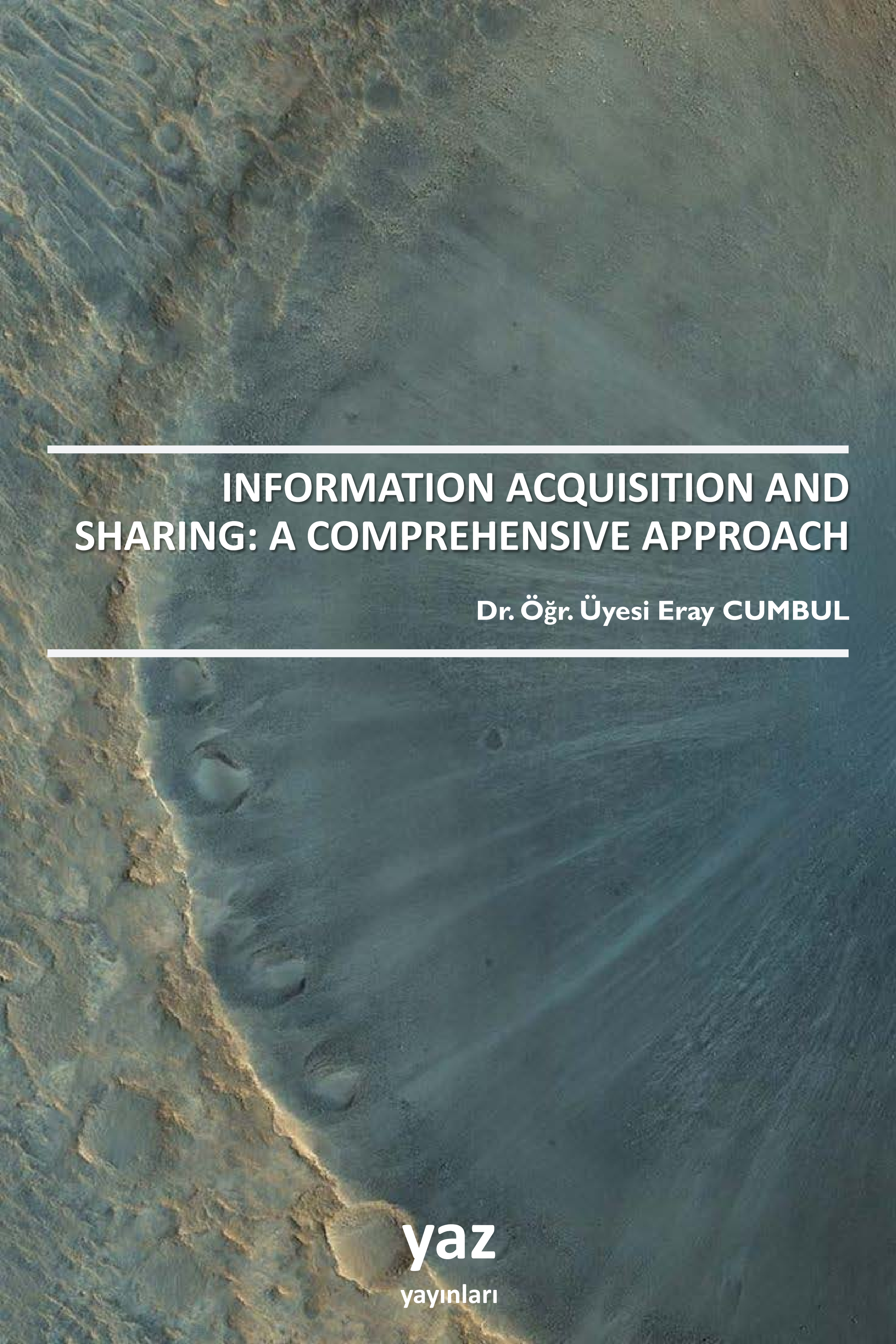

INFORMATION ACQUISITION AND SHARING: A COMPREHENSIVE APPROACH

Dr. Öğr. Üyesi Eray CUMBUL

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A Comprehensive Approach

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İÇİNDEKİLER

INTRODUCTION

In the modern economy, information is a critical resource that shapes decision-making, competitive strategies, and market outcomes. The acquisition and sharing of information have profound implications for firms, policymakers, and society at large. This book, Information Acquisition and Sharing: A Comprehensive Approach, delves into the intricate dynamics of how information is gathered, utilized, and disseminated in various economic contexts. Through a blend of theoretical models and real-world applications, the book explores the welfare implications of information acquisition and sharing, offering insights into how these processes influence market efficiency, competition, and social welfare.

The book is structured into three main sections, each focusing on a distinct aspect of information economics:

Welfare Implications of Information Acquisition: This section examines how firms in oligopolistic markets acquire private and public information to enhance their strategic decision-making. It explores the value of information in different competitive settings, such as Cournot and Bertrand models, and analyzes how information acquisition affects market outcomes, consumer surplus, and total welfare. Real-world examples from industries like airlines, oil and gas, and pharmaceuticals illustrate the practical relevance of these theoretical insights.

Welfare Implications of Information Sharing: The second section investigates the incentives for firms to share information and the resulting impact on market dynamics. It explores scenarios where firms choose to share or conceal information, depending on whether their products are complementary or substitutable. The analysis extends to both Cournot and Bertrand competition, highlighting how information sharing can either enhance or undermine market efficiency and consumer welfare.

Information Acquisition and Sharing in Aggregative Games: The final section broadens the scope to include aggregative games, where individual strategies depend on the aggregate actions of all players. This section covers a range of applications, including partnership games, public good contribution games, common resource games, and gas emission games. It examines how information acquisition and sharing influence strategic behavior and welfare in these contexts, emphasizing the role of public information and the externalities of private information.

Throughout the book, the interplay between theory and practice is emphasized, with each section providing a detailed analysis of the strategic implications of information in different economic settings. The findings are not only relevant for firms navigating competitive markets but also for policymakers seeking to design regulations that promote efficiency and fairness in the face of information asymmetry.

As industries continue to evolve with advancements in data analytics, artificial intelligence, and digital platforms, the strategic use of information will remain a cornerstone of economic decision-making. This book aims to provide a comprehensive framework for understanding the complexities of information acquisition and sharing, offering valuable guidance for both academic researchers and practitioners in the field of industrial organization and beyond.

CHAPTER 1

WELFARE IMPLICATIONS OF INFORMATION ACQUISITION

1.1. Introduction

In oligopoly markets, firms frequently seek to acquire information about market demand and their costs to improve their strategic decision-making. The acquisition of private information is critical in shaping competitive outcomes, influencing pricing strategies, production levels, and overall market efficiency. Understanding how firms obtain, use, and sometimes share private information has been a significant topic in industrial organization and microeconomic theory (Vives, 1999).

Information acquisition in oligopoly settings can take various forms, including market research, data analytics, customer surveys, and strategic investments in information gathering technologies. In some cases, firms acquire private information independently, while in others, they engage in indirect information-sharing arrangements, either explicitly or implicitly. The extent to which firms acquire and utilize such information affects the intensity of competition, potential collusion, and social welfare.

In the context of retail, businesses are increasingly leveraging alternative data sources-like credit card transactions and social media analysis-to predict performance and make informed decisions. This trend underscores the growing importance of data-driven strategies across various industries. [1]

Similarly, the integration of artificial intelligence in research and development accelerates innovation and product development, enabling companies to better meet consumer demands. For example, online retailers such as Amazon and

Walmart continuously monitor competitors' prices using web-scraping algorithms to adjust their own pricing strategies dynamically. By acquiring real-time demand and cost information, these firms optimize inventory management and price discrimination strategies.

The are several real-world examples of information acquisition in oligopoly markets in which several firms compete in prices or quantities. In the airline industry, airlines invest heavily in demand forecasting tools, using big data analytics and artificial intelligence to optimize pricing and seat allocation. Companies like Delta, American Airlines, and Lufthansa acquire information on customer preferences, competitor pricing, and demand fluctuations through advanced booking systems and customer loyalty programs [2].

In the oil and gas industry, oil companies invest in seismic surveys and geophysical data to acquire private information about oil reserves. In bidding for oil extraction rights, firms engage in strategic information acquisition to reduce uncertainty and secure competitive advantages [3].

In the automobile industry, car manufacturers such as Toyota and Volkswagen use predictive analytics and consumer preference tracking to forecast demand and optimize supply chain operations. Firms also gather data from dealership networks and after-sales services to improve cost efficiency and production planning [4].

In the pharmaceutical industry, pharmaceutical companies acquire private information through clinical trials and research collaborations before launching new drugs. By monitoring competitors' R&D investments and regulatory approvals, firms make strategic entry decisions into new therapeutic markets [5].

In the financial markets, investment firms and hedge funds acquire private market data through algorithmic trading strategies, analyzing order flows and market trends to gain a competitive edge. High-frequency traders use proprietary data acquisition techniques to predict price movements and optimize trading strategies. [6]

These real-world examples highlight the diverse mechanisms through which firms acquire and leverage private demand and cost information. Understanding these processes is crucial for policymakers and competition authorities to ensure that information acquisition enhances market efficiency without facilitating anti-competitive behavior. The subsequent part will explore theoretical models of information acquisition in oligopolies and their implications for market competition and social welfare.

Another channel in which firms can acquire information is via public information dissemination by the central bankers and government officials. This is essential for economic stability and effective policymaking. While regular communication is beneficial, excessive or inconsistent messaging can undermine confidence and create unnecessary market noise. Policymakers should aim for a balance, ensuring that their communication is transparent, consistent, and strategically delivered to support economic objectives. For instance, Morris and Shin (2002) argue that while public information is useful, excessive reliance on it can sometimes lead to inefficiencies. If agents overweight public signals relative to their private information, even small errors in public communication can propagate widely, leading to market overreactions.

To give policy recommendations about the above real-world examples, we develop a two player model. The payoffs of agents are quadratic and stochastic. Each agent receives one private signal and one public signal about the common prior. We allow for asymmetric information. The quality of the private signals of agents can be different than each other. We further assume that the conditional expectations are linear in the observable signals and agents do not share their private signals with each other.

In this model, we show that an agent's expected utility rises as it gains access to more accurate private information. When one agent acquires private information, it positively affects its competitor if their strategies are complementary, but negatively affects the competitor if their strategies are substitutable. The overall expected payoff for agents improves as the precision of public information increases. The expected total payoff of an agent increase in the precision of its own private signal and the public signal. Moreover, private information acquisition by one agent generates positive externalities on its rival if their actions are strategic complements, but negative externalities if their actions are strategic substitutes. There are a number of implications of these findings in the oligopoly models.

When firms compete in quantities in the Cournot competition, firms are motivated to gather private information about demand or costs. If firms produce complementary goods, one firm's information acquisition benefits the other; however, if they produce substitutable goods, it harms the other. When a firm obtains better information about demand or costs, the expected total surplus is expected to rise. However, the expected consumer surplus only increases if the firms produce substitutable or moderately complementary goods. Lastly, more accurate public information about demand or costs leads to higher total surplus and increased profits for firms.

When firms compete in prices in the Bertrand competition, firms have strong incentives to collect private demand information. If firms produce substitutable goods, one firm's information acquisition benefits the other; if they produce complementary goods, it harms the other. Improved demand information always boosts expected consumer surplus. However, expected total surplus only increases if the firms produce goods that are not highly substitutable. more precise public demand information increases firms' expected total profits. However, expected total surplus only rises if the firms produce close substitutes. If the goods are complementary or not highly substitutable, total surplus declines with more precise public information.

The acquisition of demand and cost information in oligopolistic markets has been a central topic in industrial organization theory. Theoretical models have explored how firms acquire, and the implications for market competition, efficiency, and welfare. The welfare effects of better-quality private information in oligopoly markets depend on the type of competition (Cournot, Bertrand, or Stackelberg), the nature of information (demand or cost), and the extent to which firms use or share their private information.

Vives (1984) defines the social value of demand information between the firms receiving signals of the same finite variance and the firms receiving no information at all. Vives (1994) shows that the social value of information is always negative under the Bertrand competition, and it is positive under the Cournot competition. This implies that if fully uninformed firms acquire symmetric quality of private signals, the expected total surplus increases under the Cournot competition, and it decreases under the Bertrand competition. Our definition of social value of information is more general than that of Vives (1984). We define that the social value of information is positive (negative) if expected total surplus increase (decrease) after a firm is privately better informed starting from any level of information. Under this more general definition, we show that the social value of information is still positive under the Cournot competition. Moreover, it can be positive under the Bertrand competition if the degree of substitutability between firms' products take some intermediate values.

More recently, Afacan, Cumbul and Colombo (2024) consider a mixed duopoly model in which a partially privatized firm compete with a fully private firm. While the former firm maximizes a weighted sum of total surplus and its own profit, the latter firm maximizes its own profit. Both firms receive a private and a public signal about a prior demand or cost value. The correlation between the prior values of firms can be perfect or imperfect. They can also be negatively or positively correlated. Firms can produce complementary or substitutable products. They investigate the value of private and public information in this model. They show that the private value of private information is always positive. However, the value of public information can be negative when the values are imperfectly correlated.

