Spatial frames and domes

STEP lecture E20
J.C. Chilton
University of Nottingham

Objectives

To develop a basic understanding of the structural principles and behaviour of three-dimensional structural forms, such as spatial frames and domes, and to describe the benefits to be gained from their use. To discuss the major design considerations and construction details for these types of structure where timber is the primary structural material.

Summary

The lecture begins by introducing the concepts of three-dimensional and two-way spanning structures and their potential advantage over planar structures such as beams, arches and trusses. For space frames this is demonstrated by examining the load sharing behaviour of two beams spanning orthogonally connected at their mid-points. From this, the influence of the aspect ratio of the two spans on the efficiency of a two-way spanning structure is demonstrated and the influence of the number and location of supports is also addressed. Some single and double layer grid types are described. For domes, the highly-efficient, load-carrying capacity which derives from their overall, three-dimensional, structural form is explained. Examples of different dome geometries such as Schwedler, geodesic, lamella and rib are described and, where appropriate, the way in which these geometries relate to basic polyhedral forms.

Subsequently, some design and construction considerations, particularly those associated with joints, supports and movement, are outlined for both spatial frames and domes, and reference is made to the relevant sections of STEP for guidance on detailed element and joint design. Finally, a few examples of timber space frame and dome structures are briefly described to illustrate the potential of solid and glued-laminated timber for this type of construction.

Introduction

Simple beams and trusses are planar structures that span in one direction and must sustain every load that may be applied to them (including any heavy point loads) transmitting these to the two end supports. However, by appropriately connecting these planar systems with elements in the orthogonal direction, an efficient, three-dimensional, load-sharing system may be produced, that ensures that a load applied anywhere in the structure is resisted by all of its component elements. If composed of beams, such a structure is classified as a single-layer grid or grillage, whereas, if formed from trusses, it is referred to as a double-layer grid, space frame or space truss. Similarly, a three-dimensional dome distributes loads more efficiently than a single arch. The behaviour of these structures is discussed in more detail below.

The benefit of using domes for efficient, three-dimensional structures has been known since antiquity, e.g the 43 *m* span, masonry dome of the Pantheon in Rome (118-128 AD). There are also many historical examples of timber domes, e.g. the outer dome of St. Paul's Cathedral in London (1705-1708) and St. Mark's in Venice (13th. century modifications) although these are not pure timber domes but work in combination with masonry domes. However, space frames are a much more recent innovation, the pioneer of this structural type being generally recognised to be Alexander Graham Bell, the inventor of the telephone, who constructed many experimental space frame structures in the 1900s. However, space frames were not widely used until the 1950s with the development of modular systems by, for example, Mengeringhausen, Buckminster Fuller, Wachsmann, du Chateau, and Fentiman, and the wider availability of the electronic computer which enabled the analysis of these complex, three-dimensional structures. Despite the wide acceptance of space frame structures today the great majority are constructed from steel or aluminium with only limited numbers being fabricated using timber members.

Geometry and stability in 3 dimensions

To form a stable pin-jointed structure in two dimensional space a fully triangulated structure must be formed. In three-dimensional, pin-jointed structures it is a necessary but not sufficient condition for static stability that,

$$n \geq 3j - 6 \tag{1}$$

where, n = number of bars in the structure
j = number of joints in the structure
6 = the minimum number of support reactions.

A study of the stability of the Platonic polyhedra (tetrahedron, cube, octahedron, dodecahedron and icosahedron) helps in the understanding of the stability of three-dimensional structures. The tetrahedron is the minimum stable, three-dimensional, pin-jointed, bar structure. It has 4 joints or nodes connected by 6 bars or members and, given the necessary support conditions, it complies with Equation 1 above having only axial forces in the bars when loads are applied at the nodes (i.e. $j = 4$, $n = 6$ and $3j - 6 = (3 \cdot 4) - 6 = 6$). The cube has 8 joints and 12 bars, thus, $n = 12$ but $3j - 6 = (3 \cdot 8) - 6 = 18$ and the pin-jointed cube is unstable unless additional bars are introduced between the nodes or further support reactions are inserted. In the case of the octahedron $n = 12$, $j = 6$ and $3j - 6 = (3 \cdot 6) - 6 = 12$ thus it is a stable pin-jointed bar structure. Following similar reasoning, the pin-jointed dodecahedron is found to be unstable as a bar structure but the icosahedron is stable. For this reason, most double-layer space grid geometries are based on linked tetrahedral or half-octahedral modules. As the vertices of the Platonic polyhedra occur on the surface of a circumscribed sphere, dome geometries are frequently based on triangulated subdivision of the faces of these polyhedra, particularly of the icosahedron.

