

RF Interferometer with Baseline Geometry and Frequency Scaling

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A Spreadsheet-Driven Method for Visualizing DOA Accuracy and Baseline Behavior in Three- and Four-Channel RF Interferometers

Introduction

Modern RF interferometer arrays trace their analytical foundation to **NRL Report 8005 (September 1976)** authored by **Robert L. Goodwin**. His work established the geometry and processing basis for four-element, three-integer-set interferometers and phase-based DF systems. Goodwin quantified channel-to-channel phase-tracking errors, demonstrated their impact on direction-finding accuracy, and documented the complete processing path from RF input through ambiguity resolution and angle computation.

This foundation enables a transparent, spreadsheet-based method for analyzing DOA behavior, baseline geometry, and unwrapped-phase functions without requiring specialized software. The approach follows Goodwin's structure: baseline spacing defines the phase-slope region, channel differences define the mid-phase geometry, and the unwrapped-phase function determines DF accuracy across the field of view.

1. Hybrid Mid-Phase Geometry (Channel 2 Reference)

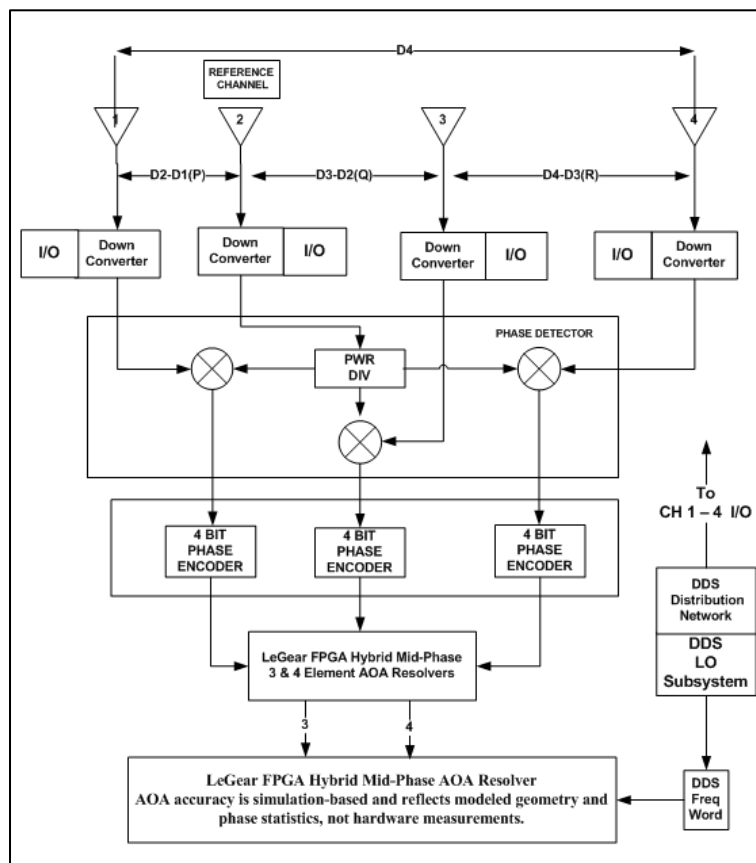


Figure 1: Hybrid Mid-Phase Geometry (Channel 2 Reference)

Figure 1 illustrates the hybrid mid-phase geometry used in a four-channel interferometer with Channel 2 as the reference. Baseline spacings $D2-D1(P)$, $D3-D2(Q)$, and $D4-D3(R)$ define the phase-slope behavior across the field of view. Each antenna feeds a down-converter and I/O module, followed by a phase detector and a 4-bit phase-encoder chain. The encoded phase values enter the **LeGear FPGA Hybrid Mid-Phase 3- and 4-Element AOA Resolver**, which performs ambiguity resolution and angle computation.

AOA accuracy shown in this document is **simulation-based** and reflects modeled geometry and phase statistics rather than hardware measurements. The spreadsheet-driven method allows rapid exploration of baseline configurations, wavelength scaling, and mid-phase behavior using only analytical expressions and controlled phase-error models.

