Plane frames and arches

STEP lecture E9
L. Mortensen
Aalborg University

Objective

To give an introduction to the design and use of plane frames and arches.

Summary

The lecture gives an introduction to the field of application of frames and arches and to some aspects of their design. The influence of the structural system and of the geometry of the structure on the moment distribution will be discussed. An example demonstrating the design of a three-hinged curved frame concludes the lecture.

Introduction

Plane frames and arches are widely used structures in timber engineering. They are used for sports halls, assembly halls, hangars, churches, halls in industry and farming and also for domestic housing. Besides covering a wide range of applications the frames and arches also cover a wide range of spans, i.e. from a few metres up to about 50 metres for frames and more than 100 metres for arches. The structures will normally have solid cross-sections but especially in the case of greater spans trussed structures may be chosen. The structures may be fabricated from glued laminated timber or LVL or wood based material combined with glued laminated timber, LVL or solid timber in box and I-sections.

Structural systems

Frames and arches are normally designed as two-hinged or three-hinged structures. Structures with fixed supports are very seldom used because they are more difficult to assemble and because they transmit moments to the foundations.

The most commonly used design is the statically determinate three-hinged structure with hinges at apex and supports. Compared with the two-hinged structure it may require more timber because of a less efficient distribution of the internal moments and, furthermore, it will be less rigid. But these disadvantages of the three-hinged structure may easily be eclipsed by the fact that the distribution of internal forces in the static determinate structure is independent of possible displacements of the supports and deformations due to possible changes in the moisture content. Furthermore, the connections at the joints will be simpler and less costly since they are not subjected to bending moments.

Frame designs

Examples of glued laminated timber frames are shown in Figure 1. The frames may either be curved, Figures 1a and 1b, or they may be designed with sharp knees, Figures 1c and 1d. The three-hinged curved glued laminated timber frame, Figures 1a and b, is a very common solution. This is not only because it is a very economical structure but also because of its aesthetic qualities which contribute to a pleasant and often graceful interior. The roofing may follow the curved shape of the structure but most often a post and rafter assembly will be employed as indicated in Figure 1a.

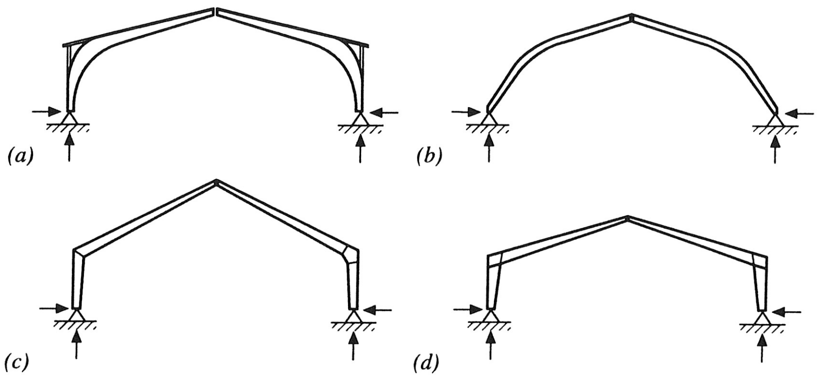

Figure 1 *Examples of glued laminated timber frames.*

Frames with inclined legs as indicated in Figure 1b will often be advantageous, e.g. in silo buildings. By fitting the inclination of the legs to the angle of friction of the material to be stored the pressure on the walls will be reduced and the structure will therefore be more efficient. The inclined leg will reduce the transport height of the frame.

Curved frames are normally manufactured with an inner radius of 3 to 5 *m* at the curved knee. The bending of the laminates will induce bending stresses and in order to limit these stresses the laminate thickness shall be chosen in accordance with prEN 386 "Glued laminated timber. Performance requirements and minimum production requirements".

Since the maximum bending moment will normally occur at the knee the largest cross-section is required here, and the rafter is therefore tapered from the curved knee to the apex, and often the straight part of the leg or part of it is tapered towards the base.

