Timber shell roof structures

STEP lecture E21
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Objectives

To give an appreciation of the opportunities to create exciting building forms in in timber, covering large clear spans without internal supports, using as a basis the structural principles of shells.

Summary

Shells are thin and stiff curved membranes which act alone to provide a complete roof, including the structure, as well as the form of the roof surface. After an introduction further explaining this concept, the lecture states the applications and advantages of timber shells. It outlines how they are constructed. The historical development of structural shells is very briefly discussed. Basic theoretical principles are given, including membrane stress theory. A numerical example reinforces this. Orthotropicity, important for timber construction, is then touched upon. In the main, the lecture concentrates upon shell forms where membrane action predominates, but shells with bending, and with ribs, are also mentioned. Constructional aspects related to the standards which support EC5 are considered. The lecture concludes with two illustrated case studies.

Introduction

In the context of this lecture, a shell may be considered to be a thin and stiff curved membrane, acting alone to provide a complete roof, including the structure, as well as the form of the roof surface. The thickness of a shell roof is small compared with the other dimensions of the surface. The geometry of the form is closely prescribed, since its strength and stiffness is enhanced by its curvature. Shell forms may be of single or of double curvature, and a considerable variety of shapes is possible. In general, forms of double curvature are likely to be stiffer, for a given material thickness, than single curved types. In all instances, bending moments perpendicular to the shell surface are intentionally minimised in at least one planwise direction, through the choice of the geometrical form. In many cases, both of the orthogonal in-plane directions are associated with a curvature that leads to minimal out-of-plane moments. Normal and shear forces acting within the membrane of the shell are collected at stiffening edge beams, and are thus transmitted to the foundations of the structure. In certain cases, a small number of principal tension members may also be required to complete the stability of the shell roof.

Applications

Timber shell roofs should be considered whenever an elegant, architecturally appealing solution is sought for the two-way spanning of considerable clear areas. They are particularly suitable for structures in which large or frequent gatherings of people occur. The form of the structure is very evident, from both without and within the building. Extreme lightness of structure is achieved by the technique, but to be successful, this must be accomplished through careful design, by meticulous attention to detail, and with first-class workmanship. Shell roofs should not therefore be entertained for the more mundane building purposes. Nor are they likely to be architecturally advantageous for situations where internal load-bearing walls are essential, since these destroy the sense of "openness" of the internal space.

To construct a timber shell, a series of thicknesses of material are laid at an angle to one another, and are nail-glued together. This forms a membrane, which in its entirety is less orthotropic than would be the case for a unidirectional layer of timber. Traditionally, timber boards laid side-by-side have been used to form each individual layer of the shell. In some instances, strips of plywood have been used as an alternative. Nowadays, other wood-based panels or structural composites such as Laminated Veneer Lumber (LVL), as described in STEP lecture A9, might offer advantages. Edge beams are usually of glued laminated timber, although solid structural timber is sometimes used, on small shell components. Here again, LVL might also now be considered for such a function.

Historical development

Early historical developments

Early historical structural developments tended to favour the use of timber as a material for beams, which are clearly unidimensional elements. Indeed, there is a semantic connection between the words "beam" and "timber". At first, stone, too, was most conveniently used in this way, as monolithic columns, for example. However, as stone vaulting succeeded stone arches, two-way spanning structural forms were born. Classical, Gothic and Renaissance vaults and domes share this important characteristic with shells. The cellular, vaulted construction of the Pantheon dome in Rome, is witness to the genius of Roman engineering. Its construction is like that of a modern gridshell. It held the world record for a free-spanning dome, at 43,2 *m*, until the birth of modern reinforced concrete techniques. With these, comparable domes can now be built with a structural mass of some 1/50th of the unreinforced concrete or stone types.

Steps leading directly to the development of modern shell roofs resulted from the convergence of two paths, nineteenth century applied mathematical progress, and the use of reinforced concrete, followed by timber for two-way spanning forms.

The Surface Tension Analogy

The theories used for the determination of membrane forces in shells and other types of structural membrane, such as tent and pneumatic structures have their roots in a very fundamental early nineteenth century theory, relating to surface tension. This was expressed by de Laplace (1806), and had considerable importance in natural science, as well as in engineering. It was discovered that a fundamental law of capillarity is that a liquid film in equilibrium assumes a form which gives it the minimum possible area under the conditions to which it is subjected. A soap-film taken upon a plane wire ring will be exposed to equal atmospheric pressure on both sides, and obviously has its minimised area in the form of a plane. The wire ring may then be bent, so that it no longer lies in a plane, and the film will become curved. However, the film will remain the smallest possible surface which can be drawn continuously across the boundaries of the ring. A quadrangular wire ring thus arranged, and warped into the outline of the edge beams of a hyperbolic paraboloid shell, for example, will take up a soap film of exactly the shape of the structural shell surface, showing this form to be a surface of minimum area.

Hence it can clearly be seen that the basis of membrane theory is that all forces are assumed to lie within the shell surface, and no bending moments are deemed to exist. Adaptation of the theory in 1826 by Lamé and Clapeyron, and its perfection by Airy and Love (1927), cannot be described here, but may be pursued by those mindful of the importance of engineering history.

Developments of modern theory and practice

Theoretical methods of analysis for engineering shells, which include components such as boilers and other pressure vessels, as well as roof structures, have attracted many famous investigators, such as Finsterwalder (1932), Lungdren (1949) and Timoshenko (1959). A comprehensive and classic textbook, written in English, is that by Flügge (1960). The same author published extensively in German. This textbook also contains a comprehensive bibliography, which is recommended for those wishing to pursue the development of modern theory.

Major advances in concrete shells began in approximately 1928, when Bauersfeld and Dischinger (1928) published papers on the theory and erection of these types. A dome shell for the planetarium was constructed in Jena, at this time, and this building was followed by other notable shells, including those in Frankfurt, Leipzig and Zürich, the latter being a hyperbolic paraboloid.

The use of timber as shuttering for in-situ cast concrete shells may well have given rise to the inspiration to use timber alone. Early examples of timber shells are stated by Tottenham (1961) to have occurred in Russia, not long after the first concrete types mentioned above. These are said to have had spans in the order of 48 *m*, and to have used boarded membranes with mechanical fasteners. Little is now known about the fate of these structures. However, it is well known and documented that timber shell roof construction was developed and promoted extensively by the Timber Development Association (TDA) in the 1950's. This work led to a number of actual building projects, as well as laboratory prototypes. Examples include the roof built in 1957 for the Royal Wilton Carpet factory, which was formed from four 17 *m* span square plan hyperbolic paraboloids, and a multiple conoid roof for a railway station in Manchester, mentioned again later in this lecture.

Following the principles explained above, the theory of structural shells developed very rapidly in the decades just after the mid-twentieth century. A large number of shells in timber, using forms including hyperbolic paraboloids, cylindrical and conoid shells, dome shells and elliptical paraboloids were built in the UK and other parts of Europe. All were multi-layer boarded, with nail/staple gluing and glulam edge beams.