The welfare effects of more precise public signals are also studied in simultaneous-move games. For example, in the beauty contest games of Morris and Shin (2002), Angeletos and Pavan (2007a, b), Chahrour (2014), and Ui and Yoshizawa (2015), a more precise public signal can be socially harmful. Angeletos and Pavan (2004), Colombo, Femminis, and Pavan (2014), Cornand and Heinemann (2008), Hellwig (2005), Myatt and Wallace (2012, 2015), Vives (2017), and Bayona (2018) examine the social value of private and public information in simultaneous-move investment, beauty-contest, and supply function competition games. Ui and Yoshizawa (2015) characterize the social value of information in simultaneous-move games with common priors and quadratic payoffs. However, we provide a detailed analyzes on both the private and social values of private and public information on profits, consumer, and total surpluses in various games, and a deeper analyzes on the effects that generate our results.

Cornand and Ferreira (2020) study the social value of information under the Cournot and Bertrand competition games when a multi-divisional company produces differentiated products. They show that the social value of public information is always positive under both types of competitions. Our set-up is substantially different than theirs. We assume that there are two firms and produce differentiated products in the context of firms. We show that the social value of information is positive under the Cournot competition. Similarly, it is negative under the Bertrand competition if firms produce complementary products or sufficiently low degree of substitutable products. However, it is positive if they produce sufficiently close substitutes.

In sequential-move games such as the leader-follower Stackelberg games, Cumbul (2022) shows that the early-mover agents may not have incentives to acquire private information because the late-mover agents infer the private signal of the early-movers from the decisions of the early-movers under the signaling effect. This effect clearly cuts incentives to acquire information.

1.2. Set-up

We consider a rich two-agent set-up that encompasses many earlier two-agent models discussed in the literature. Each agent $i \in \{1,2\}$ chooses an action $a_i \in \mathbb{R}$, and is assumed to have the following quadratic utility function:

$$u_i(a_1, a_2) = \left(A + \omega\theta - \gamma a_i - \gamma\lambda a_j\right)a_i, (1.1)$$

where $A > 0, \gamma > 0$, and $\omega \neq 0$ are known parameters, $\lambda \in [-1,1]$, and θ is a prior random variable with mean $\bar{\theta} \geq 0$ and variance $\sigma_\theta^2 > 0$. We say that the strategies of the players are considered strategic substitutes if $\partial^2 u_i / \partial^2 a_i a_j = -\gamma\lambda < 0$ or $\lambda > 0$, and strategic complements if $\lambda < 0$.

Each agent receives a noisy private signal $s_i = \theta + \epsilon_i$ of equal quality and a common pubic signal $z = \theta + \epsilon_z$, where $E(\epsilon_i) = E(\epsilon_z) = 0, \text{Var}(\epsilon_i) = \sigma^2_{\epsilon_i} \geq 0,$ $\text{Var}(\epsilon_z) = \sigma^2_{\epsilon_z} \geq 0, \text{Cov}(\epsilon_i, \theta) = \text{Cov}\left(\epsilon_i, \epsilon_j\right) = \text{Cov}(\epsilon_z, \epsilon_i) = \text{Cov}(\epsilon_z, \theta) = 0$, and all random variables are jointly normally distributed. Public information is shared by everyone and can be accessed from the communications of the government officials and central bankers or from news and media. Thus, we assume that the agents can have asymmetric private information and $\text{Var}(s_i) = \sigma^2_{\epsilon_i} + \sigma^2_\theta$. As $\sigma^2_{\epsilon_1}, \sigma^2_{\epsilon_2}, \sigma^2_{\epsilon_z} \rightarrow 0$, the private and public signals of agents become perfectly informative, and we reach the complete information scenario. As $\sigma^2_{\epsilon_1}, \sigma^2_{\epsilon_2}, \sigma^2_{\epsilon_z} \rightarrow \infty$, the signals become perfectly uninformative, leading to the fully incomplete information scenario. Let the precision of the private signal i and that of the public signal be measured by $\tau_{\epsilon_i} = 1/\sigma^2_{\epsilon_i}$ and $\tau_{\epsilon_z} = 1/\sigma^2_{\epsilon_z}$, respectively. All variance and covariance terms are common knowledge to both players.

1.3. Results

1.3.1. Value of private information

We begin by solving the following game. In Stage 0, each agent observes the public signal z and privately observes its private signal s_i. In Stage 1, both agents simultaneously choose their actions by solving $\max\limits_{a_i} E(u_i(a_1, a_2) \mid s_i, z)$. Specifically, agents do not observe each other's choices. The timeline of this game is as follows:

Stage 0: Agents observe their private signals and the public signal

Stage 1: Agents simultaneously choose a_1 and a_2.

We now seek the Bayesian Nash equilibrium of this game, which is defined as follows:

Definition 1. *Bayesian Nash Equilibrium: A strategy profile (a_1^*, a_2^*) is a Bayesian Nash equilibrium of this game if, for each $i \in \{1,2\}$, where $j \neq i$, it holds that* $a_i^* \in arg \max_{a_i} E\left(u_i\left(a_i, a_j^*\right) \mid s_i, z\right)$.

Equivalently, the strategies of the players should be best responses to each other's strategies in a Nash equilibrium. Since the payoff function is quadratic and agents have asymmetric quality of private signals, we assume that the Bayesian Nash equilibrium strategies of the players are affine and asymmetric. Thus, we let $a_i^* = F_{0,i} + F_{1,i}(s_i - \bar{\theta}) + F_{2,i}(z - \bar{\theta})$ to the equilibrium strategy of player $i = 1,2$, where $F_{0,i}, F_{1,i}$, and $F_{2,i}$ are constants to be determined. We will now derive these two constants.

Each agent i solves the following problem:

$$\max_{a_i} E(u_i(a_1, a_2) \mid s_i, z) = E\left(\left(A + \omega\theta - \gamma a_i - \gamma\lambda a_j\right)a_i \mid s_i, z\right).$$

The expected utility is strictly concave in a_i because the second derivative of the expected utility with respect to a_i is negative, i.e.,

$$\frac{\partial^2 E(u_i \mid s_i)}{\partial a_i^2} = -2\gamma < 0$$

Thus, the second-order conditions are fullfilled. The first-order condition (FOC) is given by:

$$\frac{\partial E(u_i \mid s_i, z)}{\partial a_i} = 0$$

which simplifies to:

$$a_i = F_{0,i} + F_{1,i}(s_i - \bar{\theta}) + F_{2,i}(z - \bar{\theta}) = \frac{A + \omega E(\theta \mid s_i, z) - \gamma\lambda E(a_j \mid s_i, z)}{2\gamma} \quad (1.2)$$

Observe that $E(a_j \mid s_i, z) = F_{0,i} + F_{1,i}\big(E(s_j \mid s_i, z) - \bar{\theta}\big) + F_{2,i}(z - \bar{\theta})$. It follows that the expected reaction function of each player slopes downward (upward) if $\lambda > 0$ $(\lambda < 0)$. Moreover, the conditional expectations can be derived using the projection theorem for multivariate normally distributed variables, as discussed in Vives (2011) and Rostek and Weretka (2012).

$$E(\theta \mid s_i, z) = E(s_j \mid s_i, z) = (\mathrm{Cov}(\theta, s_i) \quad \mathrm{Cov}(\theta, z)) \begin{pmatrix} \mathrm{Var}(s_i) & \mathrm{Cov}(s_i, z) \\ \mathrm{Cov}(s_i, z) & \mathrm{Var}(z) \end{pmatrix}^{-1} \begin{pmatrix} s_i - E(s_i) \\ z - E(z) \end{pmatrix} + E(\theta) = \frac{\sigma_{\epsilon_z}^2\left(\bar{\theta}\sigma_{\epsilon_i}^2 + s_i\sigma_\theta^2\right) + z\sigma_\theta^2\sigma_{\epsilon_i}^2}{\sigma_\theta^2\sigma_{\epsilon_i}^2 + \sigma_{\epsilon_z}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_i}^2\right)}$$

Thus, equation (1.2) simplifies to:

$$a_i = F_{0,i} + F_{1,i}(s_i - \bar{\theta}) + F_{2,i}(z - \bar{\theta}) = \frac{A + \omega\bar{\theta} - \gamma\lambda F_{0,i}}{2\gamma} + \frac{\sigma_\theta^2\sigma_{\epsilon_z}^2(s_i - \bar{\theta})\left(\omega - \gamma\lambda F_{1,i}\right)}{2\left(\gamma\sigma_\theta^2\sigma_{\epsilon_1}^2 + \gamma\sigma_{\epsilon_z}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_1}^2\right)\right)} + \frac{z - \bar{\theta}}{2}\left(\frac{\sigma_\theta^2\sigma_{\epsilon_1}^2\left(\omega - \gamma\lambda F_{1,i}\right)}{\gamma\sigma_\theta^2\sigma_{\epsilon_1}^2 + \gamma\sigma_{\epsilon_z}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_1}^2\right)} - \lambda F_{2,i}\right).$$

We now have two equations with six unknowns, $F_{0,1}, F_{1,1}, F_{2,1}, F_{0,2}, F_{1,2}$ and $F_{2,2}$, which we can solve. This leads to the following lemma:

Lemma 1. There exists a unique Bayesian equilibrium. The equilibrium strategy of each agent $i = 1,2$ is:

$$a_i^* = F_0 + F_{1,i}(s_i - \bar{\theta}) + F_{2,i}(z - \bar{\theta}), \quad (1.6)$$

where for $r_{1i} = \sigma_{\epsilon_z}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_i}^2\right) + \sigma_\theta^2\sigma_{\epsilon_i}^2, r_{2i} = \sigma_\theta^2\left(4\sigma_{\epsilon_z}^2(2 - \lambda) + 8\sigma_{\epsilon_1}^2\right) + 8\sigma_{\epsilon_1}^2\sigma_{\epsilon_z}^2$, and $r_3 = 4\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)\left(\sigma_{\epsilon_2}^2 r_{11} + \sigma_\theta^2\sigma_{\epsilon_1}^2\sigma_{\epsilon_z}^2\right) + (4 - \lambda^2)\sigma_\theta^4\sigma_{\epsilon_z}^4$,

$$F_0 = \frac{A + \omega\bar{\theta}}{\gamma(2 + \lambda)}, F_{1,i} = \frac{\omega\sigma_\theta^2\sigma_{\epsilon_z}^2 r_{2,j}}{4\gamma r_3} \text{ and } F_{2,i} = \frac{2\omega\sigma_\theta^2\left(2r_{1j}\sigma_{\epsilon_i}^2 - \lambda\sigma_\theta^2\sigma_{\epsilon_j}^2\sigma_{\epsilon_z}^2\right)}{\gamma(2 + \lambda)r_3}.$$

The expected equilibrium strategies are:

$$E(a_1^*) = E(a_2^*) = F_0 = \frac{A + \omega\bar{\theta}}{\gamma(2+\lambda)}, \tag{1.8}$$

because the expected value of each signal is $E(s_i) = E(z) = \bar{\theta}$ by assumption. Using this lemma, we can also derive the expected Bayesian equilibrium utilities for each agent. These utilities consist of two components: the non-covariance part and the covariance part. Let the non-covariance part be denoted by $E\left(u_{i,1}^*\right)$, and the covariance part by $E\left(u_{i,2}^*\right)$. The first component is relatively straightforward to calculate and simplifies to:

$$\begin{aligned} E\left(u_{i,1}^*\right) &= E(a_i^*)\left(A + \omega\bar{\theta} - \gamma E(a_i^*) - \gamma\lambda E\left(a_j^*\right)\right) \\ &= \frac{(A+\omega\bar{\theta})^2}{\gamma(2+\lambda)^2}. \end{aligned}$$

To compute the covariance components of the expected equilibrium utilities, we first set $A = F_0 = \bar{\theta} = 0$, as these terms do not contribute to the covariance parts. Next, we can derive that:

$$\begin{aligned} E\left(u_{i,2}^*\right) &= E(\omega\theta a_i^*) - \gamma E((a_i^*)^2) - \gamma\lambda E\left(a_i^* a_j^*\right) \\ &= \omega \mathrm{Cov}(a_i^*, \theta) - \gamma \mathrm{Var}(a_i^*) - \gamma\lambda \mathrm{Cov}\left(a_i^*, a_j^*\right) \\ &= \gamma \mathrm{Var}(a_i^*)) \\ &= \gamma\left(F_{1,i}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_i}^2\right) + \left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)F_{2,i}^2 + 2\sigma_\theta^2 F_{1,i}F_{2,i}\right), \end{aligned}$$

where the first equality follows by definition, the second equality follows from the definition of expectation, the third equality holds due to equation (1.5), and the final equality follows from the definition of variance. Noting that $\mathrm{Cov}(\theta, s_i) = \mathrm{Cov}(\theta, z) = \sigma_\theta^2, \mathrm{Var}(s_i) = \sigma_\theta^2 + \sigma_{\epsilon_i}^2$, and $\mathrm{Var}(z) = \sigma_\theta^2 + \sigma_{\epsilon_z}^2$. we obtain the expected Bayesian equilibrium utility of each agent i :

$$E(u_i^*) = \sum_{k=1}^{k=2} E\left(u_{i,k}^*\right) = \gamma\left(F_0^2 + F_{1,i}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_i}^2\right) + \left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)F_{2,i}^2 + 2\sigma_\theta^2 F_{1,i}F_{2,i}\right)$$

Using the equilibrium constants from Lemma 1, we can derive that for $i, j = 1,2$ with $i \neq j$, the following holds:

$$\frac{\partial E(u_i^*)}{\partial \sigma_{\epsilon^*}^2} = -\frac{r_{2,j}^2 \omega^2 \sigma_\theta^4 \sigma_{\epsilon_z}^4 \left(r_3 + 2\lambda^2 \sigma_\theta^4 \sigma_{\epsilon_z}^4\right)}{16\gamma r_3^3} < 0,$$

$$\frac{\partial E(u_i^*)}{\partial \sigma_{\epsilon_j}^2} = \frac{\lambda r_{1,i} r_{2,i} r_{2,j} \omega^2 \sigma_\theta^6 \sigma_{\epsilon_z}^6}{4\gamma r_3^3} \gtreqless 0 \text{ if } \lambda \gtreqless 0.$$

The inequalities arise because $\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2, \sigma_{\epsilon_z}^2, r_{1,i}, r_{2,i}, r_3, \gamma > 0$. From these calculations, we obtain the following results:

Proposition 1. i) *The expected utility of agent i increases as it acquires a more precise private signal.* **ii)** *Private information acquisition by agent i generates positive externalities on its rival if their actions are strategic complements (substitutes), that is, if $\lambda < 0 (\lambda > 0)$.*

The first result implies that both players have an incentive to acquire private information if the cost of acquiring information is sufficiently low. The second result suggests that the rival firm's expected payoff decreases when agent i acquires a higher quality private signal if and only if their actions (and expected strategies) are strategic complements.

