluding summary

The calculations are limited to those essential to the design. Only the worst beam, stringer and column cases are covered. Wind and horizontal load effects are ignored and would not normally effect the design of a low rise structure of this type.