To these theories can now be added the advantages of cheap, high powered computational facilities. Considerable experience has also been gained in the better detailing of structures to avoid biodeterioration, and in environmentally acceptable methods of ensuring durability (STEP lectures A14 and A15). The release of the engineer and architect from the constraint of having to use only those forms whose behaviour can be understood and analyzed through methods of discrete, algebraic stress analysis, has led to many exciting and very free shell roof forms, a number of good examples of these being illustrated in Holzbau Atlas (Natterer, Herzog and Volz, 1991).

Basic Theoretical Principles

Shell roofs

Shells constructed from cross-layered timber elements, in the manner described above, will always have a radius-to-thickness ratio approximately in the order of $r/t \approx 120$ and hence will qualify for the structural theories applicable to *thin shells*.

The following assumptions are generally associated with *fundamental theories:*

- The shell material is homogenous, isotropic and linearly elastic;
- The system behaves according to small deflection theory, thus deflections under load are sufficiently small that changes in the geometry do not alter the static equilibrium;
- Only static loading and static structural response are considered.

The first assumption, regarding *isotropicity,* has been modified by certain investigators, who have shown how to handle organised *orthotropic* systems, such as those found in timber structures. This will be further discussed later. Tests, both in full-scale and on models, tend to support the assumptions of homogeneity and elasticity, at the levels of normal design. Small deflection theory has also been checked by timber researchers, using models and other tests. In addition to requiring that actions (loadings) are static, many of the theoretical derivations for thin shells deal only with uniform cases of loading. Lack of symmetry, or concentrated load effects, require special consideration.

Geometry

Conventions must be adopted in order to define every co-ordinate in a structural shell in an unequivocal manner. As an introduction to these conventions of shell geometry, consider the typical cylindrical shell element of Figure 1. It will be noted that the principal axes x, y and z are denoted using a convention similar to that adopted by EC5 (EC5: Part 1-1: 5.1.6) for beams. The thickness is denoted by t. At $t/2$ there is a middle surface, which bisects the thickness of the shell. In the case of a cylindrical shell, this middle surface is straight longitudinally, and circular transversely. The middle surface has a radius r in the transverse plane. For more complex shell elements, such as bicurved surfaces, these general conventions are retained, so that the geometry is always defined by reference to the middle surface.

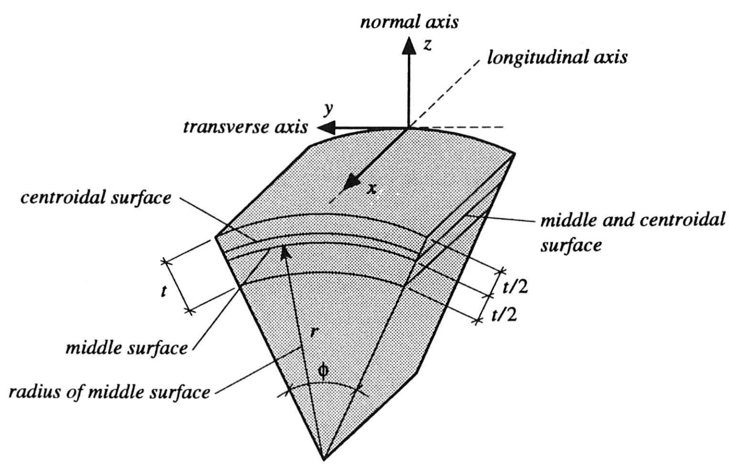

Figure 1 Cylindrical shell element.

Centroidal surfaces must also be defined. Figure 1 makes an important point, which is that in a curved shell element such as this, middle surfaces and centroidal surfaces are not necessarily coincident. In the case of an element from a cylindrical shell, the middle and centroidal surfaces are coincident in the sections (y = const) shown at the edge of the figure, but not in the sections (x = const) shown as the front face. General shell elements of arbitrary curvature will have centroidal and middle surfaces which are separate in both planes.

Stress Resultants

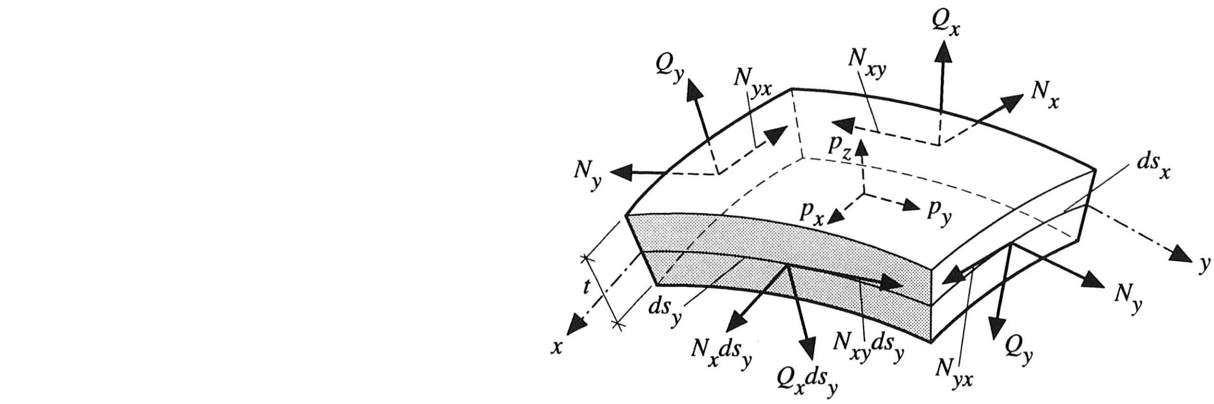

Figure 2 The loads and stress resultants acting on a shell element.

Figure 2 shows the loads and stress resultants which may act upon a bicurved shell element of arbitrary shape. The loads p_x, p_y and p_z may be considered as uniform, and unitary, for the purposes of presenting the essential theory.

The cross-hatched front edge (in Figure 2) is part of the cross section (x = const) through the shell. Since this is an element of calculus, this section is considered to have an area $ds_y\, t$. The dimension ds_y is deemed to approach zero, leading to a force divided by the length of section which reaches a finite limit. Such a quotient is termed a "stress resultant", having units such as *kN/m* for example.

For analysis, the stress resultants are resolved into three components, also shown in Figure 2.

- The normal forces N_x and N_y, considered positive if tensile and negative if compressive;
- The shear forces N_{xy} and N_{yx}. N_{xy} is considered positive if it points in the direction of increasing y on the side of the element where + N_x would point towards increasing x. N_{yx} is the reverse;
- The transverse forces Q_x and Q_y.

The three types of stress resultant which were defined as forces, and described above, give rise to corresponding normal and shear stresses, as illustrated in Figure 3. Because of the factors $(r_x + z)/r_x$ and $(r_y + z)/r_y$, the moments are not zero, even when the stresses are uniformly distributed across the thickness of the shell. Adjustment for the separation of the middle and centroidal surfaces are usually made by means of shape factors, which depend upon the geometry of the shell form. For thin shells however, such factors are seldom far from unity.

