Curved and pitched cambered glulam beams

STEP lecture E5
H.J. Larsen
Danish Building
Research Institute

Objectives

To give examples of the use of curved and pitched cambered glulam beams and to describe some special design considerations for these types of structures, especially the influence of moisture variations on the deformations and the internal stresses. Also, to describe possibilities of reinforcing the beams to reduce the influence of tensile stresses perpendicular to the grain.

Summary

Curved beams and pitched cambered beams offer many architecturally interesting possibilities. Often the design of these elements is governed by tensile stresses perpendicular to the grain. These may be caused by external loads, but moisture variations may also have a major effect: they give rise to internal stresses and increase the deformations from external loads. To reduce the risk of failure perpendicular to the grain, curved and pitched cambered beams are often reinforced. Two reinforcement systems are described: glued-in steel rods and glass fibres glued to the surfaces. The design of the reinforcement is described together with a design example.

Curved beams

Curved beams, i.e. members where the actions are predominantly carried by bending, as opposed to arches, where most of the actions are carried by axial forces, are used to achieve special architectural expression. Examples of the use of single curvature beams are shown in Figure 1.

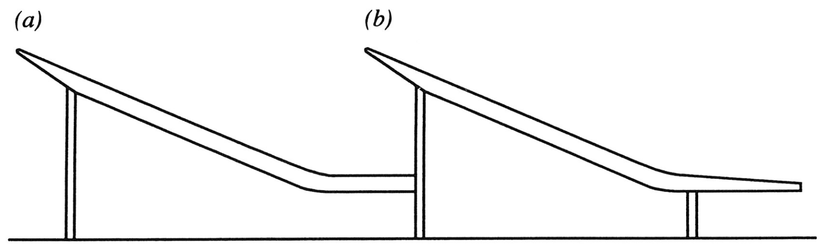

Figure 1 Example of a structure with single curvature beams.

In the structure shown in Figure 1a the main actions (dead load and snow load) will reduce the radius of curvature, i.e. give rise to compressive stresses perpendicular to the grain; the tensile stresses perpendicular to the grain (from wind) will normally be small and not cause any serious problems.

In the structure shown in Figure 1b the main action on the cantilever will cause relatively large stresses perpendicular to the grain, and these stresses may be critical for the load-carrying capacity.

It is possible to create architecturally interesting structures by using beams with multiple curvatures in a single plane; an example is shown in Figure 2. However, production of such members is difficult, they are expensive, and it is impossible to avoid relatively large stresses perpendicular to the grain.

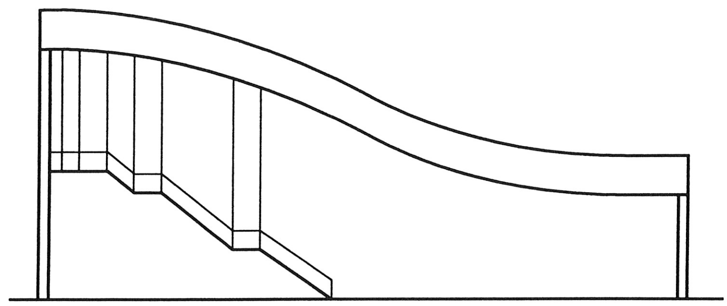

Figure 2 Example of a structure with reversed curvature beams.

Except for the problems related to the stresses perpendicular to the grain, the design of curved beams poses no problems. The strength verification is as for corresponding straight or tapered beams with a small axial force.

Pitched cambered beams

Pitched cambered beams, see Figure 3, are used for roof slopes between about 1/10 and 1/3 (α between 5° and 20°). The minimum depth h_{min} should not be less than l/30, where l is the span. The apex depth h_{apex} is normally between l/15 and l/10.

