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Focal Length, EFL, and the Eye

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The “focal length” is often called the “effective focal length” or efl instead, and although this is acceptable for a lens in air, it is not otherwise correct. The eye is used as an example here for an optical system where the object is in air and the image is in fluid. Welford (1986) was found to have paraxial equations that are consistent with historical use, while also clearly defining efl. These are based on power at a surface having to be the same for light traveling in both directions (n’/f’). The focal length f’ is the actual physical distance from the 2nd principal point to the paraxial focus, and the “equivalent focal length” or efl is the focal length divided by the image index (f’/n’). Separately, when the object is in air, the efl is shown to act at the nodal point, with the lens system represented by either an “equivalent thin lens” at the principal point with a focal length, or a different “equivalent thin lens” in air at the nodal point with an efl. The rationale for using “effective” instead of “equivalent” for efl is unclear, but efl is used more as a symbol than as an acronym. https://doi.org/10.1364/AO.481805

1. INTRODUCTION

Most lenses are used in air where there is little ambiguity about what is meant by a “focal length”, but this is not the case, if the object is in air and the image is in a different medium. Efforts to generalize calculations for computers may have led to the widespread use of “effective focal length” or efl instead of focal length, even for a lens in air. The rationale for this terminology is explored here, along with evaluations related to the cardinal points and their uses. This paper uses phrasing that is both clear in older publications, and also used routinely by the Zemax optical raytrace software (now usually a version of Zemax OpticStudio, www.ansys.com). The lack of clarity in the literature often depends on whether there is a statement about whether a lens system is assumed to be in air or not.

This topic is particularly relevant to ophthalmic optics, where the eye always forms an image in fluid, and this evaluation is a follow-up to recent work relating to the nodal points[1,2], which are only different to the principal points if the object or image medium is not air. It is also important for intraocular lenses (IOLs), which are small foldable plastic lenses that are fully immersed in fluid in the eye to replace the natural crystalline lens during cataract surgery. These are both used as specific examples here, but the principles can be applied more generally to any optical system.

One aspect that may have been used in raytrace software for decades is that the nodal point location can be found very simply by taking the focal length, and dividing it by the image refractive index when the object is in air, with that distance also being the efl. It is not clear if this was ever specifically stated in a publication.

The evaluations start by reviewing the background to the topic. The important calculations are identified, and relationships between the principal and the nodal points are discussed. Raytrace calculations are then used to cross check the terminology and the image properties.

2. BACKGROUND

The cardinal points have been used to characterize optical systems for over 150 years, and Donders summarized the six paraxial cardinal points on a drawing like Fig 1 in 1864 for the eye, with air on the left and fluid on the right[3]. The original basis for paraxial optics had been established by Gauss, who showed in 1841 that for very small angles all the power of a set of lenses in air appears to act at the principal planes[4], with collimated light being brought to a focus at a focal point. Gauss had a colleague called Johann Listing who was interested in the eye, and he published a monograph in 1845 that first described the “nodal points”[5], where a ray heading to the first nodal point NP1 is parallel to a ray that appears to come from the second nodal point NP2.

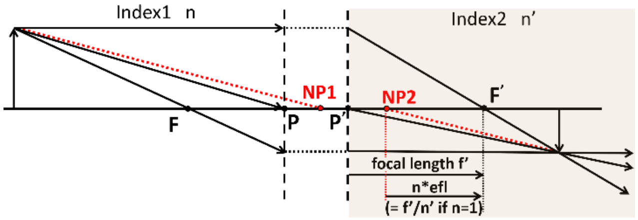

Fig. 1. Standard drawing for cardinal points with focal points F, principal points P, and nodal points NP. Notation has been added here to show that the “focal length” is from the second principal point to the paraxial focus, and the efl is from the second nodal point to the paraxial focus if n=1.

Fig. 1 captures the main concepts for general imaging, but it tends to give a visual impression that the angles can be relatively large, even though these are only paraxial rays. Discussions about this figure usually give no indication that the principal point and nodal point locations are simply related by refractive index if n=1 (described below). In fact, the nodal points are often sketched separately on a thick lens in air, with rays at large angles, even though in air the nodal points are also the principal points. The angular property is a valuable concept that can be used to evaluate magnification and field angles, yet the "nodal" word is linked to both an axial point and an angle. In practice, the pairs of principal and nodal points are each separated in Fig. 1 because the lens is not thin, and the nodal points are shifted because the object and image media are different.