The full development of the relationships between the projected membrane forces, the loads and the shape of the middle surface of the shell requires rather lengthy explanation, and the reader is recommended to study the texts for a fuller understanding of this, especially Flügge (*op. cit.* 1960). In principle, an important first step is to relate the normal forces and the normal stresses, by means of integrals of the latter. For example, the total normal force on the element $ds_y\ t$, Figure 2, is given by the expression:

$$N_x\, ds_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x\, ds_y\, \frac{r_y + z}{r_y}\, dz \tag{1}$$

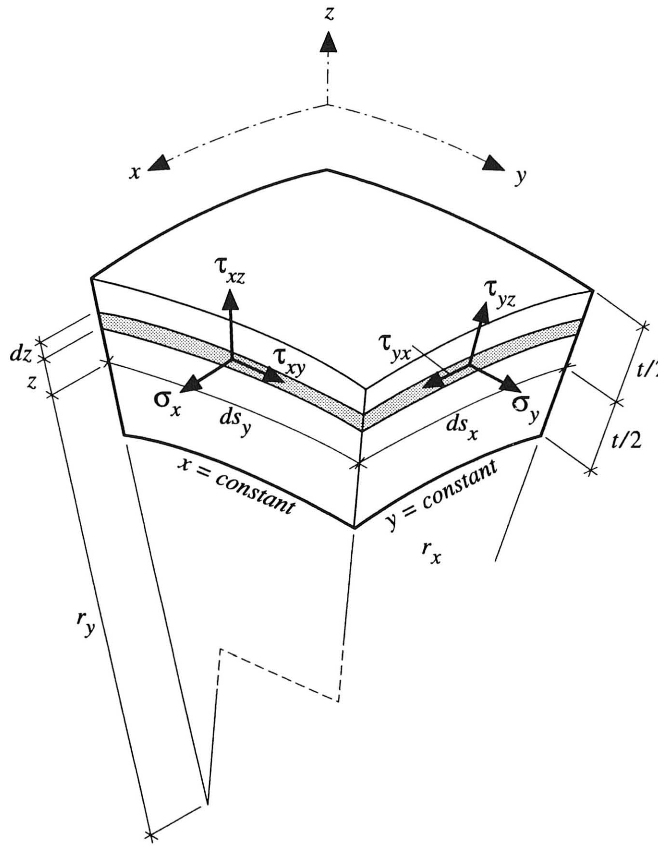

Figure 3 *The stresses acting on a shell element of curvatures r_x, r_y; cross hatched area is an element of differential magnitude.*

Corresponding relationships exist between all six in-plane stress resultants and their respective stresses. It should be noted that the integrals show that the equality of the shear stress $\tau_{xy} = \tau_{yx}$, does not necessarily imply equality of the shearing forces, although if t and z are small compared with r_x and r_y, the difference may be negligible.

When the stresses are not distributed uniformly across the thickness, some of them are in reality associated with moments with respect to the centre of the section, M_x and M_y, and there also exist two twisting, or warping moments, M_{xy} and M_{yx}. Since it is the intention of this lecture to present only the essentials of the more elementary membrane theory, these within-thickness moment effects are not further detailed. Furthermore, in the design of timber shells, (Keresztesy, 1966) analysts have generally tended to check by means of experimental stress analysis, for example using accurate, scaled structural models, in order to establish the domain of the simpler membrane theory, and thus avoid dealing with moment complexities.

Hence, in the simplest of cases, it is possible to consider in-plane forces only, Figure 4. These are considered to be distributed uniformly over the thickness. Responses of the structure to the actions which it experiences may be taken to include the stress resultants which are used in shell theory to denote the force per unit length on the middle surface. Thus the *stress resultant* N_x acting on the transverse section of the shell element in Figure 4, when divided by shell thickness t, yields an actual design stress, which would correspond to EC5 symbols such as $\sigma_{t,0,d}$ for a beam element (EC5: Part 1-1: 5.1.2).

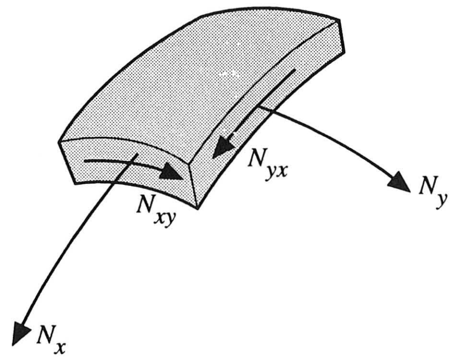

Figure 4 Membrane stress resultants.

Membrane Stress Theory

In many thin-shell analyses, the membrane theory has provided a reasonable basis for the design, except at the boundaries, where the shell is supported or stiffened. Flügge (*op. cit.* 1960) describes the adaptation of membrane theory as a "spectacular simplification". He goes on to explain that:

"the inadequacies of the membrane theory can be discovered by a critical inspection of the membrane solutions, without any need for first solving the bending problem - a task which often enough is out of reach of the practical engineer and even of the research worker".

Local restraints usually exist at boundaries; edge members need to be added to receive the reactions indicated by membrane theory, and these edge members in turn introduce their own mass and rigidities. Bending effects at such boundaries are usually evaluated by approximations, to make allowance for the effect of the edge loads and displacements on both the normal stress resultant and the introduced shell moment.

Following membrane theory it can be shown by examining the equilibrium of the differential moments that the transverse forces Q_x and Q_y cancel out. Also by equilibrium, the two shearing forces equal one another:

$$N_{xy} = N_{yx} \qquad (2)$$

Thus of the ten stress resultants belonging to the general theory, only three remain: N_x, N_y and $N_{xy} = N_{yx}$.

Three equations of force equilibrium exist, which may be used to solve these stress resultants. It should be noted that although the normal forces are indicated as positive, or tensile, in Figure 4, there are types and regions of shell where these are compressive.

Shells of Arbitrary Shape

Shells described by analysts as being 'of arbitrary shape' include all those types with bicurved surfaces, such as hyperbolic and elliptic paraboloids, often built in timber. Hyperbolic paraboloids have been especially popular in timber because certain of the generator lines giving rise to this shape can be produced as a 'ruled surface'. The development of the differential equations for the membrane stress, however, is quite complex. The principles are demonstrated on the basis of a form with simpler equations in the following.

Direct stresses in shells of arbitrary shape can be computed using a differential equation attributed to A. Pucher (1934) who discovered the usefulness of Airy's stress function for the purpose.

The following is a brief overview, succeeded by a simple worked example. The reader is referred to the classical literature for further study.