What we will do next is identify the information effects that generate these findings. We begin with our first result.

After an agent acquires more precise private information, it can better adjust its strategy in response to changes in θ. When the agent lacks information, it would set the same strategy regardless of the state of the world. However, given that $\omega > 0$, as the agent obtains more information, it would select a higher strategy when the state is high and a lower strategy when the state is low. This strategy adjustment effect increases the agent's expected payoff.

Additionally, the variance of a better-informed player's strategy is higher than that of a less-informed player's strategy. The strategy volatility effect negatively impacts the player's expected payoff.

Lastly, the players' strategies become more correlated after an agent becomes better informed. The strategy alignment effect negatively affects the expected payoff of the information-acquiring player if the expected strategies are complements, and positively impacts the payoff if the strategies are strategic substitutes. Considering these three effects, acquiring a better-quality private signal is beneficial for a player.

We now turn to the effects of having better information on the rival agents. The signs of the changes in the three effects mentioned above can be illustrated as follows:

After agent i becomes better informed, the rival agent j better adjusts its strategy to changes in the prior if and only if the expected strategies of the agents are strategic substitutes, that is, $\lambda < 0$. This is logical because when the state is high, agent i begins setting a higher strategy as it becomes better informed. If the strategies of the agents are strategic substitutes (complements), the rival agent responds by decreasing (increasing) its strategy. Overall, the strategic adjustment effect benefits the rival agent only when the strategies are strategic substitutes ($\lambda < 0$).

Similarly, the strategy volatility of the rival agent j increases after agent i acquires a higher-quality private signal if and only if the strategies of the players are strategic complements. Greater strategy volatility negatively impacts the expected payoff of the rival agent.

Lastly, the strategies of the players become more correlated after agent i becomes better informed, but only if their strategies are strategic substitutes. Since this effect intensifies competition between the players, it also negatively impacts the profitability of player j. In light of these three opposing effects, the acquisition of private information by player i is beneficial (detrimental) to the rival agent j if their strategies are strategic complements (substitutes).

1.3.2. Value of public information

Next, we investigate the value of public information. In that regard, we calculate the partial derivate of the sum of total expected payoffs of agents with respect to the variance of the public signal as follows:

$$\frac{\partial E(u_1^* + u_2^*)}{\partial \sigma_{\epsilon_z}^2} = -\frac{4T_0\omega^2\sigma_\theta^4}{\gamma(2+\lambda)^2 r_3^3} < 0.$$

This derivative is negative because $\omega \neq 0, \lambda \in [-1,1], \sigma_{\epsilon_z}^2, \sigma_{\epsilon_i}^2, \sigma_\theta^2, \gamma, r_{2j}, r_3 > 0$ and

$$\begin{aligned} T_0 =& (4-\lambda^2)\sigma_\theta^8\sigma_{\epsilon_z}^8\left((3\lambda^2+4(1-\lambda))\left(\sigma_{\epsilon_1}^2-\sigma_{\epsilon_2}^2\right)^2+2(1-\lambda)(2-\lambda)^2\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2\right) \\ &+12\sigma_\theta^4\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2\sigma_{\epsilon_z}^4\left(2\lambda^2(2+\lambda)\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2+(4(1-\lambda)+\lambda^2)\left(\sigma_{\epsilon_1}^2+\sigma_{\epsilon_2}^2\right)^2\right)\left(\sigma_\theta^2+\sigma_{\epsilon_z}^2\right)^2 \\ &+2\sigma_\theta^6\left(\sigma_{\epsilon_1}^2+\sigma_{\epsilon_2}^2\right)\sigma_{\epsilon_z}^6\left((2-\lambda)^2(8-\lambda(4+3\lambda))\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2\right. \\ &\left.+(8-2\lambda(4+\lambda))\left(\sigma_{\epsilon_1}^2-\sigma_{\epsilon_2}^2\right)^2\right)\left(\sigma_\theta^2+\sigma_{\epsilon_z}^2\right) \\ &+8\sigma_\theta^2\sigma_{\epsilon_1}^4\sigma_{\epsilon_2}^4\sigma_{\epsilon_z}^2(8-\lambda(4+\lambda))\left(\sigma_{\epsilon_1}^2+\sigma_{\epsilon_2}^2\right)\left(\sigma_\theta^2+\sigma_{\epsilon_z}^2\right)^3+32\sigma_{\epsilon_1}^6\sigma_{\epsilon_2}^6\left(\sigma_\theta^2+\sigma_{\epsilon_z}^2\right)^4 > 0 \end{aligned}$$

Thus, we obtain an important result.

Proposition 2. *The expected total payoff of agents increases in the precision of the public signal.*

The expected total payoff will measure total welfare in public economics and environmental economics applications in Chapter 3. In our Cournot and Bertrand games applications, total payoffs correspond to the total profits.

1.4. Applications of the information acquisition model

This model has a wide range of applications in oligopoly theory, public economics, and environmental economics. We only discuss the applications in oligopoly theory in this section.

The remaining applications will be discussed in Chapter 3 of our book.

1.4.1. Cournot competition and information acquisition

In this model, the agents represent firms that simultaneously choose their production quantities (q_i). Consider two sellers producing differentiated products (or services), with each firm producing only one product. Let q_i denote the production of firm i, and let $Q = \sum_{i=1}^{2} q_i$ represent the total market output. The price, net of the marginal cost of firm i 's product, is given by:

$$p_i - c_i = A + \omega\theta - \gamma\left(q_i + \lambda q_j\right)$$

where $A > 0$ is the demand parameter, $\gamma > 0$ is the slope of the inverse demand, and $\theta \sim \left(\bar{\theta}, \sigma_\theta^2\right)$ is the prior common random variable with $\bar{\theta} \geq 0$. The parameter $\lambda \in [-1,1]$ measures the degree of horizontal product differentiation. The products are imperfectly substitutable if $\lambda \in (0,1)$, unrelated if $\lambda = 0$, and complementary if $\lambda < 0$. If $\omega = -1$, demand is known, but there is cost uncertainty. Conversely, if $\omega = 1$, only demand uncertainty exists.

The profit of firm i is given by:

$$\pi_i = (p - c_i)q_i = \left(A + \omega\theta - \lambda\left(q_i + \lambda q_j\right)\right) q_i$$

Equation (1.1) coincides with this payoff after letting $a_i = q_i$. We allow for asymmetric cost and demand information between firms because the private signals may have noise levels with different variances. The demand signals can be considered as sufficient statistics derived from various sources of single observations, such as market research and sales reports, among others.

Under the Cournot competition, each firm chooses their production levels without observing their rival's choices. For example, AT&T and Verizon in the USA, and Vodafone and Deutsche Telekom in Europe invest in network capacity (spectrum, infrastructure) and compete on service quantity (number of customers served, data offered). Similarly, OPEC members such as Saudi Aramco, ExxonMobil, and Shell are oil-producing companies and they decide how much oil to extract and supply to the market. Since price depends on total supply, each firm's output decision affects the market price, leading to strategic quantity competition.

Consumer surplus is defined as $CS = \gamma\left(\frac{q_1^2+q_2^2}{2} + \lambda q_1 q_2\right)$ in this product differentiated market. Let the total profits be $\Pi = \sum_{i=1}^{i=2} \pi_i$. The expected total surplus is the sum of the expected consumer surplus and total industry profits: $E(TS) = E(CS) + E(\Pi)$. Using the equilibirum quantities from Lemma 1, we can derive the expected Bayesian equilibrium consumer surpluses in our Cournot game as follows:

$$E(CS_C^*) = \gamma F_0^2(1+\lambda) + \sum_{i=1}^{i=2} E\left(\pi_{i,C}^*\right)/2$$
$$+\gamma\lambda\left(\sigma_\theta^2\left(F_{1,1}F_{2,2} + F_{2,1}F_{1,2} + F_{1,1}F_{2,1}\right) + \left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)F_{1,2}F_{2,2}\right.$$

after substituting $a_i = q_i$ and $u_i = \pi_i$, where C denotes the Cournot competition. After inserting the equilibrium constants from Lemma 1, we can show that

$$\frac{\partial E(CS_C^*)}{\partial \sigma_{\epsilon_i}^2} = \frac{\omega^2 \sigma_\theta^4 \sigma_{\epsilon_z}^4 r_{2,2} r_4}{512\gamma r_3^3} \qquad (1.15)$$

where $r_4 = 16r_3\left(r_{2,2} + 8\lambda\sigma_\theta^2\sigma_{\epsilon_z}^2\right) - 64\lambda^2 r_{2,2}\sigma_\theta^4\sigma_{\epsilon_z}^4$. Note that $r_3, r_{2,2} > 0$ and $\lambda \in [-1,1]$. Upon some algebra, we can also show that $r_4 > 0$ when $\lambda \in [-2/3,1]$. The main driving force for these comparative statics is the increase in the volatility of aggregate output as firms obtain more precise private signals.

Next, using the total surplus formula, the partial derivative of expected total surplus with respect to the variance of the private signal of firm i is for $j \neq i$,

$$\frac{\partial E(TS_C^*)}{\partial \sigma_{\epsilon_i}^2} = -\frac{r_{2j}\left(4\left(r_{2j}\lambda^2\sigma_\theta^4\sigma_{\epsilon_z}^4 + 4r_3\left((1-\lambda)\sigma_\theta^2\sigma_{\epsilon_z}^2 + \sigma_{\epsilon_j}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)\right)\right) + r_{2j}r_3\right)}{32\gamma\omega^{-2}\sigma_\theta^{-4}\sigma_{\epsilon_z}^{-4}r_3^3} < 0$$

and that of the public signal z is

$$\frac{\partial E(TS_C^*)}{\partial \sigma_{\epsilon_z}^2} = \frac{2T\omega^2\sigma_\theta^4}{\gamma(2+\lambda)^2 r_3^3} < 0$$

Both partial derivatives are negative because $\omega \neq 0, \lambda \in [-1,1], \sigma_{\epsilon_z}^2, \sigma_{\epsilon_i}^2, \sigma_\theta^2, \gamma, r_{2j}, r_3 > 0$

and

$$\begin{aligned} T = {} & \sigma_\theta^8\sigma_{\epsilon_z}^8(4-\lambda^2)\left(6(2-\lambda)^2\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2 + (12-\lambda(4-\lambda))\left(\sigma_{\epsilon_1}^2 - \sigma_{\epsilon_2}^2\right)^2\right) \\ & +24\sigma_\theta^2\sigma_{\epsilon_1}^4\sigma_{\epsilon_2}^4\sigma_{\epsilon_z}^2\left(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2\right)\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)^3(8-\lambda^2) + 2\sigma_\theta^6\sigma_{\epsilon_z}^6\left(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2\right) \\ & \times\left((24+\lambda(8-\lambda))(2-\lambda)^2\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2 + (24-2\lambda(4+3\lambda))\left(\sigma_{\epsilon_1}^2 - \sigma_{\epsilon_2}^2\right)^2\right)\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right) \\ & +24\sigma_\theta^4\sigma_{\epsilon_1}^4\sigma_{\epsilon_2}^4\sigma_{\epsilon_z}^4\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)^2(12-4\lambda-\lambda(1-\lambda))\Big) \\ & +12\sigma_\theta^4\sigma_{\epsilon_1}^2\sigma_{\epsilon_2}^2\sigma_{\epsilon_z}^4\left(\sigma_{\epsilon_1}^4 + \sigma_{\epsilon_2}^4\right)\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)^2(12-\lambda(4+3\lambda)) + 32(3+\lambda)\sigma_{\epsilon_1}^6\sigma_{\epsilon_2}^6\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)^4 \end{aligned}$$

is positive. Together with Propositions 1 and 2, we obtain the following results:

Proposition 3. Let $\sigma_\epsilon^2 > 0$ and firms compete in production or service quantities.

i) *Firms have incentives to collect private demand or cost information. Information acquisition of firm i is beneficial to firm j if firms produce complementary (substitutable) products.*

ii) *After a firm collects a better information about the demand for its product or about its own costs, the expected total surplus always increase. However, the expected consumer surplus increases if firms produce substitutable products or not very complementary products.*
iii) *After the central bankers or the government officials provide more precise public signals about demand or costs to the firms, the expected total surplus and total profits of firms increases.*

In Cournot competition, the dissemination of more precise public information leads to an increase in total welfare because better public information reduces uncertainty about market demand or costs, allowing firms to make more efficient production decisions. As firms adjust their output levels in response to clearer signals, aggregate production aligns more closely with actual market conditions, reducing inefficiencies and enhancing both total profits. The expected consumer surplus increases firms produce substitutable products or not very complementary products. The reduction in uncertainty also mitigates the risk of over- or under-production, which can lead to price volatility and welfare losses. Consequently, in Cournot settings, public information dissemination tends to unambiguously improve total welfare by fostering more efficient market outcomes.