2. Down Converter Functional Ingredients

Each channel's down-converter provides the conditioned IF drive required for coherent phase detection. The chain begins with a passive limiter that protects the RF front end from high-power signals. A BITE coupler follows, enabling injection of calibration tones and diagnostic signals. A switched filter bank selects the appropriate RF sub-band and rejects out-of-band interference. The RF mixer translates the selected band to the IF region using a coherent LO source. An IF bandpass filter removes mixer products and establishes the IF bandwidth and group-delay characteristics. A hard-limited IF amplifier provides constant-envelope drive to the phase detector, stabilizing MOD performance. The final IF output is delivered to the phase-detector power divider and mixers. The down-converter also includes an I/O control interface that manages calibration constants, DDS frequency words, and channel-specific control settings.

3. Phase Detector Functional Ingredients

The phase-detector block receives the hard-limited IF signals from each channel and produces the differential phase measurements required by the mid-phase resolver. A power divider supplies equal-amplitude reference IF drive to the three mixers. Each mixer compares the reference channel against one of the remaining channels, generating the P, Q, and R phase-difference signals. The mixers operate in a saturated mode to ensure stable zero-crossing behavior and minimize amplitude-induced phase errors. The resulting differential IF outputs are conditioned to remove residual amplitude variation and are then applied to the 4-bit phase encoders. The phase-detector block establishes the fundamental phase-difference measurements that drive the hybrid mid-phase AOA resolver.

4. Bit Phase Encoder Functional Ingredients

The 4-bit phase encoders convert the saturated differential IF signals from the phase detector into small digital words representing instantaneous phase differences. Each encoder operates on a constant-envelope input to ensure stable zero-crossing detection and minimize amplitude-related phase errors. The encoder quantizes the mid-phase region into sixteen discrete levels, producing a compact digital representation suitable for high-speed FPGA processing. The encoded P, Q, and R phase words preserve the essential phase relationships between channels and form the primary digital inputs to the hybrid mid-phase resolver. The encoders establish the low-latency digital interface between the analog phase-detector outputs and the FPGA-based AOA computation engine.

5. Hybrid Mid-Phase Resolver Functional Ingredients

The hybrid mid-phase resolver receives the 4-bit P, Q, and R phase words and applies the combined three-element and four-element mid-phase equations to produce continuous angle-of-arrival estimates. The resolver fuses the weighted differential phase increments derived from the D2–D1, D3–D2, and D4–D3 baselines, using fixed-point arithmetic to suppress slope discontinuities and reduce mismatch sensitivity. The FPGA implements the hybrid algorithm with low-latency logic paths, ensuring stable operation across the full field of regard. The resolver outputs continuous 8-bit mid-phase values for both the three-element and four-element geometries, providing the CPU with ambiguity-aware AOA estimates and phase-derived signal metrics. This block forms the core computational engine of the interferometer, converting quantized phase inputs into coherent direction-finding information.

6. CPU Digital Processing Functional Ingredients

The CPU digital-processing block receives the continuous mid-phase outputs from the hybrid resolver and converts them into final angle-of-arrival estimates and signal-parameter products. It applies calibration constants, baseline geometry, and channel-specific corrections to generate stable AOA values across the full field of regard. The CPU also processes fine-frequency IFM data, pulse descriptors, and other signal-parameter metrics supplied by the reference channel. Additional logic performs ambiguity resolution, smoothing, and statistical validation of the resolver outputs. The CPU integrates all digital products into coherent PDWs and system-level outputs suitable for downstream analysis or reporting. This block provides the interferometer's final computational stage, transforming resolver data into complete, actionable signal information.

7. Antenna Baseline Spacing Calculator

Figure 2 shows the D2 reference antenna and the corresponding spacings D2–D1, D3–D2, and D4–D3 used in the worksheet. This diagram establishes the physical baseline layout associated with the P, Q, R, and S phase-difference columns. These geometric definitions drive the unwrapped-phase calculations and define the effective baseline relationships used throughout the system.

The Hybrid Mid-Phase geometry reduces wrap density at higher frequencies and preserves clear baseline separation across the operating band. This compact, electrically centered structure underpins all visualizations and DF-accuracy curves generated in the spreadsheet.