Two-way spanning systems

As mentioned above, load-sharing systems may be produced, that ensure that a load applied anywhere in the structure is resisted by all of its elements. This principle may be illustrated by considering an orthogonal grid system of two horizontal beams, of span l_1 and l_2, connected together at their midpoints, where a vertical point load F is applied (as shown in Figure 1). Assuming that the beams have the same material and cross-sectional properties (i.e modulus of elasticity (E) and the second moment of area (I) are the same for both) and that each beam will carry a portion of the applied load F (F_1 by beam 1 and F_2 by beam 2) the midspan deflection (u) of each beam can be calculated. For beam 1 the midspan deflection

$$u_1 = \frac{F_1 l_1^3}{48EI} \tag{2}$$

and for beam 2 the midspan deflection

$$u_2 = \frac{F_2 l_2^3}{48EI} \tag{3}$$

The beams are connected together at their midpoint, and, from consideration of compatibility, their deflections must be equal ($u_1 = u_2$). Thus

$$F_1 l_1^3 = F_2 l_2^3 \quad , \quad F_1 = \frac{F_2 l_2^3}{l_1^3} \qquad (4) \text{ and } (5)$$

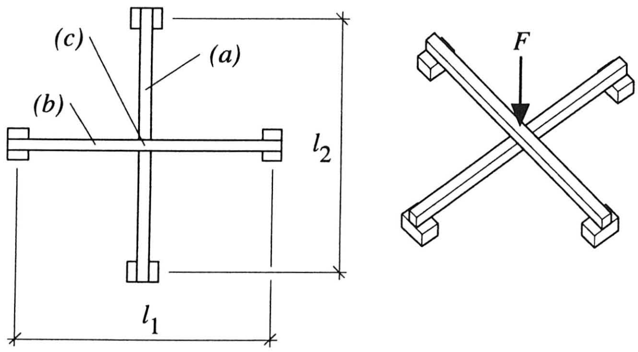

Figure 1 *Two-way orthogonal beam grid system (plan and perspective views). (a) Beam 2, (b) beam 1, (c) load F.*

Using the above equation and the fact that $F_1 + F_2 = F$, the proportion of the total load F carried by each of the two beams can be found for different span (or aspect) ratios, as shown in Table 1. This table demonstrates that, when the beams are of the same length, equal load is carried by each but that, in all other cases, the greater proportion of the load is carried by the shorter span. When the ratio of spans exceeds 2,0 most of the load is carried by the shorter beam which indicates that the benefit of two-way spanning grids is usually greatest if the structure is supported in approximately square structural bays. Alternatively, it is possible to increase the stiffness of the longer beams (usually by increasing I) to even out the load distribution in situations where one span is longer than the other.

Span ratio (l_2/l_1)		1,0	1,5	2,0	3,0
Beam 1	(F_1)	0,50 F	0,77 F	0,89 F	0,96 F
Beam 2	(F_2)	0,50 F	0,23 F	0,11 F	0,04 F

(E and I constant, l_2 longer span and l_1 shorter span)

Table 1 *Proportion of load carried by each beam for different span ratios.*

Advantages of using space frames

There are many benefits to be gained from the use of space grid structures some of which are outlined below:

- all elements contribute to the load carrying capacity,
- loads are distributed more evenly to the supports,
- there is a wide choice of support location (discussed in more detail below),
- deflections are reduced when compared with plane structures of similar weight (alternatively a lighter three-dimensional structure results in similar deflections),
- the high redundancy of space grids means that, in general, failure of one or

a limited number of elements, for instance, the buckling of a compression member, does not necessarily lead to overall collapse of the structure and consequently they have good resistance to damage caused by fire, explosion or seismic activity,

- as they are usually modular, factory-fabricated, with accurate components they are easily transportable and simple to assemble on site,
- mechanical and electrical services and air-handling ducts are easy to install within the structural depth.

Disadvantages of using space frames

There are also disadvantages to using space grids and some are given below:

- the cost, which can be high when compared with alternative structural systems, particularly when space frames are used over short spans,
- the number and complexity of joints can lead to longer erection times on site depending on the joint type and grid module chosen,
- when fire protection is required it is more expensive due to the high number and relatively large surface area of the space frame elements,
- visually, the lightweight structure can appear to be very dense when viewed from certain directions.