1.4.2. Bertrand competition and information acquisition

In this application, the agents are firms that simultaneously choose their price levels (p_i). Each seller produces a single product, and their products are horizontally differentiated. Let q_i denote the production of firm i, and let $Q = \sum_{i=1}^{2} q_i$ represent the total market output. The price of firm i 's product is given by

$$p_i = D + \theta - b\left(q_i + \eta q_j\right), \tag{1.16}$$

where $D > 0$ is the demand parameter, $b > 0$ is the slope of the inverse demand, and $\theta \sim (\bar{\theta}, \sigma_\theta^2)$ is the prior common random variable for demand, with $\bar{\theta} \geq 0$. The parameter $\eta \in [-1,1]$ measures the degree of horizontal product differentiation. The products are imperfectly substitutable if $\eta \in (0,1)$, unrelated if $\eta = 0$, and complementary if $\eta < 0$. Solving the inverse demand for quantities yields q_i. Therefore, the profit of each firm i is symmetric and is given by:

$$\pi_i = q_i p_i = \left(\frac{D+\theta}{b(1+\eta)} - \frac{p_i}{b(1-\eta^2)} + \frac{\eta p_j}{b(1-\eta^2)}\right) p_i$$

Equation (1.1) coincides with this payoff after setting $a_i = p_i, A = D\omega = \frac{D}{b(1+\eta)}$, and $\lambda = -\eta$, and $\gamma = \frac{1}{b(1-\eta^2)}$.

We use the same formulations for consumer surplus, industry profits, and total surplus as in the Cournot model. Similar calculations to those in the Cournot game yield that: When $\lambda = -\eta$,

$$\frac{\partial E(CS_B^*)}{\partial \sigma_{\epsilon_i}^2} = \frac{(1-\eta) r_{2,j} \sigma_\theta^4 \sigma_{\epsilon_z}^4 T_{2,i}}{8b(1+\eta) r_3^3} < 0$$

and

$$\frac{\partial E(TS_B^*)}{\partial \sigma_{\epsilon_i}^2} = \frac{(1-\eta) r_{2,j} \sigma_\theta^4 \sigma_{\epsilon_z}^4 T_{3,i}}{8b(1+\eta) r_3^3} < 0 \text{ if } \eta \in [-1, 2/3].$$

where B denotes the Bertrand competition and for $j \neq i$,

$$\begin{aligned} T_{2,i} =& 4r_{1,1}r_{1,2}\left((6+5\eta)\sigma_\theta^2\sigma_{\epsilon_z}^2 + 6\sigma_{\epsilon_j}^2\left(\sigma_\theta^2 + \sigma_{\epsilon_z}^2\right)\right) \\ &+ \eta^2\sigma_\theta^4\sigma_{\epsilon_z}^4\left(\sigma_\theta^2\left((2-\eta)\sigma_{\epsilon_z}^2 + 2\sigma_{\epsilon_j}^2\right) + 2\sigma_{\epsilon_j}^2\sigma_{\epsilon_z}^2\right), \\ T_{3,i} =& 4r_{1,1}r_{1,2}\left(\sigma_\theta^2\left((2-\eta)\sigma_{\epsilon_z}^2 + 2\sigma_{\epsilon_j}^2\right) + 2\sigma_{\epsilon_j}^2\sigma_{\epsilon_z}^2\right) \\ &- \eta^2\sigma_\theta^4\sigma_{\epsilon_z}^4\left(\sigma_\theta^2\left((3\eta+10)\sigma_{\epsilon_z}^2 + 10\sigma_{\epsilon_j}^2\right) + 10\sigma_{\epsilon_j}^2\sigma_{\epsilon_z}^2\right) \end{aligned}$$

The inequalities follow because $\omega \neq 0, \eta \in [-1,1), \sigma_{\epsilon_z}^2, \sigma_{\epsilon_i}^2, \sigma_\theta^2, \gamma, r_{2j}, r_3 > 0, T_{2,i} > 0$, and $T_{3,i} > 0$ if $\eta \in [-1,2/3]$. When $\eta = 1$, both partial derivatives are zero. Using similar techniques to above, we can show that the partial derivative $\frac{\partial E(TS_B^*)}{\partial \sigma_{\epsilon_z}^2} > 0$ if $\eta \in [-1,1/2]$, and it is less than zero if firms produce close substitutes, that is η is sufficiently close to 1 .

Together with Propositions 1 and 2, we obtain the following results:

Proposition 4. Let $\sigma_\epsilon^2 > 0$ and firms compete in prices.

i) *Firms have incentives to collect private demand information. Information acquisition of firm i is beneficial to firm j if firms produce substitutable (complementary) products.* **ii)** *After a firm collects a better information about the demand for its product, the expected consumer surplus always increases. However, the expected total surplus increases if firms produce products or not very substitutable products.* **iii)** *After the central bankers or the government officials provide more precise public signals about demand to the firms, the expected total profits of firms increases. However, the expected total surplus increases if firms produce close substitutes. If they produce complementary products or their products are not close substitutes ($\eta < 1/2$), the expected total surplus decreases with more precise public information.*

The welfare implications of public information dissemination in Bertrand competition are more nuanced and depend on the degree of product differentiation. First of all, the total profits of firms increases with more price public signal because the firms can better adjust their price strategies to the changes in demand. When firms produce close substitutes, more precise public information typically increases both consumer and total surpluses. However, when firms produce complementary or not very substitutable products, the loss in consumer surplus outweigh the gain in producer surplus. Hence, expected total surplus decreases in the precision of the public signal.

However, when products are complementary or not close substitutes, the dissemination of public information can have a less favorable impact on total welfare. In these scenarios, firms may use the additional information to coordinate their pricing strategies more effectively, potentially leading to higher prices and reduced consumer surplus. This is particularly concerning in markets where firms have significant market power, as the improved information could facilitate tacit collusion, thereby reducing overall welfare. Thus, in Bertrand competition, the welfare effects of public information dissemination are highly sensitive to the degree of product differentiation and the competitive dynamics of the market.

1.5. Conclusion

This section of our book has explored the welfare implications of information acquisition in oligopolistic markets, emphasizing the strategic role of private and public information in shaping competitive outcomes. By examining various theoretical models and real-world applications, we have highlighted how firms acquire and utilize information to optimize their decision-making processes, whether through demand forecasting, cost analysis, or dynamic pricing strategies. The findings underscore the dual nature of information acquisition: while it can enhance market efficiency and consumer welfare by reducing uncertainty and improving resource allocation.

The analysis of Cournot and Bertrand competition models reveals that the welfare effects of information acquisition critically depend on the nature of competition, the type of information (demand or cost), and the degree of product differentiation. In Cournot settings, improved private demand and cost information generally leads to more efficient output decisions, benefiting both firms and overall welfare. However, in Bertrand markets, the impact of enhanced information on consumer surplus and total welfare is more nuanced, depending on whether products are substitutes or complements. Public information, on the other hand, consistently enhances total welfare by reducing uncertainty and facilitating better coordination among firms in Cournot competition. However, it may reduce welfare under Bertrand competition, especially when firms produce complementary or weakly substitutable products.

Overall, this research contributes to the growing body of literature on information economics by providing a comprehensive framework for understanding the strategic implications of information acquisition in oligopolistic markets. It offers valuable guidance for firms navigating the complexities of information asymmetry and for policymakers aiming to promote competitive and efficient markets. As industries continue to evolve with advancements in data analytics and artificial intelligence, the strategic use of information will remain a critical factor in shaping market dynamics and welfare outcomes. Future research could further explore the role of information in emerging markets, the impact of digital platforms on information dissemination, and the regulatory challenges posed by the increasing availability of real-time data.

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CHAPTER 2
WELFARE IMPLICATIONS OF INFORMATION SHARING

2.1. Introduction

Firms may share their private demand information in various real-world contexts, often through industry associations, joint ventures, or informal communication. Notable examples include the early development of electric vehicles (EVs), where companies such as Toyota and Tesla exchanged demand insights and technology to accelerate market expansion. [1] Toyota even partnered with Tesla to develop EV components, indirectly sharing demand expectations. More recently, Volkswagen and Ford have collaborated on electric vehicle and autonomous driving technologies, likely involving some exchange of demand forecasts.

Similarly, airlines in global alliances frequently share demand forecasts for specific routes to optimize scheduling, coordinate code-sharing agreements, and adjust ticket pricing. For example, members of the Star Alliance, such as Lufthansa and United Airlines, exchange passenger demand data to align flight schedules and prevent overcapacity. They also share cost-related information, including fuel expenses, maintenance, and operational efficiency (Chua et al., 2005a, 2005b). Likewise, while OPEC's primary objective is to regulate oil production, its member countries also exchange market demand forecasts to guide production decisions and pricing strategies.

In this section, we analyze a two-agent framework in which each agent receives a noisy private signal regarding a common prior variable. Agents have quadratic payoffs, and their actions can be either complementary or substitutable. We investigate the incentives for information sharing and its impact on overall welfare. Our findings indicate that information sharing is a dominant strategy when agents' actions are complementary, whereas concealing information becomes optimal when their actions are substitutes. Moreover, the expected total action level is higher when both agents share their private signals, provided their strategies are not excessively substitutable.

These findings yield several novel implications for the Cournot and Bertrand games with differentiated products. Firms are incentivized to share private demand or cost information only when the products are complementary under Cournot competition, or when they are substitutable under Bertrand competition. Conversely, they tend to conceal information when the products are substitutable under Cournot competition, or complementary under Bertrand competition. When both firms share their information, the expected total surplus is higher under Cournot competition, while the expected consumer surplus is lower under Bertrand competition.

The expected consumer surplus is higher in the full-sharing game than in the nosharing game if and only if firms produce sufficiently substitutable products under Cournot competition. On the other hand, the expected total surplus is lower in the full-sharing game than in the no-sharing game if and only if firms produce sufficiently substitutable products under Bertrand competition.

The study of information sharing in industrial organization has primarily focused on firms' incentives to share demand and cost information, as well as the resulting market outcomes. Over the years, extensive research has explored the strategic considerations behind information sharing, its effects on competition and welfare, and the conditions under which firms benefit from transparency or secrecy. This essay offers a comprehensive summary of the key contributions in the field, emphasizing the main findings, underlying intuitions, and the evolution of the literature. Additionally, our findings extend earlier results to any two-agent game with complementary or substitutable actions.

Fried (1984, Prop. 3), Vives (1984), and Li (1985, Prop. 3) demonstrate that concealing demand information is a dominant strategy equilibrium in the Cournot game when values are common and products are differentiated. A common conclusion across various studies is that firms are generally better off concealing demand information, as revealing it reduces uncertainty for competitors, allowing them to adjust their production quantities more effectively. Consequently, the firm sharing the information risks losing its strategic advantage, since the competitor can use the new data to adjust its output and enhance its profitability. Spence (1985) also argues that revealing information can be harmful in highly competitive environments, as it enables rivals to respond more efficiently. Similarly, Tirole (1988) contends that by concealing demand information, firms prevent competitors from adjusting their output decisions based on that information.

Vives (1984) further demonstrates that, with an unknown common demand, sharing demand information is a dominant strategy under Bertrand competition with differentiated products. Gal-Or (1985) shows that, with unknown private costs, sharing is a dominant strategy under Cournot competition, while concealing information is the dominant strategy under Bertrand competition. However, Harrison and Wilson (1989) find that revealing demand information can have mixed effects on market outcomes and does not constitute a dominant strategy. Fudenberg and Tirole (1991) emphasize that information sharing can lead to more informationally efficient outcomes in markets, enabling firms to better align their quantities with demand. However, this does not necessarily result in higher profits for the firms involved due to the increased competition.

Raith (1996) further refined these insights by examining the role of correlation in demand shocks. He showed that when demand shocks are highly correlated across firms, concealing information becomes more advantageous, as it reduces uncertainty without significantly altering strategic interactions. In contrast, when demand shocks are independent, firms may prefer to reveal information in order to maintain an informational advantage.

Our findings extend beyond the oligopoly setting, offering more general results that apply to a broader range of contexts than the previous studies mentioned. We only present the implications of our result in the oligopoly theory. We refer to Cumbul (2022) for the other possible applications.