λ (Inches)	Design Anchor	D1(P)	D2(Q)	D3(R)	D4(S)
0.656	18 GHz	4.000	6.000	11.000	21.000
Freq GHz	Scale Factor	P ($\lambda/2$)	Q ($\lambda/2$)	R ($\lambda/2$)	S ($\lambda/2$)
18.000	4.300	2.000	3.000	5.500	10.500
	Phase Scaling $\lambda/2$	19.156	28.734	52.679	100.569
	Before Scaling	4.455	6.682	12.251	23.388
	Array Scale Factor	D2–D1	D3–D2	D4–D3	D4
	Inches	2.227	5.569	11.137	23.388

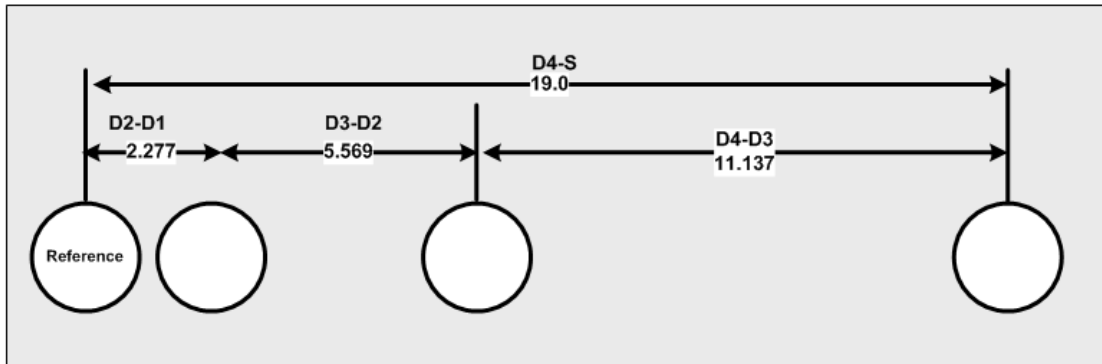


Figure 2

The antenna baseline spacing calculator expresses the Hybrid Mid-Phase geometry numerically. The four-element DF array is represented as a linear sequence of antenna positions referenced to a fixed origin. Figure 2 shows the antenna spacing, with D2–D1, D3–D2, and D4–D3 defining the incremental baseline distances, and D4 representing the total physical distance from the reference element. The physical dimensions of the antennas determine the minimum installed spacing between antennas 1 and 2, establishing the smallest allowable baseline and setting the geometric anchor for the remaining baseline layout.

The spreadsheet scaling function computes the additional array spacings by applying the user-selected scale factor to the half-wavelength phase distances at the anchor frequency. This ensures that each incremental baseline maintains the required Hybrid Mid-Phase relationships as frequency changes. Because the scaling is tied directly to wavelength, the resulting baseline geometry preserves phase-slope conditioning, maintains stable hybrid-phase formation, and guarantees that the resolver operates on properly proportioned baselines across the entire operating band.

8. Unwrapped Phase Behavior

The unwrapped interferometer phase underpins all DF accuracy expressions. Figure 2, titled Unwrapped Phase for P, Q, R, and S, shows the unwrapped phase $\varphi = 2 \cdot \pi \cdot D \cdot \sin(\text{AoA}) / \lambda$ for the four effective baselines after normalization and scaling. The hybrid mid-phase baselines P, Q, and R exhibit progressively increasing slopes as the effective aperture grows, while the absolute outer baseline S produces the largest phase excursion across the field of view. These geometric phase relationships define each baseline's sensitivity and wrap behavior and form the basis for subsequent DF-accuracy calculations.