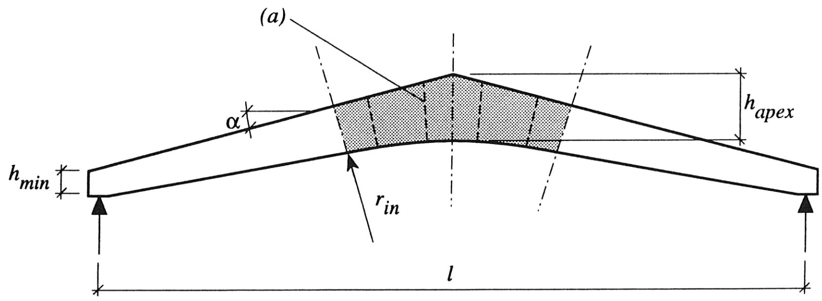

Figure 3 Pitched cambered beam comprising two tapered parts with the centre portion (the shaded apex zone) having a curved soffit. (a) Reinforcement bar.

The strength of the tapered parts is verified as described in STEP lectures B8 and E4. The strength of the apex zone is verified as described in STEP lecture B8. The strength of the beams is very often governed by tensile stresses perpendicular to the grain in the apex zone, and insufficient consideration of these stresses - or failure to consider them at all - has led to a considerable number of failures.

Stresses perpendicular to the grain may result not only from external loads but also from moisture variations. For structures in Service Class 1 dried to an appropriate moisture content before erection the effect of moisture variation may normally be assumed to be covered by the assignment of characteristic values and the normal safety factors. In some cases, however, it may be necessary to estimate the influence of moisture variations.

Assume as a rough estimate, see Figure 4, that the moisture content in the outer sixth on each side is decreased by 3% corresponding to a free strain of about $3 \cdot 0{,}002 = 0{,}006$. With $E_{90} = 300$ N/mm^2, the internal tensile stresses will be $\Delta\sigma = 2 \cdot 300 \cdot 0{,}006/3 = 1{,}2$ N/mm^2, i.e. of the same order of magnitude as the short-term tensile strength, and there is a risk of splitting. In practice, creep will reduce the stresses, but they should not be neglected.

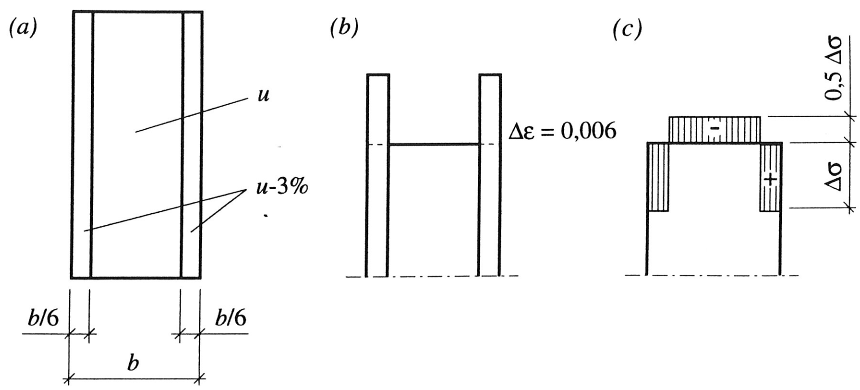

Figure 4 *The influence of a decrease in the moisture content in the outer sixth. (b) Free strain, (c) resulting internal stresses.*

Deformations

The vertical deflections of curved and pitched cambered beams are normally of no importance. The horizontal deflections at the supports may be rather large, however, and it is necessary to ensure that they can take place without giving rise to unforseen horizontal forces on the supporting structures (walls and columns) or damage to secondary elements.

Not only external forces but also moisture variations contribute to the deformations.