Grid configurations

Regular shaped grids are usually adopted for both the top and bottom layers of space grids to limit the number of different member lengths in the structure. There are only three regular polygons, the equilateral triangle, square and hexagon, that completely fill a plane with a regular tiling, thus, these are the most commonly used geometries. In square, two-way grids the grid lines have members orientated in two perpendicular directions, usually either parallel to the edges of the grid or set on the diagonal, at 45° to the edges. However, plane grids of triangles and hexagons produce three-way grids with members orientated in three directions. By combining the regular polygons or by using them in combination with other polygonal shapes (e.g. triangles with squares, triangles with hexagons, squares with octagons) more complex grid geometries may be produced.

In space grid structures, where two plane grids are separated by web members to form a double-layer grid, it is not necessary for the top and bottom grids to have the same configuration. Nevertheless, cost implications and ease of web member connection limit the number of common forms of double-layer grids. Some common configurations are shown in Figure 2 and are as follow:

- square on square - where the top grid is directly above the bottom grid and the web members connect the layers in the plane of the grid lines (Figure 2a),
- square on square offset - where the bottom grid is offset by half a grid square relative to the upper grid and web members connect the intersection points in the top and bottom grids (Figure 2b),
- square on diagonal square - where the lower grid is set at 45° to, and is usually larger than, the top grid; again with web members connecting the intersection points on the top and bottom grids (Figure 2c),
- triangle on hexagon - where the upper grid is triangular and the lower, more open, grid is hexagonal; again with web members connecting the intersection points on the top and bottom grids (Figure 2d).

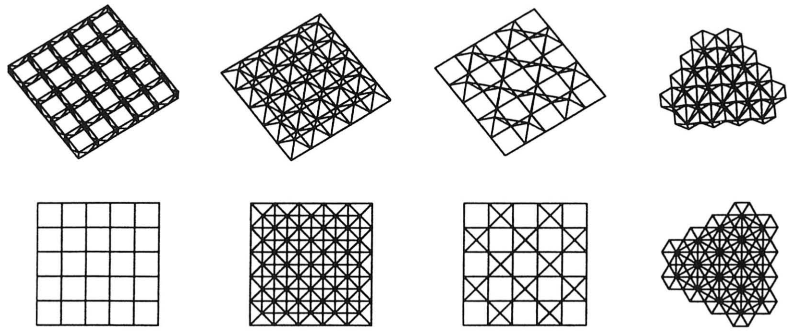

Figure 2 *Typical common double-layer grids. (a) Square on square, (b) square on square offset, (c) square on diagonal square, (d) triangle on hexagon.*

Members in the lower layer of a double-layer grid are generally in tension, i.e. not vulnerable to buckling, and may, therefore, be longer than the upper compression members, permitting a more open grid geometry in that layer. Choice of grid configuration and depth will also affect the economy of the space frame due to the cost of node joints when grid spacing is small and the larger cross sections required for compression elements to avoid buckling when grid spacing is large.

Support locations

The choice of the most favourable support locations will depend on the plan form of the structure and architectural considerations but the positions chosen will have a significant influence on structural efficiency. Either top or bottom node joints can be supported. For example, considering a space frame square in plan, providing supports for each edge node in either the top or bottom layer is most economical whilst having supports only at the corners greatly increases the maximum forces in the space frame members and the vertical deflections are also much bigger. Placing a few intermediate supports along each edge considerably improves the space grid performance compared with the corner supported condition whilst keeping the number of columns to a reasonable minimum. Single columns located at the middle of each side also produce an efficient support system, as the corners of the space frame are cantilevered and counter-balance the central area, consequently deflections and member forces in the middle are reduced (Makowski, 1981). To reduce deflections for each of the above support conditions, the supports can be brought in slightly from the edges of the space frame. This produces a cantilever around the whole structure allowing the opportunity to have column free elevations, if desired. Alternatively, both deflections and member forces in the space grid can be reduced by use of 'tree' supports instead of discrete columns.

Domes

Domes are particularly suited to covering circular or polygonal plans. Constructed from a continuous material, such as reinforced concrete, the dome is a double-curved synclastic shell. With suitable detailing of the connections between cladding and supporting structure a timber shell may also be formed, however, this lecture is confined to spatial dome structures where the cladding is assumed not to contribute significantly structurally, apart from resisting lateral buckling of compression elements. Timber shells are considered in detail in STEP lecture E21.