5. Local Phase Slope

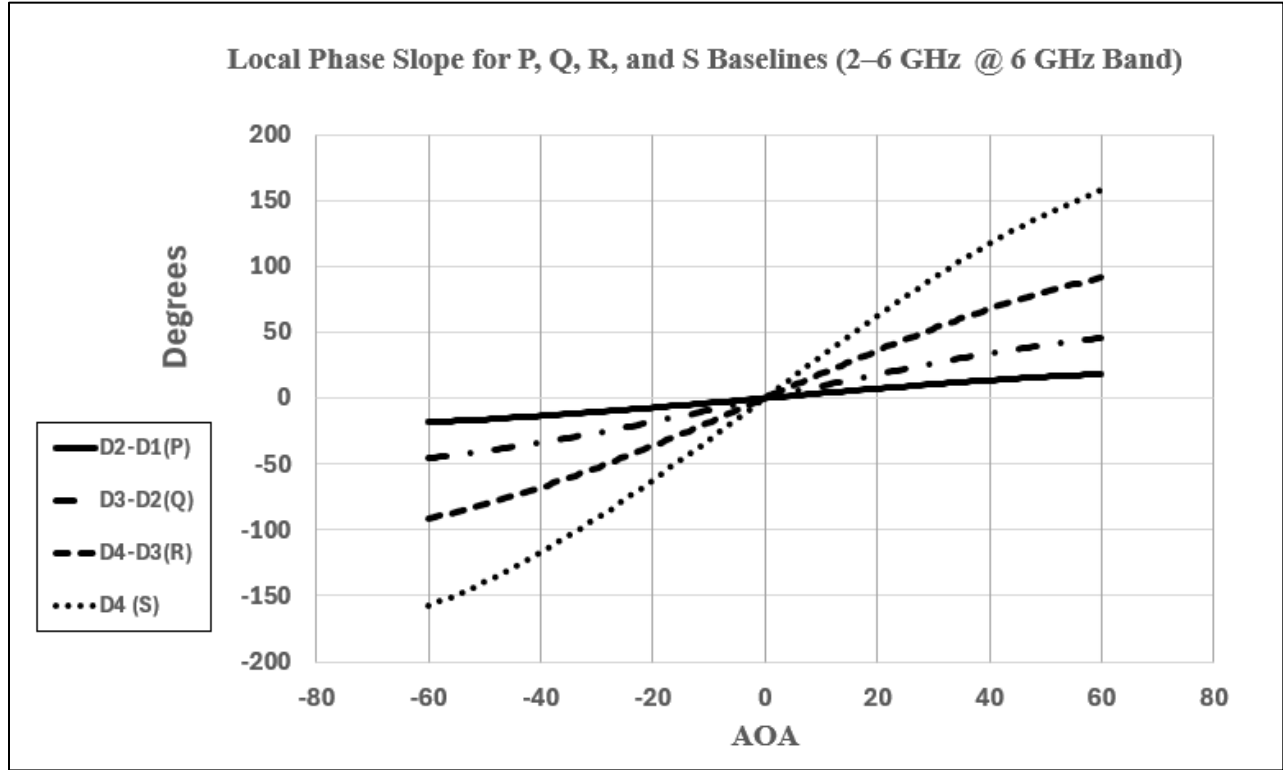


Figure 3. Local Phase Slope Magnitude for P, Q, R, and S

For illustration, the worksheet uses **6 GHz** for the AOA phase-slope plots, matching the comparison frequencies used throughout the resolver analysis. MOD behavior is not shown in the main body; a representative MOD plot demonstrating wrap spacing and baseline-dependent slope is provided in Appendix A.

At these frequencies, the physical antenna spacings correspond to a moderate number of wavelengths, and the hybrid mid-phase baselines P, Q, and R remain electrically short enough to avoid wrap compression. The outer baseline S, although larger, still produces phase slopes that Excel renders cleanly without distortion. As a result, the wrapped-phase behavior exhibits minimal crowding, maintains clear separation across the field of view, and provides an unambiguous depiction of baseline geometry. This band is therefore ideal for demonstrating wrap density, baseline separability, and the geometric relationships that govern resolver performance.

An AOA comparison at **6 GHz** and **18 GHz** is included to show resolver behavior when all embedded baselines remain fully stable. At these frequencies, the normalized phase-wrap behavior for the P, Q, R, and S baselines is well conditioned, and the hybrid mid-phase combinations do not degrade. Consequently, the four-element, three-element, and two-element resolver outputs track closely across the angle range with minimal separation between configurations. This comparison establishes the baseline reference case for resolver performance under ideal conditions and illustrates the angle behavior prior to the onset of frequency-dependent degradation.