The main effect of an increase in the moisture content will be an increase in the depth of the beam from h to $h(1 + \varepsilon)$, where ε is the strain corresponding to the increase in the moisture content. Since the moisture influence in the fibre direction is marginal, the result, see Figure 5a, is that the angle $d\varphi$ is reduced to $d\varphi'$ and the radius of curvature is increased from r to r':

$$d\varphi' = d\varphi/(1 + \varepsilon) \approx d\varphi(1 - \varepsilon) \qquad (1)$$

$$r' = r(1 + \varepsilon) \qquad (2)$$

These changes mean that the chord length is increased and the camber reduced, see Figure 5b, by

$$\Delta v = r\varepsilon(1 - \cos\varphi) + a\varepsilon\varphi/2 \qquad (3)$$

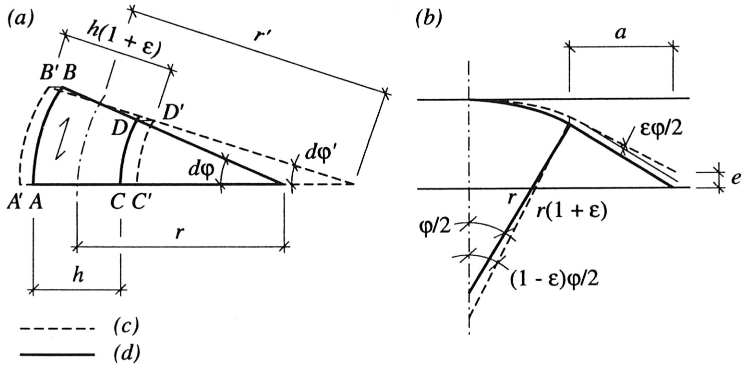

Figure 5 *Influence on the curvature of a beam of an increase in the moisture content. (c) Geometry after moisture increase, (d) undeformed state. e is the total midpoint deflection.*

Reinforcement

As shown in Figure 3 structures with tensile stresses perpendicular to the grain can be reinforced by gluing in steel rods or by gluing plywood or a fibre material to the surfaces.

Reinforcing materials

Glued-in steels rods are treated in detail in STEP lecture C14. They can be smooth or threaded; in the latter case the calculations should be based on the tensile stress area. The effective beam width is reduced by the diameter of the hole or the outer diameter of the rod, whichever is the larger, and the width or the depth may have to be increased correspondingly.

Various fibre materials - glass, aramid and carbon - offer interesting possibilities. For the time being, only glass fibres are of interest: they are cheap, easy to glue on the wood with polyester or epoxy adhesives and the reinforcement adds pleasantly to the appearance of the glulam:

Glass fibres are delivered in sheets with unidirectional fibres, as mats made of about 50 *mm* long chopped fibres with random orientations or as woven sheets. In the last mentioned case the fibres are gathered in rather thick bundles (roving) and the surface becomes rough and less attractive than in the other two cases. The unidirectional fibres are the most effective. Typical values per unit width for a reinforcement of 100 g/m^2 loaded in the fibre direction are $(tf_{t,g})_0 = 60$ *N/mm* and $(tE_g)_0 = 3000$ *N/mm*. The effective glass thickness (the sum of the layers on both sides) is denoted t, $f_{t,g}$ is the tensile strength of the glass and E_g the modulus of elasticity. If the load acts at an angle φ with the fibre direction the stiffness is reduced according to

$$(tE_g)_\varphi = (tE_g)_0 \cos^4 \varphi \qquad (4)$$

The strength reduction is less. For a mat with random fibres the stiffness is 3/8 of the stiffness of a corresponding unidirectional reinforcement with the same weight, and the strength is halved.

Stress distribution

The behaviour perpendicular to the grain is linear elastic to failure and the load perpendicular to the grain is distributed according to the stiffnesses.

When estimating the load taken by the wood it is proposed to use the short- term mean stiffness values for both wood and reinforcement, i.e. for the wood $E_{mean,90}$. Since the tensile strength perpendicular to the grain for wood is approximately proportional to the stiffness, there is no reason for taking "safe" values e.g. the 95-percentile, for the stiffness of the wood. According to EN 1194 "Timber structures - Glued laminated timber - Strength classes and determination of characteristic values", $E_{mean,90} = E_{mean,0}/30$, where $E_{mean,0}$ is the stiffness parallel to the grain.