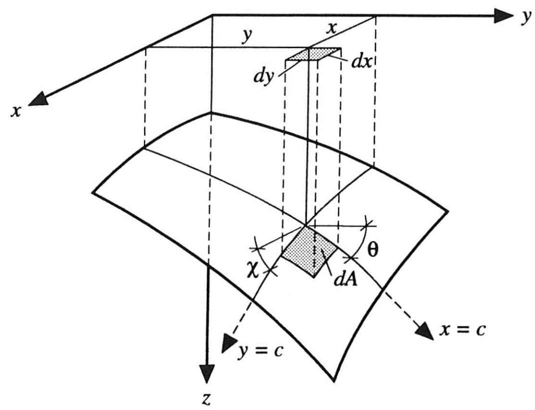

Figure 5 Shell of arbitrary shape in rectilinear co-ordinates.

Figure 5 shows a shell of arbitrary shape, defined geometrically in rectilinear co-ordinates. Vertical co-ordinates of the middle surface, the z plane, are expressed as a function of x and y.

On the middle surface, the co-ordinates $x = y =$ const do *not* meet at right angles to one another. Hence the horizontally projected plane shown at the top of the figure is constructed to reflect the co-ordinates orthogonally.

To develop the differential equations, the skewed stress resultants are also projected onto this horizontal plane as shown in Figure 6.

The projected membrane stress resultants are obtained by differentiation, giving the following:

$$\overline{N_x} = N_x \frac{\cos \chi}{\cos \theta} \tag{3}$$

$$\overline{N_y} = N_y \frac{\cos \theta}{\cos \chi} \tag{4}$$

$$\overline{N_{xy}} = \overline{N_{yx}} = N_{xy} = N_{yx} \tag{5}$$

where *tan* $\theta = dz/dy$ and *tan* $\chi = dz/dx$ (Figure 5).

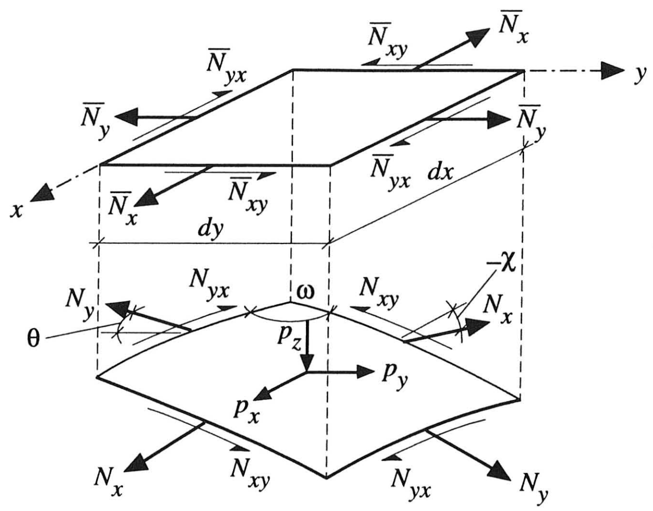

Figure 6 *Shell element with membrane stress resultants projected onto horizontal plane.*

To assist with the solution of these basic equations, expressions are written which make use of an auxiliary variable, the stress function Φ. This is the step based upon Airy's principle:

$$\overline{N}_x = \frac{\partial^2\Phi}{\partial y^2} - \int \overline{p}_x\, dx \qquad \overline{N}_y = \frac{\partial^2\Phi}{\partial x^2} - \int \overline{p}_y\, dy \qquad \overline{N}_{xy} = -\frac{\partial^2\Phi}{\partial x\,\partial y} \tag{6}$$

Hence the differential equation for Φ becomes:

$$\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - 2\,\frac{\partial^2\Phi}{\partial x\,\partial y}\,\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial^2\Phi}{\partial y^2}\frac{\partial^2 z}{\partial x^2} = q \tag{7}$$

in which

$$q = -\overline{p}_z + \overline{p}_x\frac{\partial z}{\partial x} + \overline{p}_y\frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial y^2}\int \overline{p}_x\, dx + \frac{\partial^2 z}{\partial y^2}\int \overline{p}_y\, dy \tag{8}$$

and where $\overline{P}_x$, $\overline{P}_y$ and $\overline{p}_z$ are loads per unit area of the horizontal projection. If it is necessary only to consider vertical loads, then $p_x = p_y = 0$, and $q = -\overline{p}_z$, another considerable simplification.

Example

Consider a curved paraboloid roof shell, which is triangular in plan (see Figure 7). Shells of this type have been built in timber to form part of a series of interconnected roofs over a building (Natterer *op. cit.* 1991). The reason for this choice as a worked example is that such triangular paraboloid shells can be shown to perform closely in accordance with simple membrane theory, and that the particular form of the Pucher equation from the stress resultants is also simple.

Each individual shell unit can be considered to be cut from a paraboloid surface which is developed from a square planform, that is with only one rise $h_1 = h_2 =$ const. Such a paraboloid of revolution has the equation

$$z = \frac{x^2 + y^2}{h} \tag{9}$$

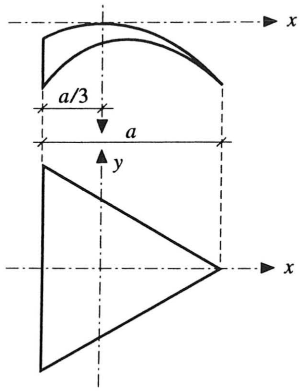

Figure 7 Triangular paraboloid shell.

In relation to the general differential equation for the stress resultants of a membrane shell, one of the terms on the left hand side vanishes, and the coefficients of the other two become constant. Furthermore, restricting consideration to $q = -\overline{p_z}$, (vertical loading only), the differential equation simplifies to:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = -\frac{1}{2}\, h\, \overline{p_z} \qquad (10)$$

This is known as the plane-harmonic equation, and its solution is stated in mathematical texts as follows:

$$\phi = -\frac{1}{8}\, p\, h\, [x^2 + y^2 + \frac{1}{a}(3\, xy^2 - x^3)] \qquad (11)$$

where a is an arbitrary distance from the co-ordinates origin.

Applying the equations shown above for the projected membrane stress resultants, together with the simplified differential equation one obtains:

$$N_x = -\frac{p\, h}{4}(1 + 3\, \frac{x}{a}) \sqrt{\frac{h^2 + 4x^2}{h^2 + 4y^2}} \qquad (12)$$

$$N_y = -\frac{p\, h}{4}(1 - 3\, \frac{x}{a}) \sqrt{\frac{h^2 + 4\, y^2}{h^2 + 4\, x^2}} \qquad (13)$$

$$N_{xy} = \frac{3}{4}\, p\, h\, \frac{y}{a} \qquad (14)$$

Numerical results for a shell of this type with $a = h$ and plotted in terms of *N/ph* are shown in Figure 8. The reader will find that if typical values for timber structures are entered into the parameters obtained from this diagram, *N/ph*, and if normal stresses are estimated using a typical thickness, say 57 *mm*, then the apparent membrane stresses are very low. This is usual for such structures, and is in agreement with tests, such as those reported by Keresztesy (*op.cit.* 1966).