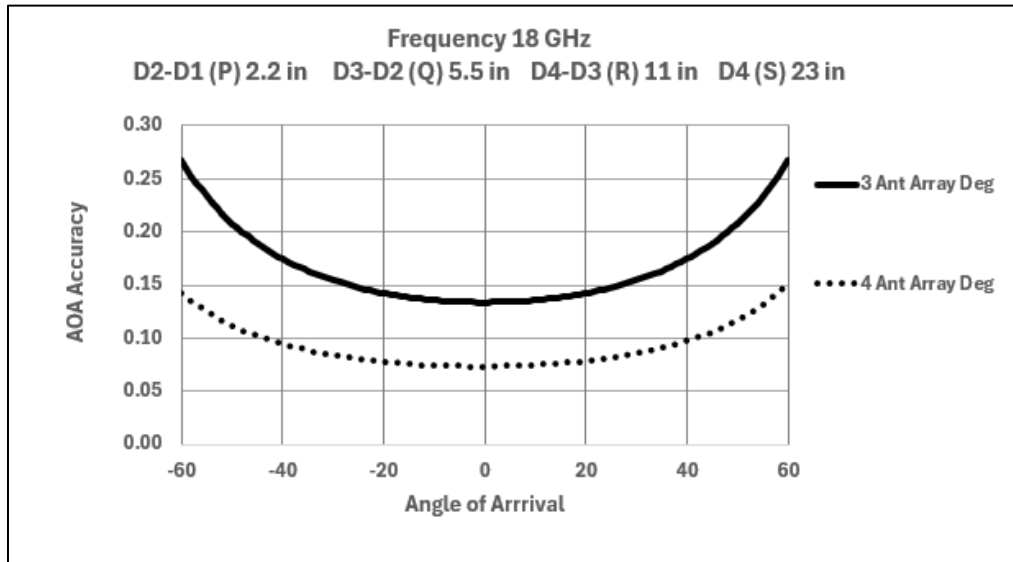
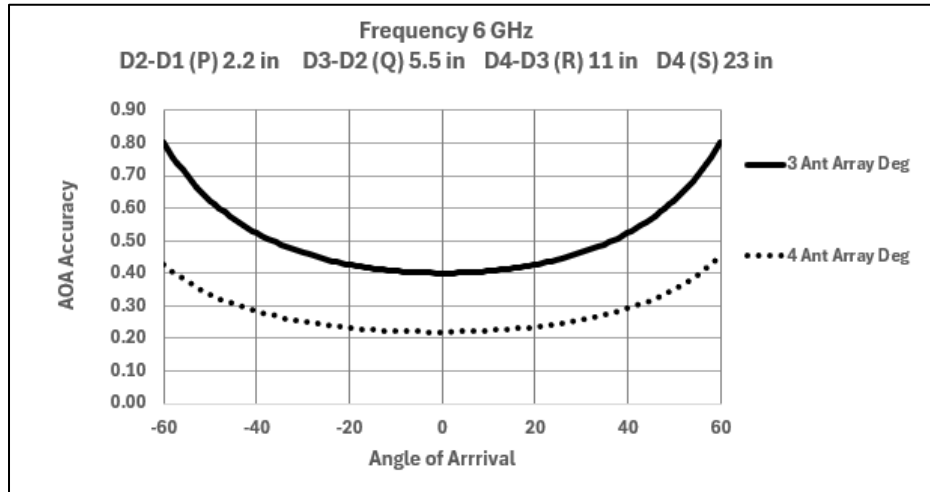


Figure 4 AOA Comparison at 6 GHz for 3 and 2-Element Resolver Configurations

A second comparison plot shows array-variance behavior versus angle of arrival at 18 GHz, where all baselines remain fully conditioned across the angle range and support continuous four-element resolver operation. At 18 GHz, the increased phase-slope variance reduces baseline conditioning and forces resolver transitions. These results demonstrate the frequency-dependent baseline behavior that drives the resolver-selection sequence described in the Summary Excel section.

This figure links the numerical hybrid-resolver analysis to the antenna array's physical baseline behavior. The combined spreadsheet outputs and performance curves show how detector behavior, hybrid mid-phase formation, composite-frequency reference scaling, and resolver selection operate together across the band to maintain stable angle reconstruction.

6. AOA Resolvers Monte Carlo Simulation

Section 6, AOA Resolvers Monte-Carlo Simulation, demonstrates the array's tolerance to random channel-to-channel phase mistracking. The Hybrid Mid-Phase AOA process remains stable under statistically applied phase error, and the resolver does not collapse or transition unexpectedly. Only the instantaneous accuracy of each Monte Carlo trial varies; the underlying resolver behavior and array geometry remain unchanged.