Dome geometry and stability

Geometrically, the surface of a dome is usually determined by the rotation of a planar arch profile about a central vertical axis. In timber domes, the three-dimensional curved surface is generally supported on some form of ribbed structure generated from curved or straight members (or both in combination) depending on the chosen structural geometry. Several different ways in which this geometry may be formed in practice are shown in Figure 3 and described below:

- radial curved ribs running continuously from a central compression ring to a perimeter tension ring and connected by a series of meridional rings and bracing members (e.g. Schwedler dome, Figure 3a),
- triangulated lattice dome (Figure 3c),
- geodesic geometry, as described by Buckminster Fuller, derived by projecting subdivisions of the faces of Platonic polyhedra (described above) onto a spherical surface (Figure 3d).

More organic curved forms can be generated using grid shells but these are beyond the remit of this lecture.

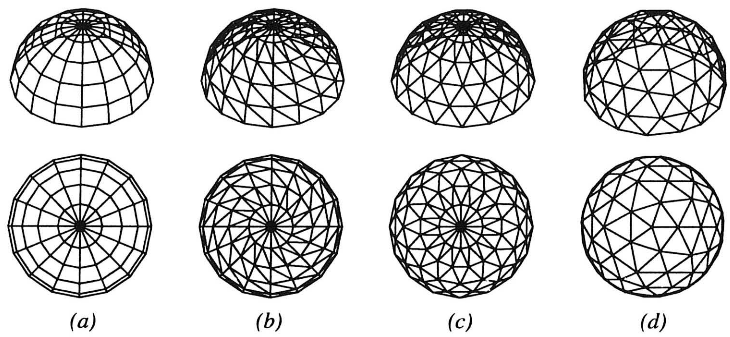

Figure 3 *Typical timber dome geometries (a) radial rib dome, (b) Schwedler dome, (c) lattice and (d) geodesic dome.*

If carefully selected for the expected load configurations, a two-dimensional arch form will be subject primarily to axial compression with relatively small in-plane bending moments. However, as with all compression elements, the arch is liable to out-of-plane buckling and will normally require lateral restraints at appropriate intervals. In a dome structure with radial ribs, the provision of suitably placed meridional rings and bracing elements between the ribs generates a three-dimensional form with high buckling resistance. Similarly, in geodesic domes, the curved surface is generally triangulated with members in axial tension or compression connected at nodes to form a stable structure.

Analysis

The analysis of spatial frames and domes is facilitated by the use of electronic computers running finite element software (usually based on the stiffness method) to evaluate member forces and deformations of the structure for critical load cases. This software can also be used to appraise the effect of initial imperfections in the structural geometry and second order effects, as required by EC5. The rise in popularity of these structural forms is undoubtedly due to developments in computer analysis.

EC5: Part 1-1: 2.3.1 & 2.3.2

Element design

In small spatial structures, members may be solid timber sections but for larger structures and curved elements glulam members are normally used. As it is quite difficult and expensive to produce rigid joints between timber members, most joints in spatial timber structures are designed (or assumed in the design process) to be pin-jointed. The individual members in most spatial frames and domes are, therefore, considered to be in axial compression or tension with only a small degree of local bending, usually associated with cladding loads. For members subject to these actions, suitable cross-sections are solid circular sections. Once the overall stability of the structure has been assessed and assured, the individual members can be designed for the combined effects of axial force and bending moment (see STEP lectures covering structural components).

Joint design

The connections between the individual elements of spatial frames and domes are normally made using steel components which must be detailed to transfer axial forces whilst minimising eccentricities which induce secondary bending in the members. Guidance on the design of joints in timber structures is given in the STEP lectures covering joints. Normally, in timber space frames, some form of metal insert is provided at each end of the timber sections and it is this that is connected to the node joint. These may be designed for a particular application or a proprietary jointing system may be used.

Construction details

Three-dimensional structures are particularly sensitive to dimensional inaccuracies in the constituent elements. For instance, in long span space frame roofs, one way of providing a fall for rainwater run-off is to generate a camber in the structure by using members of a slightly shorter length in the bottom grid. Therefore, it is essential that the overall length of members is accurately controlled during manufacture or that a means of adjustment is incorporated in the joint details.