The span of the curved frame ranges from approximately 10 *m* to approximately 50 *m* but larger spans are possible.

In frames with sharp knees the knee joint may typically be established by employing one or two large finger joints, see Figure 1c. The introduction of a knee segment and two finger joints at the knee reduces the angle between the fibres of the joining pieces of timber, and this will increase the strength when compared with the joint with only one finger joint.

Since the finger joint is located in the section with maximum internal moment the performance of the structure is very dependent on the quality of the joint. In order to ensure a reliable and durable finger joint the production shall follow prEN387 "Glued laminated timber - Production requirements for large finger joints".

As an alternative to the finger joint the joint may be produced by gluing plywood gussets on both sides of the members or by employing steel or plywood gusset plates with mechanical fasteners. If either the leg or the rafter is chosen as a double member the gussets are omitted, Figure 1d.

Designs with glued-in bolts, steel plates or steel rods represent the latest developments for this joint.

The span of the frame with sharp corners ranges from approximately 10 *m* to approximately 35 *m* but larger spans are possible.

When deciding on the geometry of the frame it is important to realize that the dimensions and thereby the economy are effected by the magnitude of the maximum bending moment in the structure which develops at the knee. The closer the centre line of the structure follows the thrust line of the load the smaller the moment that will develop at the knee resulting in a more economical structure. The extent to which the geometry of the frame can be fitted to follow the thrust line will depend on the functional requirements for the building.

It is clear that the introduction of the curved frame improves the possibility of fitting the geometry to the thrust line and thereby reduces the maximum bending moment. If the leg of the frame is inclined it will further reduce the maximum bending moment because of the reduction in eccentricity *e*, with respect to the knee as shown in Figure 2. Curved frames are therefore better suited for wide spans than the frames with sharp knees where larger moments will develop and therefore result in larger cross-sections and the consumption of more timber. For both structures the bending moment at the knee will be reduced if the eaves height is reduced (*h* in Figure 2).

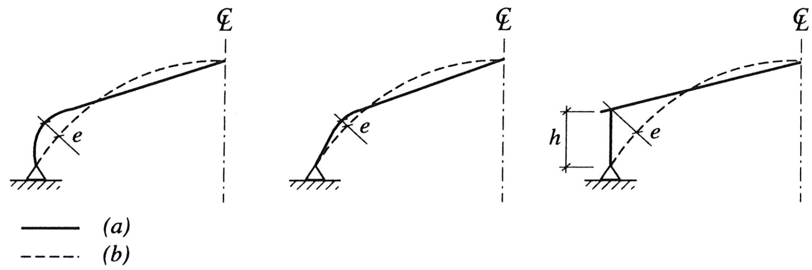

Figure 2 *Axis of structures and thrust lines for uniformly distributed load. (a) frame axis, (b) thrust line.*

Arch designs

Arches are more appropriate for larger spans than frames because the internal moments are relatively small compared to the moment that would develop in frames. This is because the geometry is fitted more closely to the thrust line of the applied load.

Ideally the geometry should follow the thrust line exactly in order to avoid internal moments but since different load combinations will produce different thrust lines it will not be possible to avoid internal moments altogether. Normally the parabolic or circular shape will give good approximations and is therefore often chosen.

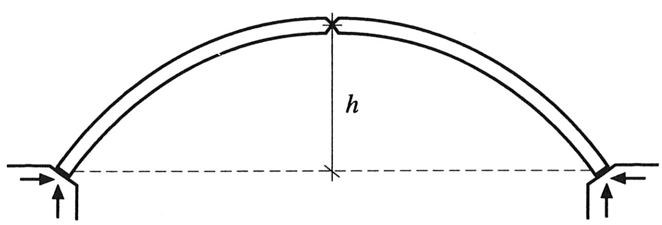

Figure 3 *Three-hinged arch.*

The arches are normally connected to the foundation via hinges. At the apex a hinged connection is also most often introduced and a statically determinate three-hinged structure is formed (see Figure 3). Alternatively, it may be chosen to establish a more rigid two-hinged arch by making all the joints between adjacent arch pieces moment resistant.