In 1905 Beck characterized the principal points in air slightly differently when he said that several lenses can be represented by a single thin lens, except that it has to receive the light at one location, and then be rapidly moved to a second location to discharge it[1,6]. This simple concept of a thin "equivalent lens" seems to have been forgotten when cardinal points are discussed today, even though that is what the principal planes represent. The focal length is usually characterized using collimated light, and a single thin equivalent lens at the 2nd principal point captures the concept very clearly.

The eye is an example of an optical system that is not all in air, and Listing simplified the eye to a single thin lens, which is illustrated in a split drawing in Fig 2. A standard model eye that has a one-surface cornea is on the top, and the simplified eye is on the bottom (based on a relatively recent drawing in Bennett and Rabbetts[7]). The two principal planes and two nodal planes are each close together in the full eye and they are superimposed for the reduced eye. This leaves an equivalent thin lens that represents the entire eye on the bottom in Fig 2. This demonstrates very clearly that the main purpose of the nodal point is to simply address the refractive index difference between object space and image space.

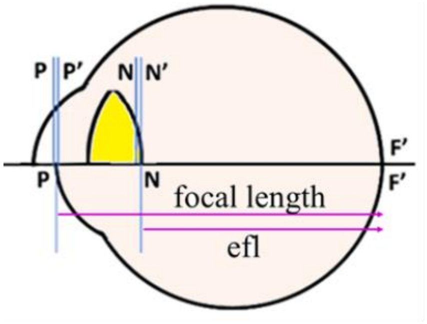

Fig 2. (top) Gullstrand-Emsley schematic eye. (bottom). Simple reduced eye that is a single lens

The single surface eye is really the most fundamental optical system that is possible, and Fig. 3(a) depicts refraction at a single surface based on a drawing in Fincham and Freeman which was originally published in 1934[8]. The paraxial imaging equation for this single optical surface is given in the same reference by

$$n'/l' - n/l = (n'-n)/r = K = \text{power} \qquad (1)$$

using l for the object distance, l' for the image distance, n' for refractive index, and a standard sign convention. If the distances l and l' are separately increased to infinity, then this leads to the related relationships

$$n'/f' = -n/f = K = \text{power} \qquad (2)$$

where the focal lengths f' and f are distances from principal planes at the surfaces to the paraxial foci.

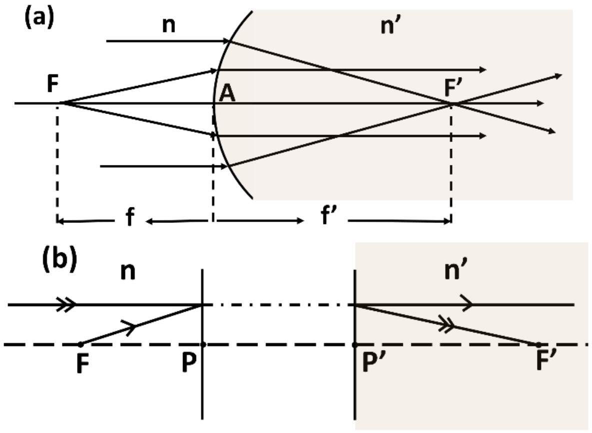

Fig. 3. (a) A single refractive surface. (b) Optical system represented by two principal planes. Optical systems have these characteristics, whether or not the nodal point or the efl are considered.

The same equations were used by Welford in 1986 for a more complex optical system that has been reduced to 2 principal planes, where the planes have a separation between them (Fig. 3(b))[9]. Welford commented that "… power is frequently used as a measure of strength of an optical system because… it occurs naturally in many equations involving raytracing and the combination of optical systems; also, its value is unchanged if the system, with terminating media, is reversed end to end."[9] He also defined the "equivalent focal length" as

$$\text{efl} = 1/K = 1/\text{power} = f'/n' \qquad (3)$$

with a comment that "[the efl] is in fact the image side focal length which a system with air on both sides would have if it had the same power as the system in question", but made no comment at all about the nodal points with respect to Fig. 3(b).

To explore this further, Fig. 4 has sketches depicting collimated light coming from the left. A thin lens that is at the interface between air and fluid is at P2, and a different equivalent thin lens is also shown that could be used in air instead at the second nodal point NP2 in order to give the same paraxial image properties. The lens in air has larger refraction angles, and these are simply scaled by the refractive index value. The rays through the lens centers can be considered to be chief rays in Fig. 4(b), where for the original situation the ray is refracted at the center, but for the nodal point situation the ray is transmitted without deviation. The ray that passes through the center of the lens in air is a nodal ray[10] because it is not deviated, and this demonstrates that the nodal point location is at f'/n' from the paraxial focus (the efl distance).