2.2 Set-up

We consider a simple two-agent setup that encompasses many earlier two-agent models discussed in the literature. Each agent $i \in \{1,2\}$ chooses an action $a_i \in \mathbb{R}$, and is assumed to have the following quadratic utility function:

$$u_i(a_1, a_2) = \left(A + \omega\theta - \gamma a_i - \gamma\lambda a_j\right)a_i \quad (2.1)$$

where $A > 0, \gamma > 0$, and $\omega \neq 0$ are known parameters, $\lambda \in [-1,1]$, and θ is a prior random variable with mean $\bar{\theta} \geq 0$ and variance $\sigma_\theta^2 > 0$. The strategies of the players are considered strategic substitutes if $\partial^2 u_i/\partial^2 a_i a_j = -\gamma\lambda < 0$ or $\lambda > 0$, and strategic complements if $\lambda < 0$.

Each agent receives a noisy private signal $s_i = \theta + \epsilon_i$ of equal quality, where $E(\epsilon_i) = 0, \mathrm{Var}(\epsilon_i) = \sigma_\epsilon^2 \geq 0, \mathrm{Cov}(\epsilon_i, \theta) = \mathrm{Cov}\left(\epsilon_i, \epsilon_j\right) = 0$, and all random variables are jointly normally distributed. [2] Therefore, $\mathrm{Var}(s_i) = \sigma_\epsilon^2 + \sigma_\theta^2$. As $\sigma_\epsilon^2 \to 0$, the signals become perfectly informative, and we reach the complete information scenario. As $\sigma_\epsilon^2 \to \infty$, the signals become perfectly uninformative, leading to the fully incomplete information scenario. All variance and covariance terms are common knowledge to both players.

2.3. Results

2.3.1. No-sharing game

We begin by solving the no-sharing game, in which no agent shares its private information with the other. In Stage 0, both agents observe their private signals s_1 and s_2. In Stage 1, both agents simultaneously choose their actions by solving $\max_{a_i} E(u_i(a_1, a_2) \mid s_i)$. Specifically, agents do not observe each other's choices. The timeline of this game is as follows:

Stage 0: Agents observe their private signals s_1 and s_2

Stage 1: Agents simultaneously choose a_1 and a_2.

We now seek the Bayesian Nash equilibrium of this game, which is defined as follows:

Definition 2. *Bayesian Nash Equilibrium: A strategy profile (a_1^*, a_2^*) is a Bayesian Nash equilibrium of this game if, for each $i \in \{1,2\}$, where $j \neq i$, it holds that $a_i^* \in \arg\max_{a_i} E\left(u_i\left(a_i, a_j^*\right) \mid s_i\right)$.*

In other words, the strategies of the players should be best responses to each other's strategies in a Nash equilibrium. Since the payoff function is quadratic and agents have symmetric quality of private signals, we assume that the Bayesian Nash equilibrium strategies of the players are affine and symmetric. Thus, we let $a_1^* = B_0 + B_1(s_i - \bar{\theta})$, where B_0 and B_1 are constants to be determined. We will now derive these two constants.

Agent 1 solves the following problem:

$$\max_{a_1} E(u_1(a_1, a_2) \mid s_1) = E\big((A + \omega\theta - \gamma a_1 - \gamma\lambda a_2)a_1 \mid s_1\big). \quad (2.2)$$

The expected utility is concave in a_1 because the second derivative of the expected utility with respect to a_1 is negative, i.e.,

$$\frac{\partial^2 E(u_i \mid s_i)}{\partial a_i^2} = -2\gamma < 0$$

The first-order condition (FOC) is given by:

$$\frac{\partial E(u_i \mid s_i)}{\partial a_i} = 0$$

which simplifies to:

$$a_1 = B_0 + B_1(s_1 - \bar{\theta}) = \frac{A + \omega E(\theta \mid s_1) - \gamma\lambda E(a_2 \mid s_1)}{2\gamma}$$

Note that $E(a_2 \mid s_1) = B_0 + B_1(E(s_2 \mid s_1) - \bar{\theta})$. Furthermore, we can derive the conditional expectations using the projection theorem for normally distributed multivariate variables, as presented in Vives (2011) or Rostek and Weretka (2012):

$$E(\theta \mid s_i) = E(s_j \mid s_i) = E(\theta) + \frac{\mathrm{Cov}(\theta, s_i)}{\mathrm{Var}(s_i)}\big(s_i - E(s_i)\big) = \bar{\theta} + \frac{\sigma_\theta^2(s_i - \bar{\theta})}{\sigma_\theta^2 + \sigma_\epsilon^2}$$

Thus, equation (2.5) simplifies to:

$$a_1 = B_0 + B_1(s_1 - \bar{\theta}) = \frac{A - \gamma\lambda B_0 + \omega\bar{\theta}}{2\gamma} + \frac{\sigma_\theta^2(\omega - \gamma\lambda B_1)(s_1 - \bar{\theta})}{2\gamma\big(\sigma_\theta^2 + \sigma_\epsilon^2\big)}.$$

We now have two equations with two unknowns, B_0 and B_1, which we can solve. This leads to the following lemma:

Lemma 2. There exists a unique no-sharing (NS) Bayesian equilibrium. The equilibrium strategies for each agent i are:

$$a_{i,NS}^{*} = B_0 + B_1(s_i - \bar{\theta}),$$

where:

$$B_0 = \frac{A + \omega\bar{\theta}}{\gamma(2+\lambda)} \text{ and } B_1 = \frac{\omega\sigma_\theta^2}{\gamma\left(2\sigma_\epsilon^2 + (2+\lambda)\sigma_\theta^2\right)}$$

The expected equilibrium strategies are:

$$E\left(a_{1,NS}^{*}\right) = E\left(a_{2,NS}^{*}\right) = B_0 + B_1(E(s_i) - \bar{\theta}) = B_0 = \frac{A + \omega\bar{\theta}}{\gamma(2+\lambda)},$$

because the expected value of each signal is $E(s_i) = \bar{\theta}$ by assumption.

Using this lemma, we can also derive the expected Bayesian equilibrium utilities for each agent. These utilities consist of two components: the non-covariance part and the covariance part. Let the non-covariance part be denoted by $E\left(u_{i,NS,1}^{*}\right)$, and the covariance part by $E\left(u_{i,NS,2}^{*}\right)$. The first component is relatively straightforward to calculate and simplifies to:

$$\begin{aligned} E\left(u_{i,NS,1}^{*}\right) &= E\left(a_{i,NS}^{*}\right)\left(A + \omega\bar{\theta} - \gamma E\left(a_{i,NS}^{*}\right) - \gamma\lambda E\left(a_{j,NS}^{*}\right)\right) \\ &= \frac{(A + \omega\bar{\theta})^2}{\gamma(2+\lambda)^2}. \end{aligned}$$

To compute the covariance components of the expected equilibrium utilities, we first set $A = B_0 = \bar{\theta} = 0$, as these terms do not contribute to the covariance parts. Next, we can derive that:

$$\begin{aligned} E\left(u_{i,NS,2}^{*}\right) &= \omega\mathrm{Cov}\left(a_{i,NS}^{*}, \theta\right) - \gamma\mathrm{Var}\left(a_{i,NS}^{*}\right) - \gamma\lambda\mathrm{Cov}\left(a_{i,NS}^{*}, a_{j,NS}^{*}\right) \\ &= \omega B_1\mathrm{Cov}(\theta, s_i) - B_1^2\mathrm{Var}(s_i) - B_1^2\gamma\lambda\mathrm{Cov}\left(s_i, s_j\right) \\ &= \frac{\omega^2\sigma_\theta^4\left(\sigma_\theta^2 + \sigma_\epsilon^2\right)}{\gamma\left(2\sigma_\epsilon^2 + (2+\lambda)\sigma_\theta^2\right)^2}, \end{aligned}$$

where we plugged in $\mathrm{Cov}(\theta, s_i) = \mathrm{Cov}(s_1, s_2) = \sigma_\theta^2$ and $\mathrm{Var}(s_1) = \sigma_\epsilon^2 + \sigma_\theta^2$. Altogether, the expected Bayesian equilibrium utility of each agent i is

$$E\left(u_{i,NS}^*\right) = \sum_{k=1}^{k=2} E\left(u_{i,NS,k}^*\right) = \frac{(A + \omega\bar{\theta})^2}{\gamma(2+\lambda)^2} + \frac{\omega^2 \sigma_\theta^4 (\sigma_\theta^2 + \sigma_\epsilon^2)}{\gamma\left(2\sigma_\epsilon^2 + (2+\lambda)\sigma_\theta^2\right)^2} \quad (2.3)$$

2.3.2. Partial and no-sharing information sharing games

We now define the information-sharing game. In Stage 0, both agents observe their private signals. In Stage 1, each agent decides whether to share their private signal with the other, assuming that information sharing is costless. Finally, in Stage 2, both agents simultaneously choose their strategies based on the available information. The sequence of events in this game follows a structured timeline, where the decision to share information influences the strategic choices made in the final stage.

Stage 0: Agents observe their private signals s_1 and s_2

Stage 1: Agents simultaneously choose whether to share their private signals.
Stage 2: Agents simultaneously choose a_1 and a_2.

There are four possible outcomes in this game: neither agent shares information, both agents share information, or only one agent chooses to share while the other does not.

Our objective is to determine the subgame-perfect Bayesian equilibrium of this game. To achieve this, we employ the backward induction method. We have already derived the equilibrium for the no-sharing case, denoted as $\left(a_{1,NS}^*, a_{2,NS}^*\right)$.

Next, we derive the Bayesian equilibrium for both the unilateral and full-sharing games. Unlike the no-sharing case, the equilibrium of the unilateral sharing game is inherently asymmetric. Without loss of generality, suppose that only firm 2 shares its signal with agent 1.

In this setting, the equilibrium strategies of agents 1 and 2 are given by $a_1 = C_0 + C_1(s_1 - \bar{\theta}) + C_2(s_2 - \bar{\theta})$ and $a_1 = C_3 + C_4(s_2 - \bar{\theta})$, where $C_0, C_1, C_2, C_3, C_4 \in \mathbb{R}$ are equilibrium constants. In the following, we derive the values of these constants.

Agent 1's problem is to solve:

$$\max_{a_1} E(u_1(a_1, a_2) \mid s_1, s_2) = E\big((A + \omega\theta - \gamma a_1 - \gamma\lambda a_2)a_1 \mid s_1, s_2\big) \quad (2.4)$$

The FOC yields that

$$a_1 = C_0 + C_1(s_1 - \bar{\theta}) + C_2(s_2 - \bar{\theta}) = \frac{A+\omega E(\theta|s_1,s_2)-\gamma\lambda E(a_2|s_1,s_2)}{2\gamma}$$

Note that the conditional expectations are $E(s_2 \mid s_1, s_2) = s_2$ and

$$E(\theta \mid s_1, s_2) = E(\theta) + \big(\mathrm{Cov}(\theta, s_1)\ \mathrm{Cov}(\theta, s_2)\big)\begin{pmatrix} \mathrm{Var}(s_1) & \mathrm{Cov}(s_1, s_2) \\ \mathrm{Cov}(s_1, s_2) & \mathrm{Var}(s_2) \end{pmatrix}^{-1} \begin{pmatrix} s_1 - E(s_1) \\ s_2 - E(s_2) \end{pmatrix} = \frac{(s_1 + s_2)\sigma_\theta^2 + \sigma_\epsilon^2\bar{\theta}}{\sigma_\epsilon^2 + 2\sigma_\theta^2}$$

by applying the projection theorem as before, and noting that $E(\theta) = E(s_i) = \bar{\theta}$, $\mathrm{Cov}(s_i, s_j) = \mathrm{Cov}(\theta, s_i) = \sigma_\theta^2$ for $j \neq i$, and $\mathrm{Var}(s_i) = \sigma_\epsilon^2 + \sigma_\theta^2$.

Since agent 1 does not share its private signal with agent 2, agent 2's equilibrium strategy must satisfy:

$$a_2 = C_3 + C_4(s_1 - \bar{\theta}) = \frac{A - \gamma\lambda C_3 + \omega\bar{\theta}}{2\gamma} + \frac{\sigma_\theta^2(\omega - \gamma\lambda C_4)(s_2 - \bar{\theta})}{2\gamma\big(\sigma_\theta^2 + \sigma_\epsilon^2\big)} \quad (2.5)$$

By applying similar calculations as in the no-sharing game, solving equations (2.4) and (2.5) simultaneously yields a system of five equations with five unknowns: C_0, C_1, C_2, C_3, C_4. By analogy, we can also determine the equilibrium strategies when only agent 1 shares its private information with agent 2 . This analysis leads to the following lemma.