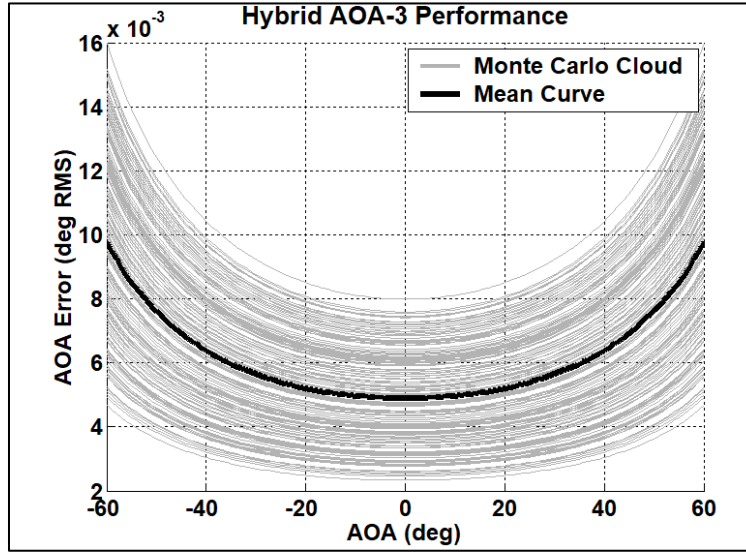


Figure 5 Monte-Carlo Hybrid Accuracy for 3 Element Resolver

The 3-element hybrid resolver shows a well-behaved Monte-Carlo cloud, with the mean curve remaining stable across the angle range. Random phase perturbations introduce small trial-to-trial variations, but the resolver maintains consistent angle reconstruction and does not exhibit slope discontinuities or baseline-dependent instability.

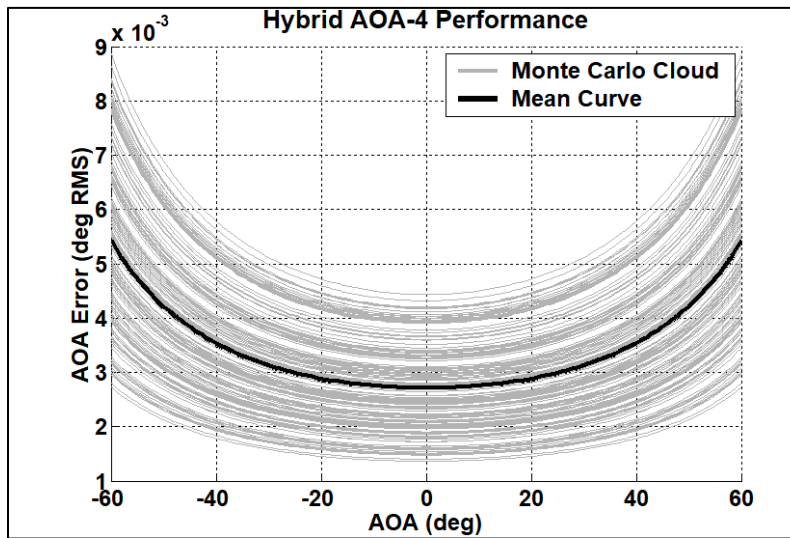


Figure 6 Monte-Carlo Hybrid Accuracy for 4 Element Resolver

The 4-element hybrid resolver exhibits even lower RMS error across the field of view. The Monte-Carlo cloud remains tightly grouped, and the mean curve shows minimal deviation. The additional baseline improves statistical conditioning and reduces sensitivity to channel-to-channel phase mismatch, demonstrating the inherent robustness of the 4-element configuration.

7. Summary Excel

Across the operating band, the phase detectors follow the wrapped-phase behavior defined by the hybrid mid-phase baselines. Increased wrap density at higher frequency affects only the visualization; it does not enter the resolver calculations. Frequency resolution is formed from two sources: the DLVA-derived IF characteristics and the instantaneous frequency estimate from the IFM. This estimate is combined with the DDS tuning word to form the composite frequency reference used by the LeGear FPGA. The composite reference is applied solely for baseline scaling and ADC-range protection and does not participate in the resolver math.

As frequency increases, the phase slope grows, causing the hybrid mid-phase value inside the FPGA to increase while the ADC input range remains fixed. The FPGA applies a frequency-based scaling factor derived from the composite reference to maintain the mid-phase output within the ADC's dynamic range. The three hybrid paths feed the LeGear resolver function, producing linearized phase values that are digitized and combined after ADC conversion to form the final angle.

This processing ladder—phase-detector behavior, hybrid-baseline formation, three resolver paths, composite-frequency scaling, ADC conversion, and angle reconstruction—maintains stable operation across the band. The Monte-Carlo accuracy curves quantify angle-error behavior for the four-element, three-element, and two-element resolvers and provide the statistical basis for resolver selection.