As noted above, pin-jointed, three-dimensional structures require a minimum of six support restraints. The location and direction of these will obviously depend on the situation of the supports within the plan of the structure and the ability of the supporting structure to provide the necessary reactions to the applied actions. Typically, long span space frame structures will have at least three vertical restraints and three horizontal restraints. For instance, if one corner of a square plan structure is restrained vertically and in both horizontal directions the space frame will need at least two additional vertical restraints to prevent rotation about a horizontal axis and one more horizontal restraint to prevent rotation about a vertical axis.

Ribbed domes will usually require a central compression ring to facilitate connection of the radiating ribs, as it is difficult to devise a joint which permits all of the ribs to be joined easily at the centre of the dome. A tension ring or radial buttresses are required to resist any outward thrust from the ribs at the perimeter.

Examples of timber spatial structures

To illustrate the construction details discussed above examples of some timber spatial structures are included. The sophisticated proprietary Mero steel ball node joint is shown connecting members of glulam timber in a roof structure at Mittelstadt (Figure 4a) and a bridge in Munich (Figure 4b). As a contrast a system using roundwood poles, developed by Huybers (1987) at Delft University, is also shown (Figures 5a and 5b). In this case, the joints are formed from components fabricated from steel plate and subsequently galvanised for corrosion protection. Individual components are inserted into the roundwood poles and fixed by hollow steel dowels, retained in position by wire lacing. Then the metal connectors are bolted together at the nodes. The structure shown is the roof of a 10,8 by 16,2 *m* span agricultural building at Lelystad in the Netherlands (Huybers et al., 1987).

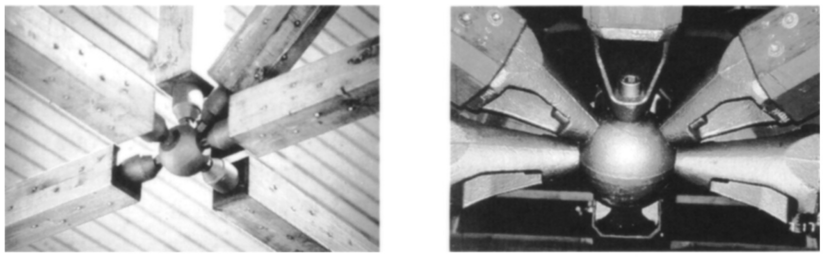

Figure 4 *Joint details of a roof at Mittelstadt (left) and a bridge in Munich (right).*

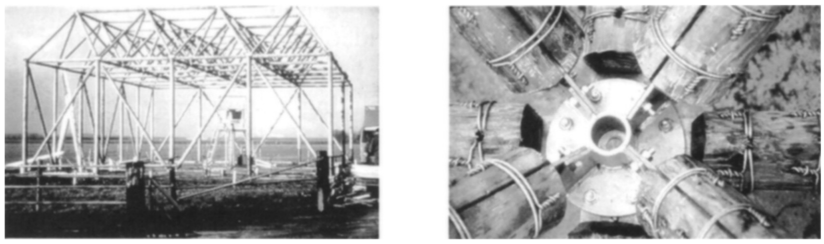

Figure 5 *Agricultural building at Lelystad (left) and joint detail (right).*

Recent examples of large span timber domes have been constructed at Northern Michigan University in Marquette (timber decked, 162,5 *m* span) and the Asemia Dome in Japan (140,7 *m* span, covered with a Teflon coated fabric membrane).

Concluding summary

- Timber spatial frames and domes provide three-dimensional, highly-efficient, load-distributing, lightweight structures for medium to long spans.
- In the case of spatial frames and for some dome configurations, short, timber elements are connected by metal connectors to generate large structures.
- The ratio of spans in two directions and the location of supports greatly influence the efficiency of spatial frames and domes.
- Most timber members within these structures can be designed as pure tension or compression elements. Bending normally only being produced by local cladding loads or secondary effects due to joint eccentricities.
- Connections between elements may be made with either proprietary node joints or purpose designed metal fasteners.

References

Makowski, Z.S. (1981). Analysis, Design and Construction of Double-Layer Grids, Applied Science Publishers, England.

Borrego, J. (1968). Space Grid Structures, MIT Press, Cambridge, Massachusetts, USA.

Huybers P., Ende, G. van der, Lanser J., and Reijken, S.Th. van der (1987). Opbouw en beproeving van een bouwkonstruktie uit larix rondhout te Lelystad, Report C4-86-03, Technische Universiteit Delft, Netherlands.