Orthotropicity

As is evident from the basic theoretical principles explained above, the membrane stress resultants in a shell may be either tensile or compressive, depending upon the form of the shell and the region of the membrane under consideration. It has been shown that the loads and stress resultants acting on a shell element must come into equilibrium, in accordance with the fundamental theory. The membrane analogy suggests that an ideal shell material would be *isotropic*, whereas of course the individual boards of timber or wood-based material forming a timber shell roof are significantly *orthotropic* in their behaviour. Textbooks such as that by Bodig and Jayne (1982) give a clear introduction to the equations for the orthotropic elasticity of wood and wood composites.

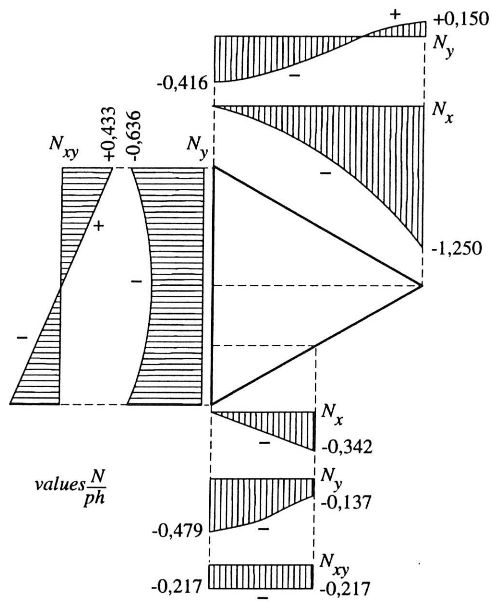

Figure 8 *Numerical values (N/ph) of the stress resultants of the paraboloid shell example.*

Orthotropic composite systems have long been recognised in timber technology, one of the best known and longest established being plywood. Special plywoods are also known, with certain veneer types arranged such that the longitudinal axis (grain direction) is neither at 0° nor at 90°, but is positioned at some intermediate value, aimed at establishing an overall composite sheet with near-isotropic properties. These have been used in aircraft manufacture. The individual boards in a shell roof have to be laid up in a workshop, or even possibly on site, rather than in a plywood press. However they can be arranged in a similar fashion to plywood, thus providing the degree of orthotropicity required. With particular types of shell, such as hyperbolic paraboloids for example, the form generator directions may also be taken into account in planning the layup, so that each layer of boards may for example require to be either twisted or bent, but not both.

Those boards forming the individual layers which receive most tensile stress are able to resist such forces without difficulty, timber being a material of high tensile strength and stiffness to mass ratio, particularly when defects such as sloping grain are eliminated by strength grading. However, unless the individual layers are thoroughly connected to one another, there would be a tendency for the pieces which are in compression to buckle about their less-stiff axis, causing a distorted and possibly unserviceable roof form. There would also be problems if layers were insufficiently joined at the edges of the shell, where strong shear connections to the edge beams may be required. Unconnected layers could inadvertently introduce twists into such beams. For these reasons, it is recognised practice substantially to join the individual layers to one another, at least with mechanical fasteners, and in most instances with both fasteners and adhesive. It should be noted that in this second case, the adhesive should be regarded as the primary joining medium, and the fasteners only as a means of ensuring glue pressure, since the difference in stiffness of fasteners and adhesives renders the two systems non-additive.

The following explains briefly how the effective modulus of elasticity E_s at a specified angle θ from any convenient axis can be computed, for a layered shell composite, thus enabling it to be considered as a thickness-homogenous material, obeying Hooke's Law. This procedure considerably simplifies the relationship between stresses and strains in the membrane. It depends, of course, upon the layers being properly connected, as described above.

Rectilinear orthotropicity is assumed, that is to say the effect of growth ring orientation is ignored. Hence $E_R \approx E_T$, and two of the Poisson's ratios are taken to be similar, namely $\nu_{LR} \approx \nu_{LT}$. This is a simplification of practical significance, since the method of sawing to define growth ring orientation cannot usually be specified, in any case.

Then, for each single, orthotropic, straight grained layer, denoting E_1 and E_2 as the moduli of elasticity parallel and perpendicular to the grain respectively, the effective modulus of elasticity at an arbitrary angle θ to the grain is given by:

$$E_\theta = \frac{E_1}{\cos^4\theta + \left(\frac{E_1}{G} - 2\,\nu\right)\sin^2\theta\,\cos^2\theta + \frac{E_1}{E_2}\sin^4\theta} \qquad (15)$$

Suitable approximate relationships are:

$$\frac{E_1}{G} = 16 \qquad \frac{E_1}{E_2} = 20 \qquad \nu = 0{,}4 \qquad (16)$$

Then, denoting the thickness of one layer and the number of layers by t and n respectively, the effective modulus of elasticity at angle θ of the complete shell is given by:

$$E_s = \frac{t_1\,E_{\vartheta,1} + t_2\,E_{\vartheta,2} + \ldots.\; t_i\,E_{\vartheta,i}}{t_1 + t_2 + \ldots.\; t_i} \qquad (17)$$

Where experimental stress analysis has been undertaken, either on models or on a full-sized structure, it will be necessary to relate the measured strains to equivalent membrane stresses following the above principles. This can be carried out by a simple averaging process as follows.

If $\sigma_t = \varepsilon_t\,E_s$ is the stress in the top fibre, and $\sigma_b = \varepsilon_b\,E_s$ is the stress in the bottom fibre, at an arbitrary measurement co-ordinate, where ε_t and ε_b are the measured strains, then the equivalent membrane stress is given by:

$$\sigma = \frac{\sigma_t + \sigma_b}{2} = \frac{E_s}{2}\,(\varepsilon_t + \varepsilon_b) \qquad (18)$$

Shells involving bending

As explained in the introduction, shell forms are provided with curvature, in order to obtain increased stiffness, within a relatively thin membrane. In general, forms having double curvature are stiffer than single-curved types. In barrel vaulted and conoid shell forms ideal, moment-free behaviour cannot possibly be attained, since there is substantial longitudinal beam action. This is illustrated qualitatively for a barrel shell of the proportions that might be built in timber in Figure 9. Similar bending effects in timber conoids have been noted (Tottenham, *op. cit.* 1961). In such shells, the actions may be considered to create a response of compression, tension and shear, all in the plane of the middle surface.

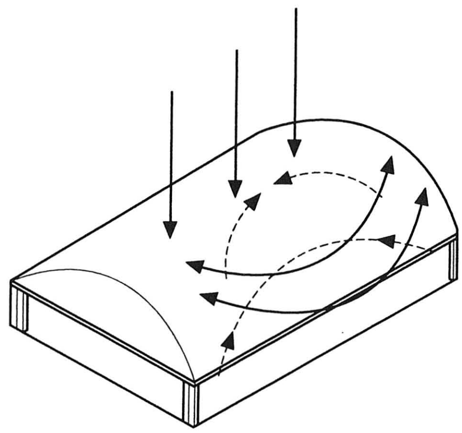

Figure 9 In long barrel shells, beam action is significant.