The height of arches (h in Figure 3) is normally chosen in the interval 0,13 to 0,20 times the span but greater heights are used when required.

The arches are often built up of a constant rectangular, glued laminated timber cross-section but other materials may be chosen, and it may also be decided to vary the cross-sectional depth for structural or architectural purposes.

Design of frames and arches

The design procedure is essentially by trial-and-error.

Normally the geometry of the structure is decided on the basis of functional and architectural considerations. The cross-sectional dimensions are then estimated and the design will show if they are adequate. A revision of the originally estimated values may be necessary.

Design example. Preliminary design of a three-hinged curved frame

Figure 4 shows a three-hinged frame.

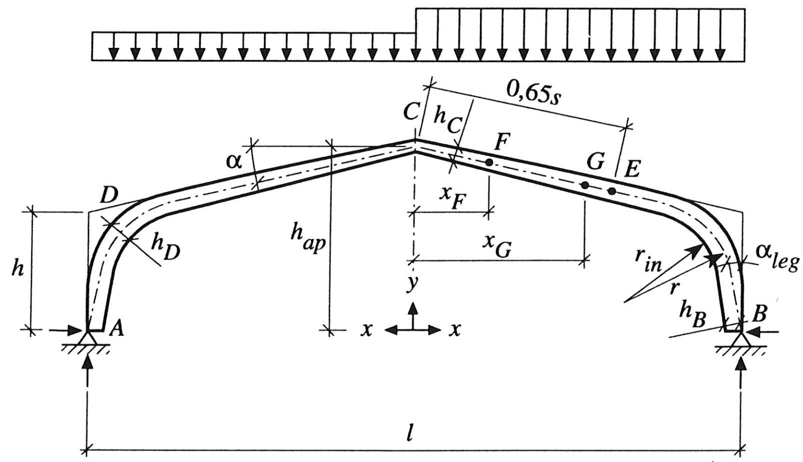

Figure 4 Symmetrical three-hinged frame.

The following dimensions are given:
l = 25 m, h = 4 m, h_{ap} = 7,2 m, α = 16°, r_{in} = 3 m, lamination thickness t = 22 mm, width of cross-section b = 160 mm, inclination of leg α_{leg} = 14°.

The cross-sectional dimensions required will depend on the specific loading conditions, frame geometry and frame spacing and also on the strength class of the glued laminated timber.

The guidelines for estimating cross-sectional depths given below should therefore be considered as rough estimates.

depth of curved section $h_D \sim 0{,}03\, l$, where l is the span
depth at supports $h_B \sim 0{,}9\, h_D$
depth at apex $h_C \sim 0{,}5\, h_D$

In this example the following cross-sectional depths are chosen:

curved section	h_D = 748 *mm* (34 laminations of 22 *mm*)
supports	h_B = 500 *mm*
apex	h_C = 300 *mm*

The frame geometry is hereby defined.

In this example only dead load and snow load are considered but it should be noted that this is only one of many cases that must be checked. It is assumed that the design load from the combination of dead load and snow load produces a line load of q_1 = 9,00 *kN/m* on the left half and q_2 = 10,13 *kN/m* on the right half of the frame. This is an approximation with regard to the loading in the corner because the rafter assembly will distribute the load here as point loads to the post and the rafter.

The load duration class is short term since the snow load is short term. The structure is assumed to be assigned to service class 1.

The strength class GL36 is chosen according to prEN 1194:1993 "Timber structures - Glued laminated timber - Strength classes and determination of characteristic values".

The design strength properties are found as:

EC5: Part 1-1: 2.2.3.2, 2.3.3.2 and 3.1.7

$f_{m,g,d}$ = 24,9 N/mm^2 $f_{c,o,g,d}$ = 21,5 N/mm^2 $f_{c,90,g,d}$ = 4,36 N/mm^2

$f_{v,g,d}$ = 2,40 N/mm^2

In the following three cross-sections will be examined.