Lemma 3. *Suppose only firm j shares its private signal with agent i. There exists a unique Bayesian equilibrium for this unilateral sharing game (US_j). The equilibrium strategies of agents i and j are $a_{i,US_j} = C_0 + C_1(s_i - \bar{\theta}) + C_2(s_j - \bar{\theta})$ and $a_{j,US_j} = C_3 + C_4(s_j - \bar{\theta})$, respectively, where the equilibrium constants are $C_0 = C_3 = \frac{A+\omega\bar{\theta}}{\gamma(2+\lambda)}$, $C_1 = \frac{\omega\sigma_\theta^2}{\sigma_\epsilon^2+2\gamma(2\sigma_\theta^2)}$, $C_2 = \frac{\omega(2\sigma_\theta^2\sigma_\epsilon^2+(2-\lambda)\sigma_\theta^4)}{2\gamma(2+\lambda)(\sigma_\theta^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+2\sigma_\theta^2)}$, and $C_4 = \frac{\omega\sigma_\theta^2}{\gamma(2+\lambda)(\sigma_\theta^2+\sigma_\epsilon^2)}$.*

Similar to the no-sharing game, the expected Bayesian equilibrium utility for each agent i and j in the unilateral sharing game US_j can be expressed as follows:

$$E\left(u_{i,US_j}^*\right) = \frac{A+\omega\bar{\theta}}{\gamma(2+\lambda)} + \frac{\omega^2\sigma_\theta^4\left(8(\sigma_\epsilon^2+\sigma_\theta^2)+\lambda(4+\lambda)\sigma_\epsilon^2\right)}{4\gamma(2+\lambda)^2(\sigma_\theta^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+2\sigma_\theta^2)}. \quad (2.6)$$

and

$$E\left(u_{j,US_j}^*\right) = \frac{A+\omega\bar{\theta}}{\gamma(2+\lambda)} + \frac{\omega^2\sigma_\theta^4}{\gamma(2+\lambda)^2(\sigma_\theta^2+\sigma_\epsilon^2)}. \quad (2.7)$$

Finally, we analyze the full-sharing game, where both agents share their private signals with each other. Due to symmetry, let $a_i = D_0 + D_1(s_i - \bar{\theta}) + D_2(s_j - \bar{\theta})$ represent the equilibrium strategy of agent i in the full-sharing game. Similar to the maximization problem in (2.4), each agent i solves the following:

$$\max_{a_1} E(u_i(a_1,a_2) \mid s_1,s_2) = E\left((A+\omega\theta-\gamma a_i-\gamma\lambda a_j)a_i \mid s_1,s_2\right).)$$

The FOC yields that

$$a_i = D_0 + D_1(s_i-\bar{\theta}) + D_2(s_j-\bar{\theta}) = \frac{A+\omega E(\theta \mid s_1,s_2) - \gamma\lambda E(a_j \mid s_1,s_2)}{2\gamma} \quad (2.8)$$

Solving for the equilibrium constants yields the following lemma.

Lemma 4. *There exists a unique full-sharing (FS) Bayesian equilibrium. The equilibrium strategies of each agent* $i \in \{1,2\}$ *are* $a_{i,FS}^* = D_0 + D_1(s_i - \bar{\theta}) + D_2\big(s_j - \bar{\theta}\big)$, *where* $D_0 = \frac{A+\omega\bar{\theta}}{\gamma(2+\lambda)}$ *and* $D_1 = D_2 = \frac{\omega\sigma_\theta^2}{\gamma(2+\lambda)\left(\sigma_\epsilon^2+2\sigma_\theta^2\right)}$.

Similar to our earlier derivations, we can deduce the expected equilibrium payoffs of each agent i as

$$E\left(u_{i,FS}^*\right) = \frac{A+\omega\bar{\theta}}{\gamma(2+\lambda)} + \frac{2\omega^2\sigma_\theta^4}{\gamma(\lambda+2)^2\left(\sigma_\epsilon^2+2\sigma_\theta^2\right)}. \quad (2.8)$$

The corresponding payoffs for the agents in the different sharing and no-sharing scenarios are presented in the following game matrix.

		Agent 2	
		No-Share	Share
Agent 1	No-Share	$\left(E\left(u_{1,NS}^*\right), E\left(u_{2,NS}^*\right)\right)$	$\left(E\left(u_{1,US_2}^*\right), E\left(u_{2,US_2}^*\right)\right)$
	Share	$\left(E\left(u_{1,US_1}^*\right), E\left(u_{2,US_1}^*\right)\right)$	$\left(E\left(u_{1,FS}^*\right), E\left(u_{2,FS}^*\right)\right)$

It follows from (2.3), (2.6), (2.7) and (2.8) that

$$E\left(u_{1,US_1}^* - u_{1,NS}^*\right) = E\left(u_{2,US_2}^* - u_{2,NS}^*\right) = -\frac{\lambda\omega^2\sigma_\epsilon^2\sigma_\theta^4\left(2(2+\lambda)\sigma_\theta^2 + (4+\lambda)\sigma_\epsilon^2\right)}{\gamma(2+\lambda)^2(\sigma_\epsilon^2+\sigma_\theta^2)(2\sigma_\epsilon^2+(2+\lambda)\sigma_\theta^2)^2}$$

$$E\left(u_{1,FS}^* - u_{1,US_2}^*\right) = E\left(u_{2,FS}^* - u_{2,US_1}^*\right) = -\frac{\lambda\omega^2\sigma_\epsilon^2\sigma_\theta^4(4+\lambda)}{4\gamma(2+\lambda)^2(\sigma_\epsilon^2+\sigma_\theta^2)(\sigma_\epsilon^2+2\sigma_\theta^2)}.$$

Note that $\lambda > 0$ and $\sigma_\theta^2 > 0$. Let $\sigma_\epsilon^2 > 0$ so that agents have incomplete information. Thus, we obtain the following comparisons: $E\left(u_{1,US_1}^*\right) \gtreqless E\left(u_{1,NS}^*\right)$, $E\left(u_{2,US_2}^*\right) \gtreqless E\left(u_{2,NS}^*\right)$, $E\left(u_{1,FS}^*\right) \gtreqless E\left(u_{1,US_2}^*\right)$, and $E\left(u_{2,FS}^*\right) \gtreqless E\left(u_{2,US_1}^*\right)$ if $\lambda \gtreqless 0$. This leads to part i) of our first proposition.

In our second result, we compare the expected total payoffs of the agents between the full and no-sharing games, deriving that

$$\sum_{i=1}^{i=2}\left(E(u_{i,NS}^{*})-E(u_{i,NS}^{*})\right)=\frac{2\omega^2\sigma_\theta^4\Gamma_0}{\gamma(2+\lambda)^2\left(2\sigma_\theta^2+\sigma_\epsilon^2\right)\left(2\sigma_\epsilon^2+(2+\lambda)\sigma_\theta^2\right)^2}$$

where $\Gamma_0=-\lambda^2\left(3\sigma_\theta^2\sigma_\epsilon^2+\sigma_\epsilon^4\right)\Big)-4\lambda\sigma_\epsilon^2\left(\sigma_\theta^2+\sigma_\epsilon^2\right)+4\sigma_\epsilon^2\left(\sigma_\theta^2+\sigma_\epsilon^2\right)$. Note that $\lim_{\lambda\to-\infty}\Gamma_0=-\infty<0, \Gamma_0(\lambda=-1)=5\sigma_\theta^2\sigma_\epsilon^2+7\sigma_\epsilon^4>0, \Gamma_0(\lambda=0)=4\sigma_\epsilon^2\left(\sigma_\epsilon^2+\sigma_\theta^2\right)>0,\quad \Gamma_1(\lambda=1)=-\sigma_\epsilon^2\left(3\sigma_\theta^2+\sigma_\epsilon^2\right)>0$, and $\lim_{\lambda\to\infty}\Gamma_1=-\infty<0$. Moreover, by applying Descartes' Rule of Signs, there exists exactly one positive root and one negative root for λ. The positive root lies between 0 and 1 , and is given by $\lambda_1=\frac{2\left(\sigma_\epsilon^2+\sigma_\theta^2\right)}{\sigma_\epsilon^2+\sigma_\theta^2+\sqrt{4\sigma_\theta^4+6\sigma_\theta^2\sigma_\epsilon^2+2\sigma_\epsilon^4}}\in(0,1)$. The negative root is less than -1 and thus not relevant for our analysis.

Proposition 5. i) *Sharing information is a dominant strategy for both agents when* $\lambda<0$, *while no-sharing information is a dominant strategy when* $\lambda>0$. **ii)** *The expected total profit is higher in the full-sharing game than in the no-sharing game if and only if* $\lambda<\lambda_1$.

As we transition from the no-sharing game to the full-sharing game, the following changes occur in the equilibrium payoff of an agent.

$$\Delta E(u_i^*)=\underbrace{\mathrm{Cov}(\omega\theta_i,a_i^*)}_{\substack{\text{Strategy}\\ \text{adjustment effect}\\ +}}\ \underbrace{-\gamma\mathrm{Var}(a_i^*)}_{\substack{\text{Output volatility}\\ \text{effect }(-)}}\ \underbrace{-\gamma\lambda\mathrm{Cov}(a_1^*,a_2^*)}_{\substack{\text{Strategy alignment effect}\\ +/-}}$$

First, agents are able to better adjust their strategies to changes in the prior θ in the full-sharing game than in the no-sharing game. For example, if $\omega > 0(\omega < 0)$ and the state is high, the agent's strategy will be higher (lower) in the full-sharing game compared to the no-sharing game. Second, the output volatility of an agent is greater in the full-sharing outcome than in the no-sharing outcome. Lastly, the strategies of the agents are more correlated when $\lambda > 0$ and less correlated when $\lambda < 0$ in the full-sharing game compared to the no-sharing game.

When $\lambda < 0$, the strategy adjustment and alignment effects lead to an increase in the expected payoff of the agents in the full-sharing game, while the output volatility effect causes a decrease compared to the no-sharing game. However, when $\lambda > 0$, only the strategy adjustment effect results in higher expected payoffs in the full-sharing game compared to the no-sharing game.

2.4. Applications of the information sharing model

This model has a wide range of applications in oligopoly theory, public economics, and environmental economics. We will be discussing the application on public and environmental economics in Chapter 3 of our book.

2.4.1. Cournot model and information sharing

In this model, the agents represent firms that simultaneously choose their production quantities (q_i). Consider two sellers producing differentiated products, with each firm producing only one product. Let q_i denote the production of firm i, and let $Q = \sum_{i=1}^{2} q_i$ represent the total market output. The price, net of the marginal cost of firm i 's product, is given by:

$$p_i - c_i = A + \omega\theta - \gamma\left(q_i + \lambda q_j\right),$$

where $A > 0$ is the demand parameter, $\gamma > 0$ is the slope of the inverse demand, and $\theta \sim \left(\bar{\theta}, \sigma_\theta^2\right)$ is the prior common random variable with $\bar{\theta} \geq 0$. The parameter $\lambda \in [-1,1]$ measures the degree of horizontal product differentiation. The products are imperfectly substitutable if $\lambda \in (0,1)$, unrelated if $\lambda = 0$, and complementary if $\lambda < 0$. If $\omega = -1$, demand is known, but there is cost uncertainty. Conversely, if $\omega = 1$, only demand uncertainty exists.

The profit of firm i is given by:

$$\pi_i = (p - c_i)q_i = \left(A + \omega\theta - \lambda\left(q_i + \lambda q_j\right)\right) q_i$$

Equation (2.1) coincides with this payoff after letting $a_i = q_i$. The demand signals can be considered as sufficient statistics derived from various sources of single observations, such as market research and sales reports, among others.

The consumer surplus is defined as $CS = \gamma\left(\frac{q_1^2+q_2^2}{2} + \lambda q_1 q_2\right)$. Let the total profits be $\Pi = \sum_{i\in N} \pi_i$. The expected total surplus is the sum of the expected consumer surplus and total industry profits: $E(TS) = E(CS) + E(\Pi)$. We can further derive the expected Bayesian equilibrium consumer surpluses in both the no-sharing and full-sharing Cournot games as follows:

$$E\left(CS_{C,NS}^*\right) = \frac{(1+\lambda)(A+\omega\bar{\theta})^2}{\gamma(2+\lambda)^2} + \frac{\omega^2\sigma_\theta^4\left(\sigma_\epsilon^2 + (1+\lambda)\sigma_\theta^2\right)}{\gamma\left(2\sigma_\epsilon^2 + (2+\lambda)\sigma_\theta^2\right)^2}$$

and

$$E\left(CS_{C,FS}^*\right) = \frac{(1+\lambda)(A+\omega\bar{\theta})^2}{\gamma(2+\lambda)^2} + \frac{2\omega^2\sigma_\theta^4(1+\lambda)}{\gamma(2+\lambda)^2\left(\sigma_\epsilon^2 + 2\sigma_\theta^2\right)}$$

after substituting $a_i = q_i$ into Lemmas (2) and (4), where C denotes the Cournot competition, it follows that

$$E\left(CS_{C,FS}^*\right) - E\left(CS_{C,NS}^*\right) = \frac{\omega^2\sigma_\epsilon^2\sigma_\theta^4\Gamma_1}{\gamma(2+\lambda)^2\left(2\sigma_\theta^2 + \sigma_\epsilon^2\right)\left(+2\sigma_\epsilon^2(2+\lambda)\sigma_\theta^2\right)^2},$$

where $\Gamma_1 = -\lambda^3\sigma_\theta^2 + \lambda^2(\sigma_\theta^2 - \sigma_\epsilon^2) + \lambda(8\sigma_\theta^2 + 4\sigma_\epsilon^2) + 4(\sigma_\theta^2 + \sigma_\epsilon^2)$. Note that $\lim_{\lambda\to-\infty}\Gamma_1 = \infty > 0, \Gamma_1(\lambda = -1) = -2\sigma_\theta^2 - \sigma_\epsilon^2 < 0, \Gamma_1(\lambda = -1/2) = 3\sigma_\theta^2/8 + 7\sigma_\epsilon^2/4 > 0, \Gamma_1(\lambda = 0) = 4(\sigma_\theta^2 + \sigma_\epsilon^2) > 0, \Gamma_1(\lambda = 1) = 12\sigma_\theta^2 + 7\sigma_\epsilon^2 > 0$, and $\lim_{\lambda\to\infty}\Gamma_1 = -\infty < 0$. Moreover, applying Descartes' rule of signs, there is only one positive root and two negative roots for λ. The positive root is greater than 1 , while one of the negative roots is smaller than -1 , and the other lies between -1 and $-1/2$. Similarly, we can derive the differences in total surplus between the two games as follows:

$$E(TS_{C,FS}^*) - E(TS_{C,NS}^*) = \frac{\omega^2\sigma_\theta^4\sigma_\epsilon^2\left((12-\lambda(4+3\lambda))\sigma_\epsilon^2 + (12-\lambda^2(5+\lambda))\sigma_\theta^2\right)}{\gamma(2+\lambda)^2(2\sigma_\theta^2+\sigma_\epsilon^2)\left((2+\lambda)\sigma_\theta^2 + 2\sigma_\epsilon^2\right)^2} > 0$$