The equivalent thin lens in air can be represented by re-expressing the main terms in Eq. 2:

$$1/(f'/n') = -1/(f/n) = K = \text{power} = 1/\text{efl} \qquad (4)$$

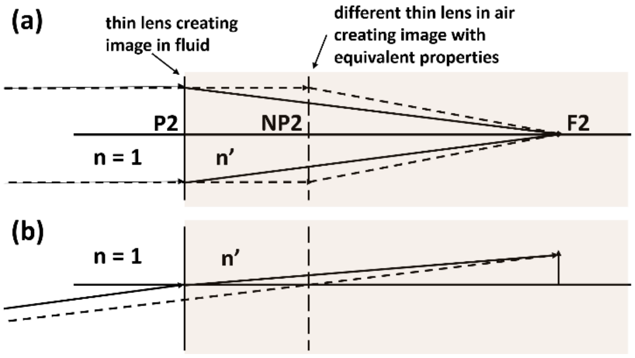

Fig 4. (a) Collimated on-axis light with an equivalent thin lens at the principal point P2 for air in object space and fluid in image space. A different equivalent thin lens could be placed at the nodal point NP2 if the system was in air instead, with focal length f'/n'. (b) Collimated light at an angle, illustrating that the central ray in each case is like a chief ray.

The power is the same in both cases, but whereas Eq. 2 requires both focal length and refractive index, Eq. 4 simply uses power =1/efl, with efl having a distinct meaning of its own when the image medium is not air. This indicates that the discussions about angles that are normally used to define the nodal points may be a distraction from the underlying optical concepts. The nodal points are located where an equivalent lens would be used in air to give the same image properties.

An "intraocular lens" (IOL) represents an alternative situation where neither the object nor the image are air[11]. The power of an IOL is given by the standard thick lens equation [12], with a focal length f' in fluid, and a power that is n'/f'. This is not evaluated in detail here, but it provides an additional example of the terminology, for a type of lens that is widely used (IOLs are used by perhaps 7% of the overall population when readily available, though this is mostly in older eyes, with over 50% of people aged 75 or older having at least one IOL[13]). The angular scaling of the wide-angle retinal image is also discussed below, since this may have contributed to limiting the exploration of the nodal point[2].

3. METHOD

The Zemax raytrace software Optic Studio Professional v22.2.1 was used to model a single refracting surface model eye, and raytrace drawings were used to compare different conditions. In particular, the effect of the refractive index at the image was evaluated separately from the optical characteristics of the lens itself, by setting up a thick fluid "window" on the image side. The values found from raytracing were compared to the standard cardinal point and efl values provided separately by the software. A raytrace plot for a schematic eye with rays entering the eye at a large visual angle was also created.

4. RESULTS

Fig. 5(a) has a ray drawing for the single surface model eye from Zemax. This has r1 = 5.56 mm, a 2 mm diameter aperture at 5.56 mm, and an image distance of 22.24 mm. The nodal point is at the center of curvature of the optical surface. The field angle is set to 5°, and with "ray aiming" on in Zemax, the chief ray traverses the optical axis at the stop location. The aperture and field angle were chosen arbitrarily for their relatively modest aberrations, and it is clear that the main characteristics are depicted in this single drawing. The focal length is at 22.24 mm in fluid from the optical surface, and the nodal point is at 5.56 mm from the lens, which is 22.24/1.333… or 16.68 mm from the paraxial image, and this is also the distance that is commonly termed the efl (EFFL in Zemax). Zemax gives these distances for the focal length and for the nodal point location.

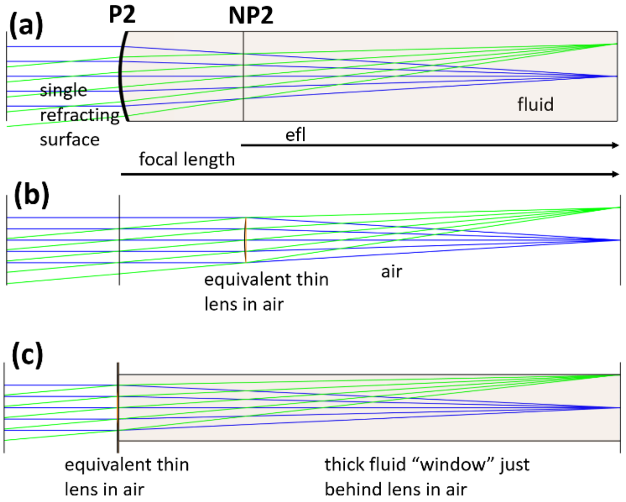

Fig. 5. (a) Raytrace diagram from Zemax for the simple one-surface eye. The stop is set at the nodal point location at 5.56 mm from the lens apex, and the chief ray passes through the center without deviation. (b) Thin lens in air with focal length of 16.68 mm located at 5.56 mm from original lens surface. (c) Thin lens in air re-positioned at original anterior surface, with fluid box inserted.