First section D at the frame corner (see Figure 4) where the maximum bending moment occurs. Then section F (see Figure 4) where the maximum bending stress in the straight part of the rafter occurs. Both cross-sections are examined for the combined action from the bending moment and the axial compressive force acting at these sections. Finally, section A at the support (see Figure 4) is examined for the shear force acting here.

Calculation of internal forces

EC5: Part 1-1: 5.4.4

The geometrical and structural imperfections and induced deflections shall be taken into account when determining the internal forces. This may be done by carrying out a second order analysis on an imperfectly shaped structure.

EC5: Part 1-1: 5.2.1

In this example the influence of imperfections and deflections will be taken into account by employing the column equations. As a first step the internal forces are therefore found on the basis of the perfect and undeformed structure. They are found from hand calculations or more conveniently obtained from computer programs.

Frame corner

In this example the maximum internal moment occurs at the left-hand corner. (Section D in Figure 4).

At section D the Internal forces are found as:

M_D = 218 *kNm* N_D = 147 *kN* V_D = 0

As mentioned above the increase of the moment due to imperfections and deflections will be taken into account by considering the frame corner as a column with the effective length l_{ef} according to STEP lecture B7, Equation (16).

$$l_{ef} = h\sqrt{4+3{,}2\frac{I_s}{I_0 h}+10\frac{EI}{hK_r}}$$

h	= 3830 *mm* (length of column between A and D)
s	= 11820 *mm* (length of rafter between D and C)
I	$= 160 \cdot 748^3/12 = 5{,}58 \cdot 10^9\ mm^4$
I_0	$= 160 \cdot 642^3/12 = 3{,}53 \cdot 10^9\ mm^4$
K_r	$\sim \infty$

$$l_{ef} = 3830\sqrt{4+3{,}2\frac{5{,}58 \cdot 11820}{3{,}53 \cdot 3830}}$$

$$l_{ef} = 3830 \cdot 4{,}43 = 17000\ mm$$

With respect to the lateral stability of the frame it is assumed that it is supported laterally at the support and by purlins spaced at 2 *m*. The purlins closest to the frame corner are assumed to be located where the assembly rafter is connected to the rafter at $(x, y) = (9{,}66,\ 4{,}43)$ and the effective length with respect to deflection out of the frame plane is therefore calculated as $l_{ef,z} = 5300\ mm$ (see Figure 4).

The stresses at the corner section, D, of the undeformed structure are calculated as:
$\sigma_{c,0,d} = 1{,}23\ N/mm^2$ and $\sigma_{m,y,d} = 1{,}11 \cdot 14{,}6\ N/mm^2$.

EC5: Part 1-1: 5.2.4

Because of the curvature the bending stress is multiplied by the factor $k_l = 1{,}11$.

EC5: Part 1-1: 5.2.4

Because of the strength reduction caused by bending of the laminates the bending strength is reduced by the factor $k_r = 0{,}90$ (see the column equations below).

EC5: Part 1-1: 5.2.1

The strength of the frame corner is sufficient if the stresses satisfy the column equations.

$k_{c,y}$ and $k_{c,z}$ to be employed in the equations are found as:

$k_{c,y} = 0{,}53 \qquad k_{c,z} = 0{,}27.$

By inserting in the column equations the following is found:

$$\frac{1{,}23}{0{,}27 \cdot 21{,}5} + 0{,}7\frac{1{,}11 \cdot 14{,}6}{0{,}90 \cdot 24{,}9} = 0{,}72 < 1$$

$$\frac{1{,}23}{0{,}53 \cdot 21{,}5} + \frac{1{,}11 \cdot 14{,}6}{0{,}90 \cdot 24{,}9} = 0{,}83 < 1$$

The conditions are satisfied.

Rafter
Similarly to the frame corner the rafter is examined for the combined action from the axial compressive force and the bending moment. The rafter on the

right-hand half of the frame will be subjected to the heaviest load and will therefore be considered.