Together with Proposition 3.1, we obtain the following results:

Proposition 6. *Let* $\sigma_\epsilon^2 > 0$.
i) *Firms have incentives to share their private demand or cost information with each other only when they produce complementary products. If they produce substitutable products, no-sharing is optimal.*
ii) *If both firms share their private signals with each other, the expected equilibrium consumer surplus increases if and only if* $\lambda > \lambda^*$ *for some* $\lambda^* \in (-1, -1/2)$.
iii) *If both firms share their private signals with each other, the expected equilibrium total surplus always increases.*

2.4.2. Bertrand Model and information sharing

In this model, the agents are firms that simultaneously choose their price levels (p_i). Each seller produces a single product, and their products are horizontally differentiated. Let q_i

denote the production of firm i, and let $Q = \sum_{i=1}^{2} q_i$ represent the total market output. The price of firm i 's product is given by

$$p_i = D + \theta - b(q_i + \eta q_j) \tag{2.29}$$

where $B > 0$ is the demand parameter, $b > 0$ is the slope of the inverse demand, and $\theta \sim (\bar{\theta}, \sigma_\theta^2)$ is the prior common random variable for demand, with $\bar{\theta} \geq 0$. The parameter $\eta \in [-1,1]$ measures the degree of horizontal product differentiation. The products are imperfectly substitutable if $\eta \in (0,1)$, unrelated if $\eta = 0$, and complementary if $\eta < 0$. Solving the inverse demand for quantities yields q_i. Therefore, the profit of each firm i is symmetric and is given by:

$$\pi_i = q_i p_i = \left(\frac{D + \theta}{b(1 + \eta)} - \frac{p_i}{b(1 - \eta^2)} + \frac{\eta p_j}{b(1 - \eta^2)}\right) p_i \tag{2.30}$$

Equation (2.1) coincides with this payoff after setting $a_i = p_i, A = D\omega = \frac{D}{b(1+\eta)}$, and $\lambda = -\eta$, and $\gamma = \frac{1}{b(1-\eta^2)}$.

We use the same formulations for consumer surplus, industry profits, and total surplus as in the Cournot model. Similar calculations to those in the Cournot game yield that: $E(CS_{B,FS}^* - CS_{B,NS}^*)$

$$= \frac{-\sigma_\epsilon^2 \sigma_\theta^4 (1 - \eta)\left((12 - \eta^2(5 - \eta))\sigma_\theta^2 + (12 + \eta(4 - 3\eta))\sigma_\epsilon^2\right)}{b(2 - \eta)^2(1 + \eta)(2\sigma_\theta^2 + \sigma_\epsilon^2)(2\sigma_\epsilon^2 + (2 - \eta)\sigma_\theta^2)^2}$$

$< 0,$

where B denotes the Bertrand competition. Similarly, we can derive the differences in total surplus between the two games as follows:

$$E(TS_{B,FS}^*) - E(TS_{B,NS}^*) = \frac{\sigma_\theta^4 \sigma_\epsilon^2 (1 - \eta)\Gamma_2}{b(2 - \eta)^2(1 + \eta)(2\sigma_\theta^2 + \sigma_\epsilon^2)((2 - \eta)\sigma_\theta^2 + 2\sigma_\epsilon^2)^2},$$

where $\Gamma_3 = \eta^3\sigma_\theta^2 + \eta^2(\sigma_\theta^2 - \sigma_\epsilon^2) - 4\eta(2\sigma_\theta^2 + \sigma_\epsilon^2) + 4(\sigma_\theta^2 + \sigma_\epsilon^2)$. Using similar techniques to other calculations we can show that there is only one root of Γ_2 in η when $\eta \in [-1,1]$. This root is between 0 and 1 . Together with Proposition 3.1, we obtain the following results.

Proposition 7. Let $\sigma_\epsilon^2 > 0$ and firms compete in prices. *i) Firms have incentives to share their private demand or cost information with each other only when they produce substitutable products. If they produce complementary products, no-sharing is optimal.*
ii) If both firms share their private signals with each other, the expected equilibrium consumer surplus always decreases, while the expected equilibrium total surplus decreases if and only if $\eta < \bar{\eta}$ at some $\bar{\eta} \in (0,1)$.

2.5. Conclusion

In conclusion, this section of our book highlights the strategic role of information sharing between firms in contexts of both complementary and substitutable products, emphasizing its impact on market outcomes and overall welfare. We find that firms have strong incentives to share private demand or cost information when their products are complementary, as this allows them to better coordinate their actions and improve their collective welfare. However, when products are substitutes, firms are more likely to withhold information to preserve their competitive advantage and reduce the potential for increased rivalry.

The study builds upon a rich body of literature that has explored the complexities of information sharing in oligopolistic markets. Our findings contribute to this literature by providing a clearer understanding of when firms are likely to benefit from transparency or secrecy, depending on the nature of their products and the competitive environment. Additionally, we extend previous results in the context of Cournot and Bertrand competition, demonstrating that information sharing can either increase or decrease consumer and total surplus, depending on the degree of substitutability or complementarity in the products being offered.

Overall, the implications of this research are far-reaching, offering insights not only for firms operating in differentiated markets but also for policymakers and regulators considering the potential effects of information sharing in various industries. By examining the dynamics of demand and cost information exchange, this study provides a foundation for future research on the strategic use of information in different competitive settings. As firms continue to navigate the challenges of information asymmetry, our results offer valuable guidance on when and how they can leverage information sharing to their advantage, while also considering the broader effects on market welfare.

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[2] Alternatively, we can assume that the conditional expectations are linear in the observable signals. In this case, the prior-posterior $(\theta - s_i)$ distribution pairs that satisfy our linearity assumption include Normal-Normal, Gamma-Poisson, and Beta-Binomial distribution pairs (De Groot, 1970).

CHAPTER 3

INFORMATION ACQUISITION AND SHARING IN AGGREGATIVE GAMES

Information acquisition and sharing can also occur in contexts other than oligopoly markets. For example, in common resource games, agents compete for access to a shared resource, such as fisheries, groundwater, forests, or fossil fuels. The strategic decisions of these agents depend heavily on their knowledge of the resource's size and the costs associated with extraction. Without accurate information, agents may overexploit the resource, leading to depletion, inefficiencies, and potential long-term losses for all stakeholders. Thus, acquiring precise information about resource availability and extraction costs is crucial for sustainable management and optimal decision-making.

One of the primary reasons why information acquisition is important is that it helps agents assess the sustainability of their extraction levels. If firms or individuals underestimate the total resource size, they may engage in overly cautious extraction, leading to underutilization and economic inefficiencies. Conversely, if they overestimate availability, excessive extraction can lead to resource exhaustion, as seen in the case of overfishing in the world's oceans. For example, the collapse of the Atlantic cod fishery off the coast of Canada in the early 1990s was largely due to poor information about fish stock levels, leading to unsustainable catch limits and ultimately the industry's near-total collapse (Myers et al, 2003).

Similarly, understanding extraction costs is vital for efficient resource allocation. In oil and gas industries, companies conduct extensive geological surveys and seismic testing to determine extraction costs before investing in drilling operations. If a firm underestimates extraction costs, it may commit excessive resources to a project that proves unprofitable, as seen in the case of some shale oil operations in the United States. On the other hand, if extraction costs are overestimated, potentially viable resources may remain untapped, leading to missed economic opportunities.

The role of information in groundwater management also highlights its importance in common resource games. Farmers and municipalities that rely on underground aquifers for irrigation and drinking water need accurate data on water table levels and replenishment rates. In regions such as California's Central Valley, lack of precise information has led to over-extraction, causing land subsidence and long-term damage to the aquifer system (Stevens et al, 2003). Improved monitoring systems and realtime data acquisition can help manage water use more effectively and prevent such negative consequences.

In public good contribution games, individuals or groups allocate resources between private consumption and the provision of a shared good, such as environmental protection, infrastructure, or public health. A key factor influencing their decisions is the quality of the public good and the aggregate income available for contributions. Acquiring accurate information about these factors is crucial for efficient decision-making, as it affects individual incentives, collective welfare, and overall resource allocation.

Public goods often exhibit uncertainty in their effectiveness. For example, in climate change mitigation efforts, countries and corporations must assess the impact of their contributions toward reducing carbon emissions. If the benefits of emission reductions-such as temperature stabilization or disaster prevention-are unclear, contributors may underinvest due to perceived ineffectiveness. Similarly, in vaccine distribution programs, uncertainty about vaccine efficacy can lead to suboptimal funding and participation.

When reliable information about the quality of public goods is available, individuals and governments can make more informed decisions, ensuring better provision and resource allocation.

Another critical aspect of public good contribution games is the uncertainty surrounding aggregate income. When individuals or governments make decisions on how much to contribute, they often rely on expectations about the total financial capacity of the group. In international aid, for instance, donor countries determine their contributions based on their own budget constraints and the expected support from others. If aggregate income is underestimated, overall contributions may fall short of what is needed to provide an optimal level of the public good. Conversely, overestimations may lead to inefficient allocations. This is also evident in disaster relief efforts, where the uncertainty of total available funds can affect the speed and effectiveness of aid distribution.

One of the most prominent real-world examples is climate finance, where countries pledge to contribute to global climate funds such as the Green Climate Fund. The effectiveness of these contributions depends on accurate assessments of both the impact of green projects and the financial commitments of other nations. Another example is education funding in developing countries, where uncertainty about public and private sector contributions can lead to under-investment in schools and infrastructure. Furthermore, public health initiatives, such as global vaccination campaigns, depend on accurate information about both disease prevention effectiveness and the financial commitments of donor organizations.

There are also several real-world examples in which agents share their private information in these types of games. Oil and gas companies often publicly announce the estimated reserve sizes of their discoveries, and NASA provided public data on reduced water reserves in Turkey in 2021. Moreover, private information regarding the quality of public good provision is frequently exchanged through social interactions (Scharf, 2014). In addition, agents collect and report data on their gas emission inventories under mandatory programs-such as the US Toxics Release Inventory and the European Pollutant Release and Transfer Register-or through voluntary initiatives like the EU Eco-Management Audit Scheme and various environmental labeling and information schemes (Elnaboulsi et al., 2018). In this chapter, we first introduce several models derived from our earlier work, and then discuss the implications of our previous findings for these models.

3.1. Aggregative game applications

Our two-agent foundational models are aggregative games, where each player's payoff depends not only on their own strategy but also on the aggregate of all players' strategies. This is evident when we rewrite equations (2.1) and (1.1), which leads to the following form:

$$u_i(\theta, a_i, M) = \big(A + \omega\theta - \gamma a_i - \gamma\lambda(M - a_i)\big)a_i \quad (3.1)$$

where $M = a_1 + a_2$. Table 1 summarizes the parameter interpretations for the various applications discussed in the following subsections. The Cournot and Bertrand game examples have already been addressed. Next, we introduce additional applications.

Models	Agents	Action of agents (a_i)	Type of uncertainty (θ)	
			$\omega = 1/2$	$\omega = -1/2$
Cournot	Firms	Production level (q_i)	Demand	Cost
Bertrand	Firms	Level level (p_i)	Demand	
Partnership games	Individuals or partners	Effort level (f_i)	Effort benefit	Effort cost
Public good contribution	Individuals	Private good consumption level (x_i)	Aggregate income	Public good quality
Common resource	Individuals or firms	Common resource extraction level (r_i)	Stock size	Resource extraction cost
Gas emission	Firms or countries	Gas emission level (e_i)	Gas abatement cost, Emission inventory, Emission benefit	Gas abatement benefit, Emission cost

Table 3.1. Interpretation of Parameters in Different Applications of Aggregative Games

3.1.1. Partnership games

First, we introduce the concept of "partnership games," which refer to any collaborative effort where players contribute resources or effort toward a common goal, and the resulting benefits are shared. Examples of such games include (a) partners co-owning a small business, (b) firms involved in a joint venture, (c) students collaborating on a group project, and (d) coworkers working together on a project team.