Long barrel shells

Long barrel shells may be discussed as a typical example of such forms, in which these types of bending action are important. Barrel shells have transverse sections which are segments of cylinders, or which are occasionally of other closed curved profiles, such as ellipses. They may be considered to transfer load by a combination of longitudinal beam action, and transverse arch action. Short barrels have been constructed from materials other than timber, to build aircraft-hangers for example, but in these types the structural action is predominately arch-like. In long barrels on the other hand, defined arbitrarily by a ratio in the order of $r/L \leq 0,6$ any remaining arch action occurs only near the crown, and the bending is very significant.

For such long barrels, in-plane stresses are approximated well by

$$\sigma_m = \frac{N_x}{h} = \frac{M_x\, y}{I} \qquad (19)$$

$$\tau_{mean} = \frac{N_{x\phi}}{h} = \frac{V\,\theta}{I\,b} \qquad (20)$$

where

M_x is the bending moment about centroidal axis.

I is the moment of inertia of shell cross section.

V is the total shear at cross section.

b is the total cross-sectional thickness of shell measured horizontally.

Figure 10 compares the bending stresses calculated by this approximation, with those given by more comprehensive barrel shell theories. It can be seen that the correspondence, for r/L = long = 0,2 is good, and that as the proportion of the shell becomes shorter, the maximum tensile stress is progressively underestimated by the simpler theory.

Barrel shells have been of considerable economic importance in concrete construction, and a number of publications are available explaining their design more fully, and providing tables of coefficients for the stress resultants. These may provide a first approximation for proposed solutions in timber, bearing in mind the necessary adjustments for orthotropicity. Explanations of edge beam design procedures, and means for the satisfactory design of multiple roof forms, can also be found in such literature.

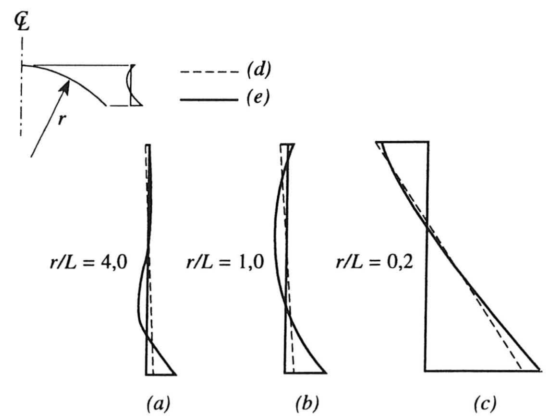

Figure 10 *The bending theory approximation for barrel shells. (d) Beam method, (e) shallow-shell theory.*

Conoids

Conoid shells in timber were introduced in the early 1960's. Following the first example, which was a railway station in Manchester, and which is currently under consideration for listed building status and for major refurbishment, a number of similar types were built elsewhere in Britain, and also in the Netherlands, Natterer (*op. cit.* 1991). In the town of Yeovil, England, for example, a market was constructed using six pairs of conoids, each varying in length from 5 to 7 metres, in span from 12 to 18 metres, and in span/rise ratios ranging from approximately 5 to 1, to 10 to 1 (Booth, 1966). Substantial research including load testing of 1:2.5 scale models, and prototype testing of laminated chord trusses, was undertaken, prior to these projects.

Stiffened Shells

Diverse forms of timber shell are possible, in which the membrane is stiffened by means of ribs, arches or gridwork arrangements. Some discussion of the theories for gridwork shells may be found in the classic texts such as Flügge (*op. cit.* 1960). In general, the approach is via anisotropic elasticity principles. Naturally however, with such unique forms of structure, each case must be considered on its own merits, and if favoured, then analysed as a special project. Specialist software is now available to ease the path of such investigations.

For an exhibition hall in Lausanne, Switzerland, (Natterer and MacIntyre, 1993), a timber shell named as a 'polydome' was chosen. This had a 25 *m* span form based upon a shallow slice taken from the crown of a circular dome. It was supported at four abutment positions, which were tied through the foundations. The gridwork was formed from intersecting laminated boarding, which was assembled at ground level on site, and subsequently lifted onto simple falsework for completion of the membrane.

Site assembly considerations often lead to a choice of forms other than the purer membrane types, which were described initially. The difficulties of erecting shell components on an inaccessible site, over piers in the middle of the river Thames, and the consequent necessity to break the form into prefabricated sections, was one of the reasons for the choice of stiffened timber shells in the Thames Barrier project, described in the first case study below.

Architecture

There is no doubt that timber shell roofs offer the possibility of creating exciting building forms that can cover large spans, without internal supports, and with openings or glazed areas in the perimeter walls. Equally, there are the general advantages of timber structures, such as high strength and stiffness to weight ratios, ease of fabrication, and energy saving benefits both in production and in use (STEP lecture A16).

Armed with a knowledge of EC5, which is a most modern design code, containing the best of information derived from tremendous international co-operation, it should be a realisable challenge for the structural engineer to participate in teamwork with the architect and others to revive the best features of these elegant and slender structural forms.

Choice of form

The choice of shell form will depend upon a number of factors. Some of the more important are as follows:

- the general plan shape of the area to be covered;
- the number and position of supports that can be accepted;
- the position and extent of any roof lighting which is required;
- for certain shell types or combinations, the acceptable position of ties.

Table 1 gives geometrical definitions for three bicurved shell forms commonly used for roofs.

Shell form	*Geometrical Equation*	*Description*
Elliptic Paraboloid	$z = \frac{x^2}{h_1} + \frac{y^2}{h_2}$	Intersections with planes, x = const and y = const, both parabolae, of two different sizes
Hyperbolic Paraboloid, edges bisecting directions of generators	$z = \frac{x^2 - y^2}{2c}$	May be stretched in x direction to provide rectangular plan, with $z = \frac{x^2}{h_1} - \frac{y^2}{h_2}$
Hyperbolic Paraboloid, edges parallel to generators	$z = \frac{xy}{c} = \frac{xy}{ab}h$	Intersections with vertical planes, x = const and y = const are straight lines, the generators

Table 1 Geometrical definitions of three bicurved shell forms commonly used for roofs.

A range of shell types which are known to have been used for timber construction is shown in Figure 11, whilst Table 2 shows the approximate range of economically feasible dimensions for these forms.

Individual shell-form components can also be combined into multi-shell roofs. A range of possible types is illustrated in Figure 12.