Normally, and on the safe side, the reinforcement is designed to take the total tensile force perpendicular to the grain, i.e. corresponding to the situation where the wood has failed in tension perpendicular to the grain.

Where a more detailed calculation of the reinforcement is thought appropriate it is proposed that a low estimate be used for the stiffness of the wood, i.e. the 5-percentile reduced by creep.

Tests on glulam beams with glass fibre reinforcement - see Larsen and Enquist, (1993) and Traberg and Larsen, (1993) - show that if the reinforcement is designed to take the full load the influence of volume and stress distribution may be disregarded, i.e. $k_{dis}(V_0/V)^{0,2} = 1$. The explanation may be that the reinforcement arrests small initial cracks in weak zones which in an unreinforced beam, would have led to brittle, catastrophic failure.

In some cases the adherence strength of the reinforcement to the wood may limit the strength of the reinforcement. According to Gustafsson and Enquist (1993) the adherence strength per unit width can be determined as

$$F_{adh} = 2\sqrt{(tE_g)_0\, G_f}\ \sqrt{1 + \frac{(tE_g)_0}{bE_{mean,90}}} \qquad (5)$$

where G_f is the fracture energy of adherence, which may be taken as $G_f = 0{,}35$ Nmm/mm^2.

For tensile failure in steel rods or in the glass fibre a partial safety factor for $\gamma_M = 1{,}1$ is proposed; for the adherence strength $\gamma_M = 1{,}3$ is proposed.

Strengthening and repair

There are many examples of failures in curved or pitched cambered beams but few total collapses, and experience and tests have shown that it is rather easy to repair the beams by filling the crack with glue, jacking the structure up and reinforcing it with a reinforcement capable of taking the full load perpendicular to the grain in the original beam.

Design example

Structure

Pitched cambered beams spaced at 4,8 *m* and with a width $b = 0{,}165$ *m* are made of GL 28 (see STEP lecture A8). They have the following geometry, see Figure 3:

$l = 16{,}00\ m$ $\quad r_{in} = 13{,}50\ m$ $\quad h_{min} = 0{,}60\ m$ $\quad h_{apex} = 1{,}50\ m$ $\quad \alpha = 15°$

Strength verification

A dead load of 0,5 kN/m^2 and a snow load (short-term) of 0,8 kN/m^2 give the following design load:

$$q_d = (1{,}35 \cdot 0{,}5 + 1{,}50 \cdot 0{,}8) \cdot 4{,}8 = 9{,}00\ kN/m$$

$$M_{apex,d} = 9{,}00 \cdot 16^2/8 = 288\ kN/m$$

The shear strength at the support and the bending strength in the straight (tapered) parts and at the apex are sufficient.

The maxiumum tensile stress perpendicular to the grain is calculated according to STEP lecture B8, Equations (14) to (18):

$$r = r_{in} + h_{apex}/2 = 13{,}50 + 1{,}50/2 = 14{,}25\ m \qquad h_{ap}/r = 0{,}105$$

$$k_5 = 0{,}2 \tan\alpha = 0{,}0536 \qquad \text{B8-(16)}$$

$$k_6 = 0{,}25 - 1{,}5 \tan\alpha + 2{,}6 \tan^2\alpha = 0{,}0347 \qquad \text{B8-(17)}$$

$$k_7 = 2{,}1 \tan\alpha - 4 \tan\alpha = 0{,}2755 \qquad \text{B8-(18)}$$

$$k_p = k_5 + k_6 \cdot (h_{ap}/r) + k_7\, (h_{ap}/r)^2 = 0{,}603 \qquad \text{B8-(15)}$$

$$\sigma_{t,90,d} = k_p \cdot 6\, M_{apex}/(b\, h_{apex}^2) = 0{,}0603 \cdot 6 \cdot 288/(0{,}165 \cdot 1{,}5^2) \cdot 10^{-3} = 0{,}286\ N/mm^2 \qquad \text{B8-(14)}$$

The volume of the apex zone is $V = 1,1\ m^3$, $k_{dis} = 1,7$ and $f_{t,90,d} = 0,9 \cdot 0,45/1,3 = 0,312\ N/mm^2$.