Fig. 5(b) depicts an alternative system that uses a thin lens in air instead, with r1 = 8.35 mm, thickness 0.06 mm, and refractive index of 1.5. The lens has the same efl of 16.68 mm, and because it is in air this is also the focal length. The lens is positioned at approximately the location of the original nodal point (with a small difference due to the finite lens thickness). The image size is the same as for the original image in fluid.

Moving the lens used with air forwards to the location of the original lens, but inserting a thick "window" of fluid that fills the entire thickness from 0.001 mm behind the lens to the approximate image location, the rays are refracted at the air/fluid boundary just behind the lens, and the image height remains the same. There is no change to the power of the system because the surfaces are flat. When the final image medium is air, both the focal length and efl in Zemax are 16.68 mm, as though the fluid is just acting like a thick window that relays the vergence from the front surface to the back surface. Changing the final image medium to fluid, the focal length in Zemax changes to 22.24 mm, but the efl remains as 16.68 mm. This corresponds to the normal behavior of the original system in Fig. 5(a). These calculations demonstrate that there are 2 characteristics; a focusing effect due to the lens, and an axial displacement effect due to a change in refractive index (without an additional power change).

Fig. 6 depicts light entering a schematic eye at a large angle of 60°, and it illustrates something that has been an additional source of confusion regarding the nodal point. The internal pupil diameter is 3mm, and although the image is increasingly aberrated moving away from the fovea, the main image location is indicated by the chief ray that goes through the center of the stop. If a line is drawn through the paraxial nodal point that is parallel to the input beam, it approximately identifies the image location on the retina over a range of more than 90°[1,2] (with 60° just used here as an illustration). This characteristic of the eye is generally known, and similar behavior has been shown for other eye models that have a gradient refractive index lens[14], and an IOL[2], but this is due to the optical design of the eye rather than to the paraxial properties of the nodal point.

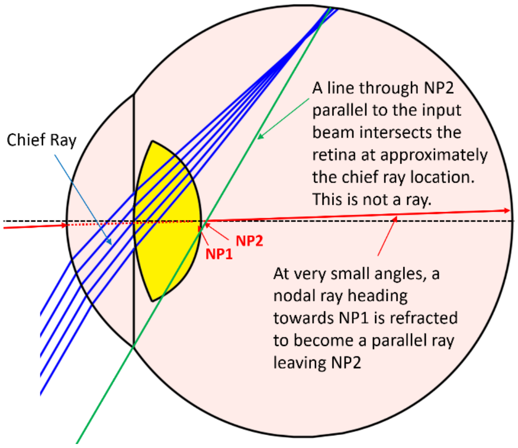

Fig. 6. Rays entering the Gullstrand-Emsley schematic eye at 60°. The line that passes through NP2 at the angle of the input beam correctly identifies the image location, but it is not a nodal ray.

5. DISCUSSION

The "focal length" is one of the most fundamental parameters for a lens, yet although the concept seems relatively clear for a lens in air, it is often described using an efl value instead. The complexities surrounding this really go back to the introduction of the "nodal point" in 1845 for the eye, where the effect of an image medium that differed from air was not really evaluated completely.

The term "equivalent focal length" or efl was used originally for a thick or compound lens in air. In a book originally published in 1936 Fincham had this comment: "The equivalent focal length of a system may therefore be defined as the focal length of a thin lens, which will produce an image of a distant object of the same size as that produced by the system"[8]. This summarizes the topic nicely, by indicating that the most important thing is the physical size of the image. In 1978 Kingslake essentially said that the word "equivalent" was redundant for a lens in air, and that it should simply be called the focal length[15].

However, the term efl was re-introduced almost immediately in 1986 by Welford, who said that the "equivalent focal length" or efl is f'/n'[9]. This deals with the image medium not necessarily being air, and it is consistent with the historical use of the term, even though the image medium may not have been previously considered. Welford had earlier worked on "bubble chambers", which had perhaps led to noticing that textbooks did not deal very clearly with lenses that were not used completely in air.