Consider a partnership game with two players. Let's imagine these players as partners in a small firm, with f_i representing the effort of partner i. Suppose the firm's accounting profit is stochastic and depends on the total level of effort provided by the partners. The aggregate profit function is given by:

$$\Pi(f_1, f_2) = (A + \theta)(f_1 + f_2) - \lambda(f_1 + f_2)^2, \qquad (3.2)$$

where θ follows a continuous normal distribution with mean $\bar{\theta}$ and variance σ_θ^2. If $\lambda < 0$ ($\lambda > 0$), the partners' efforts exhibit complementary (or substitutive) relationships. The partners split the profit in proportion to their individual effort levels, so each partner i receives a share of the benefit:

$$\pi_i = \Pi \times f_i/(f_1 + f_2) = (A + \theta)f_i - \lambda f_i(f_1 + f_2)$$

The cost of effort for partner i is $C_i(f_i) = cf_i^2$. Therefore, the net surplus for each partner is the difference between their share of the profit and their cost of effort:

$$u_i(\theta, f_i, f_1 + f_2) = (A + \omega\theta)f_i - \lambda f_i(f_1 + f_2) - cf_i^2$$

where $\omega \in \{-1,1\}$ is a dummy variable. The payoff function in (3.1) coincides with (3.4) when we let $a_i = f_i, \gamma = 1$, and $c = 1 - \lambda$. When $\omega = 1$, there is effort benefit uncertainty. However, when $\omega = -1$, there can be effort cost uncertainty and the cost of partner i is stochastic and given by $C_i(f_i) = \theta f_i + cf_i^2$ whereas the firm's profit is $\Pi = A(f_1 + f_2) - \lambda(f_1 + f_2)^2$.

In this games, public information refers to any knowledge that is commonly available to all partners and can influence their effort decisions, coordination, and overall outcomes. Some possible forms of public information include the following.

Information about market trends, consumer demand, or industry forecasts can help partners align their efforts with expected profitability. Moreover, if a partnership relies on technological advancements, publicly available updates on new tools, methods, or efficiency improvements can impact joint decision-making.

3.1.2. Public good contribution games

Consider that each individual or country $i \in N = \{1,2\}$ has an income $I_i > 0$ which can be allocated between private consumption, x_i, and contributions to a public good, g_i. The budget constraint for each player is given by

$$I_i = yx_i + g_i,$$

where $1/y$ represents the price of the public good in terms of the private good. Defining the total public good as. Let

$$G = \sum_{i \in N} g_i = \sum_{i \in N} (I_i - yx_i)$$

and the payoff of player i be given by

$$u_i(\theta, x_1, x_2) = (\Upsilon\theta + G)x_i = \big(\Upsilon\theta + (I_1 + I_2) - y(x_1 + x_2)\big)x_i.$$

Equation (3.1) becomes equivalent to (3.7) if we set $a_i = x_i$, -define $M = I_1 + I_2$, let $\omega = \Upsilon, \lambda = 1$, and choose $\gamma = y > 0$. The agent derives utility from the private good consumption (x_i) and total public good contribution of all agents (G). In this framework, an agent's utility derives from both private consumption (x_i) and the aggregate public good contributions (G). When private consumption is defined as $x_i = (I_i - g_i)/k$, the marginal benefit of agent i 's contribution to the public good is given by

$$\partial U_i / \partial g_i = x_i - (G + \Upsilon\theta)/k$$

is the marginal benefit of the public good contribution of agent i by (3.7). as specified in (3.7). Assume that θ is a random variable with distribution $\theta \sim \left(\bar{\theta}, \sigma_\theta^2\right)$ and $\Upsilon \in \{-1,1\}$. If $\Upsilon = -1$, the marginal benefit $\partial U_i / \partial g_i$ increases with θ, indicating uncertainty in the quality of the public good. Conversely, when $\Upsilon = 1$, the uncertainty lies in the aggregate income.

3.1.3. Common resource games

In many real-world scenarios, agents share a common-pool resource. For example, residents living near a lake or countries bordering a sea share access to a common stock of fish or natural gas. It is reasonable to assume that these agents lack complete information about the total resource stock, represented as $S+\theta$, where S is the observable stock and θ is a random variable (with mean $\bar{\theta}$ and variance σ_θ^2) capturing the unobserved portion. In numerous cases, agents obtain private signals s_i regarding θ. For example, fishermen use fish finders to locate fish aggregations, and oil companies search for oil reserves. An agent's utility is derived from both the resource he extracts (r_i) and the remaining resource in the environment, $(S+\theta-\sum_{i\in N} r_i)$ with $N=\{1,2\}$. Alternatively, one could assume a known stock size but uncertain, linear extraction costs given by $C_i(r_i)=\theta r_i$. Let agent i 's payoff be

$$u_i(\theta,r_1,r_2)=\left(S+\Upsilon\theta-\sum_{i\in N} r_i\right)r_i.$$

Equation (3.1) becomes equivalent to equation (3.9) when we set $a_i=r_i, A=S,\ \omega=\Upsilon, \gamma=\lambda=1$. When $\Upsilon=1$ the uncertainty pertains to the total stock size, whereas $\Upsilon=-1$ indicates uncertainty in the resource extraction costs.

3.1.4. Gas emission games

Greenhouse gas emissions provide benefits by serving as inputs in both production and consumption. However, emissions released by an agent (whether a firm or a country) also incur environmental damages. Each agent $i\in N=\{1,2\}$ can reduce its emissions below its baseline level $\bar{E}_i$ by implementing an abatement level $t_i>0$. Consequently, the actual emission level is given by $e_i=\bar{E}_i-t_i$. Define the total abatement as

$$T=\sum_{i\in N} t_i=\sum_{i\in N} (\bar{E}_i-e_i).$$

An agent derives utility from both the aggregate abatement (T) and its own emission level e_i. Its payoff is expressed as

$$U_i(\theta, e_1, e_2) = (\Upsilon\theta + T)e_i = \left(\Upsilon\theta + \sum_{i\in N} \bar{E}_i - \sum_{i\in N} e_i\right)e_i$$

Since $e_i = \bar{E}_i - t_i$, the marginal benefit of emission abatement is

$$\partial U_i/\partial t_i = e_i - (T + \omega\theta)$$

Equation (3.1) aligns with (3.11) when we set $a_i = e_i, M = \sum_{i\in N} \bar{E}_i, \omega = \Upsilon$, and $\gamma = \lambda = 1$. Let $\Upsilon \in \{-1,1\}$ and θ be a random variable. When $\Upsilon = -1, \partial U_i/\partial t_i$ is increasing in θ, and therefore, there can be total gas emission-abatement benefit uncertainty. Alternatively, $C_i(e_i) = \theta e_i$ can be agent i 's environmental tax cost, and there is emission cost uncertainty. When $\Upsilon = 1$, there is total gas emission abatement cost uncertainty, aggregate baseline emission inventory uncertainty, or emission benefit uncertainty.

3.1.5. Implications of our findings in these models

As we show above, the strategies of players are strategic substitutes or complements in partnership games, and they are perfect substitutes in public good, common resource, and gas emission (PCG) games because $\gamma = \lambda = 1$. The measure of welfare is the sum of expected payoff of agents. Using our results from Propositions 1-6, we obtain a corollary.

Corollary 1. *Consider the partnership game. i) Each partner has an incentive to acquire private information about the benefits and costs associated with their effort levels. If the partners' skills are substitutes, one partner's information acquisition negatively impacts the other. However, if their skills are complementary, the opposite occurs, and information*

acquisition benefits both partners.
ii) Expected welfare increases when agents receive a more precise public signal.
iii) Sharing private information about effort or cost levels is a dominant strategy for each partner if their skills are complementary. However, if their skills are substitutable, withholding information is optimal.
iv) When partners share their private information, expected welfare increases if their skills are complementary or only weakly substitutable. However, if their skills are close substitutes, expected welfare decreases after information exchange.

We now examine the implications of our earlier findings in the remaining games where agents' strategies are perfectly substitutable.

Corollary 2. ***i)*** *Each agent has an incentive to acquire relevant private information. However, one agent's information acquisition negatively affects the other.*
ii) *Expected welfare improves when agents receive a more precise public signal.*
iii) *Withholding private information is optimal for each agent. Furthermore, if agents share their private information, expected welfare declines after the exchange.*

3.2. Conclusion

This section of our book has demonstrated how various two-agent models fit within the framework of aggregative games, where each agent's payoff depends on both their individual strategy and the aggregate strategy of all players. By expressing the general payoff function in a unified form, we have shown that diverse economic interactions-ranging from Cournot and Bertrand competition to partnership, public good contribution, common resource, and gas emission games-can be analyzed using a consistent approach.

Our findings highlight the crucial role of information acquisition and sharing in these settings. In partnership games, the substitutability or complementarity of agents' efforts determines whether acquiring private information benefits or harms the other partner. In contrast, in public good, common resource, and gas emission (PCG) games-where agents' strategies are perfect substitutes-information acquisition always imposes a negative externality on others. This distinction has significant implications for strategic behavior and welfare outcomes.

A key insight from our analysis is that the precision of public information universally improves expected welfare across all models. However, the incentives for private information sharing differ: while partners with complementary skills benefit from information exchange, those with substitutable skills prefer to withhold their private knowledge. Similarly, in PCG games, agents strategically avoid sharing private information, as doing so leads to lower welfare by intensifying strategic competition.

Overall, our results emphasize the importance of information structure in aggregative games, shaping both individual strategies and collective welfare. The findings have broad applicability to economic and policy contexts, particularly in settings where strategic interactions involve competition over shared resources, cooperative production, or environmental externalities.

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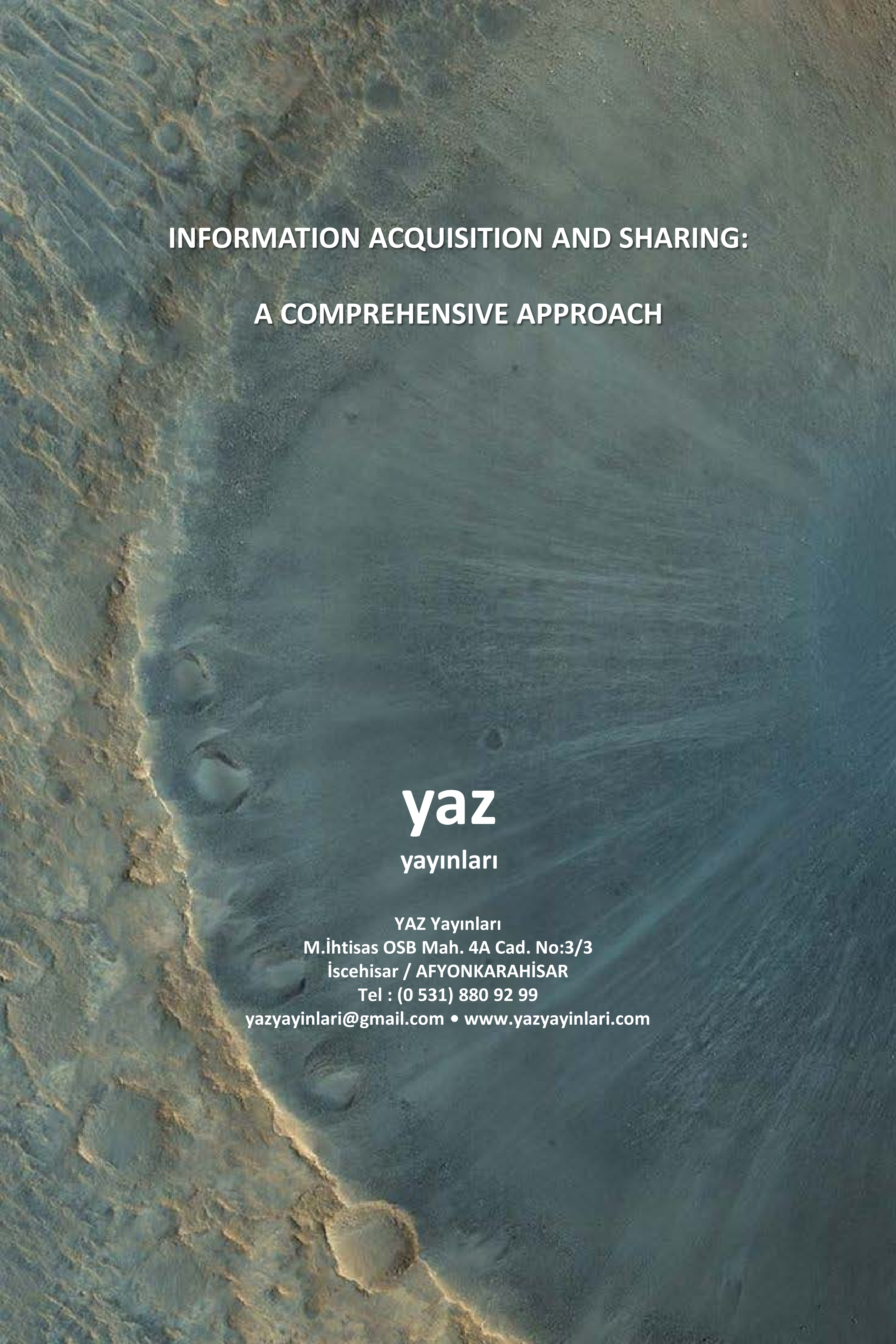

INFORMATION ACQUISITION AND SHARING:

A COMPREHENSIVE APPROACH

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