The tensile strength - see STEP lecture B8, Equation (21) - is not sufficient since $\sigma_{t,90,d} > k_{dis}\ (V_0/V)^{0,2}\ f_{t,90,d} = 1,7\ (0,01/1,1)^{0,2}\ f_{t,90,d} = 0,664 \cdot 0,312 = 0,207\ N/mm^2$.

Reinforcement

It is proposed that the beam be strengthened with a reinforcement that can take the full tensile force perpendicular to the grain, i.e. per unit length:

$$F_{t,90,d} = 0,286 \cdot 165 = 47,2\ N/mm \qquad (6)$$

The modulus of elasticity perpendicular to the grain is $E_{90,mean} = E_{0,mean}/30 = 12000/30 = 400\ N/mm^2$, giving a stiffness per unit length of $400 \cdot 165 \cdot 10^{-3} = 66,0\ kN/mm$.

Reinforcement with steel rods

The reinforcement consists of 14 *mm* threaded rods (tensile stress area 118 mm^2) per 500 *mm*. Assuming a yield stress of 235 N/mm^2 the design strength per unit length is $118 \cdot 235/(1,1 \cdot 500) = 50,4 > 47,2\ N/mm$.

With $E = 210\ kN/mm^2$ the stiffness per unit length is $118 \cdot 210/500 = 49,6$ *kN/mm*, and the stresses in the wood perpendicular to the grain are reduced by the factor $66,0/(66,0 + 49,6) = 0,57$, i.e. to $0,57 \cdot 0,286 = 0,164\ N/mm^2$, which is acceptable.

Reinforcement with glass fibres

The reinforcement is made with 200 g/m^2 unidirectional glass fibre sheets on each side with properties as given above. The average angle between the force and fibre direction is less than 5° and its influence on strength and stiffness is marginal.

The stiffness per unit length is $(tE_g)_0 = 4 \cdot 3000 \cdot 10^{-3} = 12,0\ kN/mm$ and the design strength is sufficient since

$$F_{g,d} = \min \begin{cases} 4 \cdot 60/1,1 & = 218\ N/mm \\ \dfrac{2}{1,3}\sqrt{12000 \cdot 0,35}\sqrt{1 + \dfrac{12000}{165400}} & = 100\ N/mm \end{cases}$$

The stiffness per unit length is $4 \cdot 3000 \cdot 10^{-3} = 12,0\ kN/mm$, i.e. the stress in the wood perpendicular to the grain is reduced by the factor $66,0/(66,0 + 12,0) = 0,85$:

$$\sigma_{t,d} = 0,85 \cdot 0,286 = 0,243\ N/mm^2 < f_{t,d} = 0,312\ N/mm.$$

References

Larsen, H.J. and Enquist, B. (1993). Fibre Reinforcement of Glulam, Report No. 2, Small reinforced beams. Div. of Struct. Mech., Lund Institute of Technology, Report TVSM-7083, Sweden.

Traberg, S. and Larsen, H.J. (1993). Fibre Reinforcement of Glulam, Report No. 3, Medium-sized and large reinforced beams. Div. of Struct. Mech., Lund Institute of Technology, Report TVSM-7083, Sweden.

Gustafsson, P.J. and Enquist, B. (1993). Fibre Reinforcement of Glulam, Report No. 7, Adherence of reinforcement to wood. Div. of Struct. Mech., Lund Institute of Technology, Report TVSM-7083, Sweden.