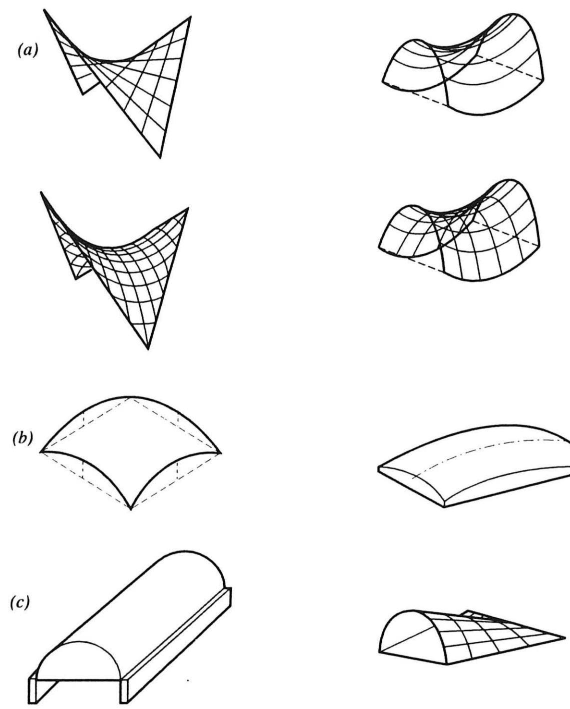

Figure 11 *Range of shell types commonly used in timber construction. (a) Hyperbolic paraboloids, (b) elliptical paraboloids, (c) long barrel vault (left) and conoid (right).*

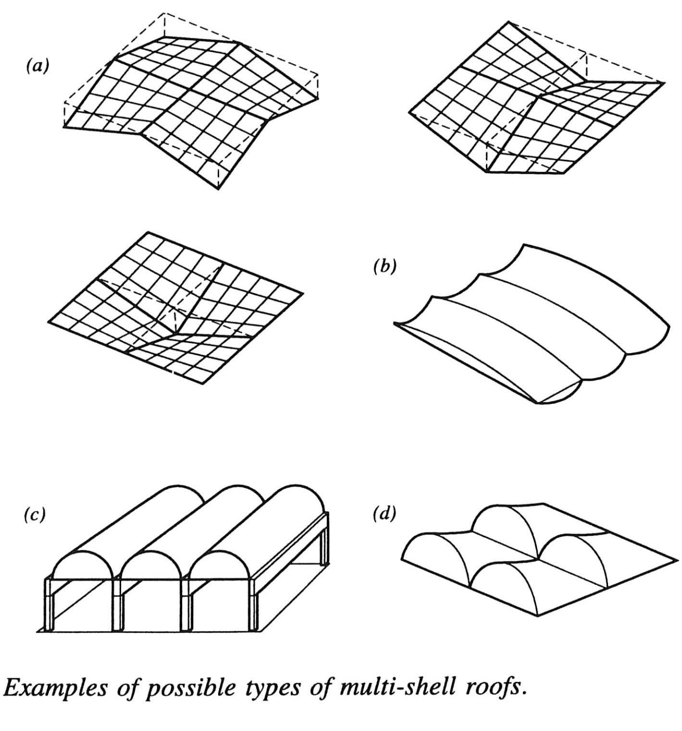

Figure 12 *Examples of possible types of multi-shell roofs.*

Shell form	Width (span) *m*	Length *m*
Hyperbolic paraboloid	7,2 - 24	7,2 - 24
Elliptical paraboloid	7,2 - 30	7,2 - 30
Barrel-vault shell	7,2 - 20	12 - 36
Conoid	12 - 27	7,2 - 12

Table 2 *Approximate range of economically feasible dimensions for the forms shown in Figure 11.*

Construction and specifications

Timber shell components normally consist of two fundamental parts. These are the shell membrane itself, and the edge beams. In some types, stiffening ribs may also be present.

The membrane is constructed from two or more layers of boarding, plywood or other composite. These layers are fastened together, either in the workshop or on site, using a combination of adhesive and mechanical fasteners. Figure 13 illustrates this taking place. Glued laminated timber (glulam), other structural composites, or in the case of small shells, solid timber may be used for the edge beams.

Figure 13 *Fastening of the membrane layers.*

Stages and details of construction to consider are as follows.

Solid timber

This is most commonly used for the cross-layered boarding of the membrane. The solid timber should be visually or machine strength graded (see STEP lecture A6). Visual grading should be carried out according to standards which fulfil the minimum requirements of EN 518 "Structural timber - Grading - Requirements for visual strength grading standards". Machine strength grading must meet the requirements given in EN 519 "Structural timber - Grading - Requirements for machine strength graded timber and grading machines". Boards of a thickness of only approximately 20 *mm* may be required, and not all visual grading rules which conform to EN 518 or grading machines which comply with EN 519 may be capable of interpretation/operation to produce such a small thickness. Enquiries of suppliers should therefore be made at an early stage in the design. The specialist timber engineering fabricator may undertake this grading himself.

Strength classes
A strength class system has been established in EN 338 (see STEP lecture A7). Softwood boarding for the membrane, and if solid strength graded timber is used, the softwood for the edge beams, will be selected from one of these strength classes. This will enable the characteristic values of the material properties to be determined by the designer.

Glued laminated timber
The production of glued laminated timber (glulam) and its strength classification system is described in STEP lecture A8. European standards are either available already, or shortly to be published, covering a number of essential aspects. These include classification and performance of the glulam adhesives; finger jointing standards for laminations and for complete glulam members; and production requirements including delamination and glue line shear tests for quality control purposes.

The five strength classes for glulam, defined in EN 1194, will provide a means of determining the characteristic values of strength, stiffness and density for this material.

Additional aspects
Shrinkage and distortion effects within the elemental constituents of the shell (membrane and edge beams) are likely to give rise to unserviceablility and even possibly to instability unless materials are correctly dried prior to stress grading, and other production processes are correctly carried out. Small deflection theory was mentioned in *Basic Theoretical Principles*. The performance of shell structures may be sensitive to departures by the actual constructed form from the initial, theoretical shape which was assumed in the design. It is important that throughout both the workshop prefabrication stages (if any, see below), and the construction work on site, recognition is given to this factor. Suitably strict quality control measures are thus called for, at all stages in the work processes. The significance of moisture in timber is thoroughly explained in STEP lecture A4. At least the outer layers of the shell boarding, and preferably all layers, should be formed from boards which are end-jointed in accordance with EN 385 "Finger jointed structural timber". Serious consideration should be given to the specification of preservative treatment for both the solid timber, and also for the glulam elements. This is a topic giving rise to a number of aspects and requiring cross-references to other European standards. It is well covered however in STEP lecture A15. Both glulam and shell membrane elements may be treated by vacuum, or vacuum pressure methods, after manufacture, but enquiries should be made as to the size of the facility. An alternative may be to consider the choice of a species which is sufficiently durable for the performance required under the relevant service class and exposure conditions. Cases have occurred where the choice has been to treat the shell boarding timbers, but to use a naturally durable hardwood for the glulam, and all such possibilities should be carefully investigated at the initial design stage.

Manufacture
Where possible, it is recommended that manufacture in a workshop should be considered, with the shell components broken down into prefabricated portions

of a size capable of being delivered to site and erected. Sometimes, it may be possible to arrange a temporary covered area which is sufficiently well controlled for manufacture adjacent to the site. Obviously, such decisions should be contemplated at the design stage, since they influence important details.