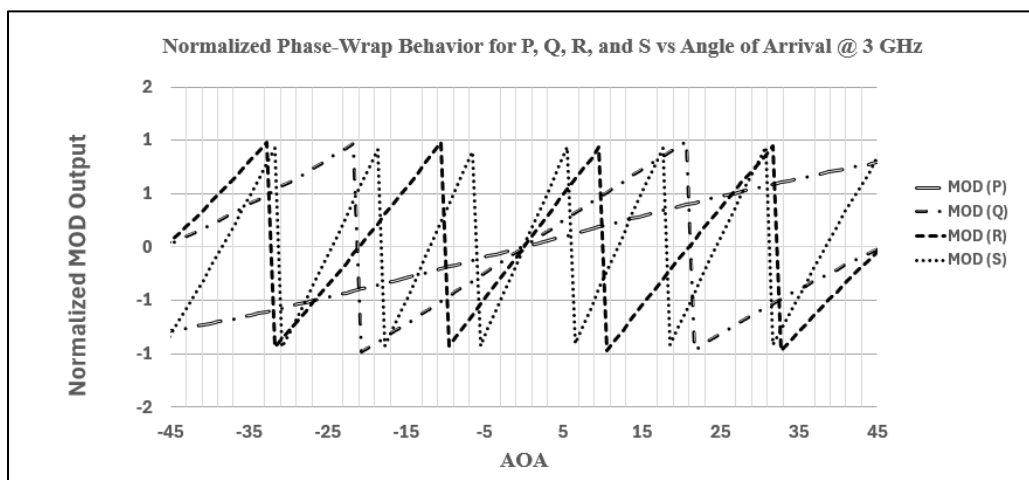
As frequency increases and the longest baselines exhibit higher phase-slope variance, the three-element resolver maintains lower error. At still higher frequencies, when only two embedded baseline combinations remain reliable, the software transitions to the two-element resolver.

The Hybrid LeGear FPGA AOA resolver enables a self-referencing wideband interferometer architecture that requires no chamber measurements or calibration tables. Hybrid AOA values are determined directly from array geometry, and the operational Hybrid AOA values inherently contain the channel-to-channel RMS mistrack scaled by the DDS tuning word.

Because the Hybrid resolver collapses all baseline errors into a single observable angle bias, the difference between the initial Hybrid AOA and the operational Hybrid AOA becomes the real-time correction factor. The system is therefore self-correcting: it compares its instantaneous state to the geometric reference and applies the resulting bias directly. This eliminates traditional calibration, provides a technical solution for YIG-filter RF preselectors, and removes the need for frequency-by-frequency mapping.

Appendix A — Illustrative MOD Response (FYI Only)

Figure A-1 shows a normalized MOD response for baselines P, Q, R, and S over a $\pm 45^\circ$ angle-of-arrival window at a mid-band frequency. The plot illustrates how baseline length governs phase-wrap spacing and slope: shorter baselines produce slower MOD variation with wider wrap intervals, while longer baselines generate steeper slopes and more frequent wraps. This MOD response is included solely to demonstrate the underlying phase-detector geometry that motivates the multi-baseline architecture.



The interferometer processing algorithm is built on the Hybrid Mid-Phase resolver architecture implemented in the LeGear FPGA. The FPGA accepts the hybrid mid-phase detector outputs, applies frequency-dependent scaling derived from the composite frequency reference, linearizes the hybrid baseline combinations, and computes the final angle of arrival through three resolver paths. The resolver operates on the hybrid baselines P, Q, and R and selects the appropriate two-, three-, or four-element combination based on baseline stability and phase-slope variance.

This processing sequence is identical for both the generic interferometer front end and a YIG-based front end; only the RF-preselector behavior differs. The complete operational ladder is described in the Hybrid Mid-Phase Processing Ladder section of the main body.

The FPGA's hybrid-baseline normalization step removes all front-end-induced variances prior to resolver selection. These variances may arise from filter-bank dispersion, channel mismatch, or tuning-state offsets in YIG-based preselectors. The normalization process is generic and applies uniformly to any RF-preselector technology, ensuring that the hybrid baselines P, Q, and R enter the resolver in a linearized and frequency-scaled form independent of front-end implementation.