The rafter is considered as a single tapered beam ($\alpha = 2{,}6°$).

The section of the rafter where the maximum bending stress occurs will be examined. Because of the taper of the beam the maximum bending stress will not occur where the bending moment is maximum but at the section located at a distance x_F from the apex where

$$X_F = \frac{H_C}{H_C + H_G} X_G$$

(Larsen and Riberholt, 1994).

x_G is the x-coordinate to the section of zero moment in the rafter. It is found as $x_G = 6{,}57\ m$. The corresponding cross-sectional depth is found as $h_G = 605\ mm$ and the cross-sectional depth at the apex is $h_C = 300\ mm$.

Therefore

$$X_F = \frac{300}{300 + 605} 6{,}57 = 2{,}18\ m$$

The corresponding cross-sectional depth is $h_F = 401\ mm$.

The internal moment at F is calculated as $M_F = 48{,}5\ kNm$.

EC5: Part 1-1: 5.2.3

The bending stresses in the outmost fibres at this section are calculated on bending stress as $\sigma_{m,0,d} = 11{,}4\ N/mm^2$ (10,1) and $\sigma_{m,\alpha,d} = 11{,}2\ N/mm^2$ (9,93). The figures indicated in brackets are the stresses at the section with maximum bending moment ($x = 3{,}29\ m$).

The axial compressive force at F is found as $N_F = 105\ kN$ and the corresponding stress is calculated as $\sigma_{c,0,d} = 1{,}64\ N/mm^2$.

EC5: Part 1-1: 5.2.3

Due to the combined action from the axial force and the moment the stress at the tapered edge will exceed that at the bottom edge and the tapered edge will therefore be considered. At the tapered edge the bending strength shall be reduced. The reduction will, however, be negligible in the case of compressive stresses parallel to the tapered edge. It is found that $f_{m,\alpha,d} = 24{,}7\ N/mm^2$.

The strength of the rafter is sufficient if the stresses satisfy the column equations.

EC5: Part 1-1: 5.2.1

$k_{c,y}$ and $k_{c,z}$ to be employed in the equations are found as:
$k_{c,y} = 0{,}46$ and $k_{c,z} = 0{,}97$.

When calculating $k_{c,y}$ the depth of the rafter cross-section was taken as $h = 642\ mm$ which is the depth at 0,65 s (see Figure 4) and the effective length of the rafter was calculated according to Equation (17). l_{ef} was found as $l_{ef} = 16000\ mm$.

The column equations may now be employed:

$$\frac{1{,}63}{0{,}97 \cdot 21{,}5} + 0{,}7 \cdot \frac{11{,}2}{24{,}7} = 0{,}40 < 1$$

$$\frac{1,63}{0,46 \cdot 21,5} + \frac{11,2}{24,7} = 0,62 < 1$$

And it is seen that the conditions are satisfied.

Having considered the frame corner and the straight part of the rafter it still remains to consider the sections at support and apex. Both cross-sections are subjected to an axial compressive force and a shear force. Here only the investigation at the support will be demonstrated.

Support
It is assumed that the leg has been cut vertically and the cross-section at the support thereby reduced to 160 x 500 mm^2. The axial force (113 kN at A and 119 kN at B) can easily be absorbed. The shear force is greatest at A. Because of the inclination of 14° of the leg it is found that V_A = 72,7 kN, and therefore

$$\tau_A = \frac{3}{2} \cdot \frac{72,7 \cdot 10^3}{160 \cdot 500} = 1,36\ N/mm^2 < 3,5\ N/mm^2$$

Conclusion

As mentioned earlier the design is a trial-and-error procedure and the results of the first trial above indicate that it will be possible to reduce the estimated cross-sectional dimensions of the frame and still satisfy the strength requirements. A second trial based on reduced cross-sectional dimensions will not be demonstrated here.

It should be remembered that requirements for the frame stiffness in the serviceability limit state might be determining for the cross-sectional dimensions.

References

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