Cross-layered membrane boarding is laid up on formers, which may be constructed from items such as solid timber, plywood, or tubular scaffolding. The costs of providing this formwork are not insignificant, and these must also be considered at the design planning stage. The formwork must be constructed in a sufficiently stiff, rigid manner to ensure that the correct shell shape is created, and that tight contact is obtained when the successive layers of boarding are nail-glued together.

The need to achieve a reasonably balanced construction has been mentioned under *Orthotropicity*. Suitably chosen angles between the successive layers of boarding will ensure a membrane which is approximately isotropic through the total thickness. Not only is this desirable to comply with the structural theory, but it also ensures that the shell does not warp when changes in moisture content occur through seasonal variations. The use of thin, and relatively narrow boards in a succession of layers is also advisable, since the boards, unlike the infinitely thin theoretical generator lines, have a distinct cross-section. Even with straight line generator forms such as hypars therefore, they require to be twisted during the lay up process, in order to achieve the desired shape. For these several reasons, thicknesses of boards usually range from 12 *mm* to 30 *mm*, and widths are generally less than 100 *mm*. Edges may either be loosely tongued and grooved, or plain.

Examples of Timber Shell Roofs

Case Study 1

The Thames Barrier Shell Roofs
To safeguard London from the possibility of flooding, a moveable tidal barrier was constructed in the early 1980's (Johnson, 1980). The Thames Barrier is located on the river at Woolwich Reach, below the City of London. It takes the form of a series of rising sector gates, which are normally swung down, beneath the water, and retained in semi-circular cills on the river bed. Ships are able to pass between the piers above these cills. When there is a danger of surge tides flowing up the river, the 200 *m* span steel sector gates can be raised to check the level of the water.

The project incorporates seven timber shell roofs, Figure 14, which form a major architectural feature seen everyday by many thousands, both from the river and also from the air when approaching London Heathrow. The roofs cover the flood-gate operating machinery, and are positioned on each of the concrete piers. Five of the roofs measure 19 *m* in height, and are 11 *m* wide and 24 *m* long. Two smaller roofs of a similar form also exist. The shells are of an arch-stiffened type, rather than being pure membrane shells. This was a structural decision resulting from both geometrical and constructional considerations, as explained below. The roof surfaces are of a complex double curvature, and the shape is generated by the rotation of the plan of each pier about two centres, which are defined on the elevations. The shells are of triple boarded construction, and are clad on the exterior with stainless steel. When viewing the structures from the river, it is possible to see the laminated timber arches, and also part of the undersides, which are unclad, and obviously of timber construction.

A requirement of the design brief was that prefabrication should be maximised, in order to speed the site work, since expensive crane barging, critical shipping closures and tide timetables were involved in the operation. Furthermore, it was envisaged that certain parts of the structures might be damaged by shipping collisions. In this event, components were to be demonstrable and replaceable. This proved a wise decision, since on one occasion it was necessary to put the plan into effect. Such considerations, together with the inappropriateness of the required geometry for any of the pure shell types which are described in the literature, led to the choice of the arched-ribbed method of shell consideration.

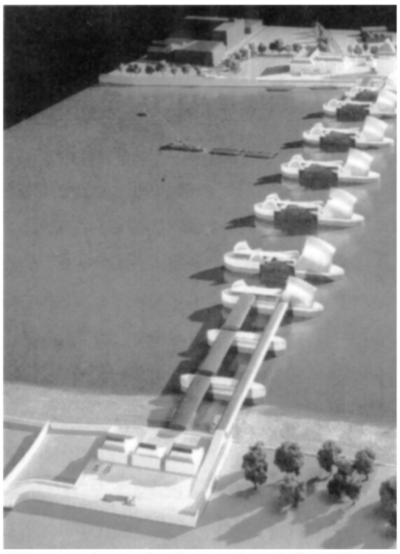

Figure 14 The timber shell roofs of the Thames Barrier.

Each roof contains four main glulam arches, which are made from a hardwood timber which is classified as 'very durable', and which has been proven by experience to be suitable for such rigorous exterior exposure conditions. Twist-laminated edge beams, having double curvature, are used to frame the shells. The roof membrane is formed from three layers of European redwood boarding. The first layer, 22 *mm* thick, was applied at an angle of 45° to the arches. The second layer, 32 *mm* thick, was laid horizontally, and the third (outer) layer, also 22 *mm* thick, was crossed at an opposing 45°. These softwood boards had been pressure treated with preservative, and a number of details were incorporated in the design, in order to ensure the free circulation of air in certain areas where decay might otherwise occur.

As a means of confirmation and support for the several stages of structural analysis and design, which included numerous computer runs, a structural model was built and tested, Figure 15. This 1/5 scale model was an exact replica of the real structure, with everything scaled down in minute detail, including the use of the same types of timber, scaled-down fasteners, and replicas of all other important aspects. The model was load tested using dead-weight and pulley systems to represent both the permanent and the variable actions. A series of serviceability load combinations were first examined, and these were followed by heavier loadings representing several important, potentially dangerous, ultimate limit state conditions, including icing.

The shell roofs have now been in service on this important and very exposed site for approximately fifteen years. They have been functioning entirely correctly, and in one case a particular component of a shell was put to the ultimate test, requiring partial replacement/repair, after being hit by a ship. Needless to say, no disproportionate collapse occurred.

Architects	Greater London Council.
Consulting Engineers	Rendel, Palmer & Tritton.
Specialist Timber Consultants	Timber Research and Development Association.

Figure 15 *Structural model of a roof.*

Case Study 2

Nursery School, Stainz, Austria

A small, but very imaginative and attractive project, using timber shell roofs, was nominated for the 1994 Awards Scheme of 'GLULAM', the European trade organisation for glued laminated timber.

This nursery school is of square-cruciform plan shape. It consists of an atrium area 8 *m* square, to which is linked, in each corner, four 'classic' hyperbolic paraboloid shell-roofed classrooms, see Figure 16. The atrium is framed from horizontal perimeter beams in glulam, which are supported off braced, spaced-column posts of similar material. This post and beam system supports a glazed roof of pyramidal hipped form. The hypars themselves, which are clad in copper roofing, are formed with the normal cross-boarded construction. They use several layers, some of which run parallel to the edge beams, whilst others lie at 45° to these. Glulam is also used for the edge beams. Each roof is cross-tied at the lower points, using unobtrusive round steel linking bars.

Areas of high-level lighting are incorporated into the timber-framed walls, which have sufficient stiffness to provide the necessary edge support and overturning resistance to the shells themselves. This high-level window lighting reflects onto several staged areas, which are reached by means of wooden staircases, with further space beneath.

A warm, restful interior is created by means of large areas of softwood, in natural colour, together with light which is reflected from the plain wall surfaces.

The whole is brightened by means of the nursery furniture and play objects, which are finished in primary colours. The exterior, of white walls, timberwork with natural pine finishes, and attractive patinated roofing, blends perfectly with the beautiful Austrian countryside.

Figure 16 *The nursery school, Stainz.*

Architect	Helmut Hafner.
Engineers	Walter Wörle and Dr. Pius Wörle.

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