The term efl also came to stand for "effective focal length", rather than "equivalent focal length". The origin is unclear, but it is possible that the very influential book "Modern Optical Engineering" by Warren Smith was involved in its widespread acceptance, where the book essentially stated in 1966 that efl = "effective focal length" = "focal length", without clarifying whether that was for a lens system in air or not[16]. The word "effective" had been used elsewhere for special situations, such as for the "effective power" at a location that is not at the principal point, but for the efl it has the same meaning as "equivalent".

Over time, it became widely accepted that a "focal length" might be listed for a lens in air as an efl, and this acceptance may have come from the use of computer software for lens design, where 3-letter codes were used for many optical parameters. Even in 1967, a brief paper by Smith described software that had EFL, BFL, and FFL parameters[17]**.** Raytracing moved to personal computers in the 1980s, and it may have seemed natural to use efl instead of focal length everywhere. When the image was not in air it was perhaps found that dividing by the image refractive index always gave the correct efl, with n' always 1 for a lens in air anyway. However, there seems to have been little discussion about this, and the fact that there are actually two different parameters with very similar names is not always clear.

Alternative terminology might also sometimes be used, such as the definitions by Grievenkamp et al, where the focal length is always identical to the efl, but the physical distance from the principal plane to the focus is called the "rear focal length"[18]. This does not really take into account the long history of the terms however, where the "focal length" is almost always defined to be the distance from the principal plane to the focus, even though that was originally specified only for a lens in air. Welford's equations are consistent with that historical use, while also addressing different surrounding media in a very straightforward manner.

There is also a distinction between the terminology for paraxial and practical properties of a lens. Traditionally, with light traveling from left to right, the cardinal points are called the 1st points (focal, principal, and nodal) for the object side, leading to the 2nd points on the right. With a physical lens, however, the "front focal length" is the physical distance from front focal point to the anterior lens surface, and the "back focal length (bfl)" is from the physical posterior surface to the back focal point. Both the bfl and efl are specifically mentioned as methods for measuring the power of an IOL (with efl described using the word "effective")[12]

The recognition that the efl also identified the nodal point location when the object is in air probably also happened several decades ago, though references like Smith[16] and Welford[9] do not mention any link between nodal points and the efl. Raytrace software pragmatically traces "parabasal rays" to find the focal length, using extremely small angles, and this enables a value to be found even when the optical system does not have simple spherical surfaces. Presumably the software then simply divides the focal length by the index to give the nodal point location, though this is not typically stated. The nodal points continue to be described using the traditional angle concept, but the efl method provides a simpler and more fundamental description.

A more recent book by Kingslake and Johnson illustrates the more general situation when the object medium is not air, though it does not include the term efl as part of the discussion. This is essentially a depiction of the same information as equations 2 and 3 above, with efl = f'/n'=-f/n, but where n is not 1. In their Figs. 3.13 and 3.14, the distance from the 2nd nodal point to the paraxial focus is shown to be -f, or the negative of the first focal length[19], and this is n*efl, rather than simply being the efl value. This gives another perspective to the overall topic, with efl being constant for light traveling in the two directions for the equivalent lens in air, but with both surrounding index values having an effect on the cardinal points.

It is also worth noting that cardinal point discussions do not mention aperture stops, but in practice the rays through both the principal points and the nodal points are like chief rays for two separate optical systems, even though this is not how they are normally described. For a lens system in air, the nodal point terminology is unclear, because the nodal points match the principal points, yet there is still a nodal ray.

The raytrace plot at a large angle for the eye in Fig. 6 is included here because this optical system has contributed to the confusion surrounding the nodal points[20], which were first described when paraxial optics was new, and when the more complete concepts relating to optical design were still unknown. Fig. 6 illustrates the optical design characteristics of the eye, with the cornea curving around to face incoming rays, the iris limiting the rays that are transmitted, and the retina curving around very steeply to meet the image. The limiting aperture (the stop) is now known be an important feature of lens systems, but although the pupil diameter is discussed for specific eye properties like the depth of focus, the combination of large angle imaging, the iris, and a highly curved retina do not appear to have a clear literature of their own. Fundamentally, if the light rays do not go through the aperture, then there isn't an image, yet the usefulness of the nodal point for scaling retinal image locations to more than 90 degrees may have obscured the fact that this is not due to paraxial characteristics[21].

6. CONCLUSION

The term "focal length" has a long history as the physical distance from the principal point to the focus (f'), and the term "efl" has more recently been used for the distance from the nodal point to the focus for an object in air. The two distances are related by the refractive index (efl=f'/n'), and computerized raytracing appears to have utilized this simplification with little discussion. Thin equivalent lenses at either location can represent paraxial optical properties, but the equivalent thin lens in air tends to be obscured by the use of a nodal ray in fluid to characterize the nodal point.

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