The Circle

Equation of a Circle

with centre (0,0) $x^2+y^2=R^2$

with centre (h,k) $(x-h)^2+(y-k)^2=R^2$

General Equation

$x^2+y^2+2gx+2fy+c=0$

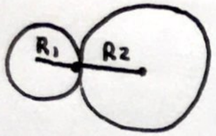
Centre = $(-g,-f)$ Radius = $\sqrt{g^2+f^2-c}$

points inside, outside or on a circle

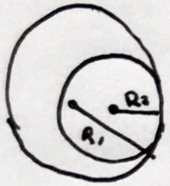
Slot point into equation.

- If answer is less than R^2 inside
- If answer is more than R^2 outside
- If answere is equal to R^2 on

Touching Circles



R_1+R_2 = Distance between centres



R_1-R_2 = Distance between centres

Common Question

Find the equation of the tangents

Circles touching X or Y axis

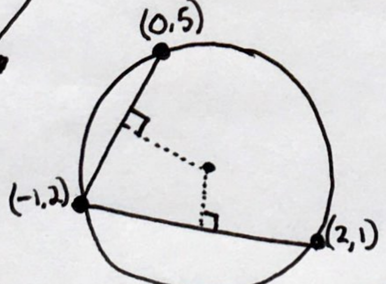
Let $x=0$ and solve

OR

Let $y=0$ and solve

Common Question

* Find the equation of a circle when given three points on the Circle



Step 1: Make two cords and find their equations

Step 2: Now find the equations of the lines that are the perpendicular bisectors of these cords.

Step 3: Now ~~go~~ use sim. equations with these to find where they cross each other. This is the centre of the circle

May also be given:

* Equation of tangent to circle, its point of contact and one other point

OR

* two points and the equation of a line which the centre of the circle lies on.

Financial Maths

$$F=P(1+i)^t$$

Mortgages

Formula in Log book

$$A=P\frac{i(1+i)^t}{(1+i)^t-1}$$

we use this to find monthly repayment on a mortgage

A: Montly repayment
P: Mortgage amount
i: interest
t: time [usually in months]

yearly to montly

eg 12% AER to montly

$$112=100(1+i)^{12}$$

work this out to find i

Contineous Investments

eg €200 p.m. @ 0.0049%
for 2 years [start of month]

$$F=200(1+0.0049)^{24}+200(1+0.0049)^{23}\ldots\ldots 200(1+0.0049)^1$$

$$F=200(1+0.0049)^1+200(1+0.0049)^2\ldots\ldots 200(1+0.0049)^{24}$$

$$F=200\left[\frac{(1.0049)(1-1.0049^{24})}{1-1.0049}\right] \qquad \boxed{S=\frac{a(1-R^n)}{1-R}}$$

* Use Calculator to solve
* If payment is at end of month, series will go from $(1.0049)^0$ to $(1.0049)^{23}$

Pensions

If I wat the €20,000 starting as soon as I invest the mony

eg. I want €20,000 per year for 20 years on retirement @ 4% interest P.A. How much do I need

Now	year 1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19

Now = €20,000

year 1 = $\frac{20000}{(1+i)^1}$

year 2 = $\frac{20000}{(1+i)^2}$

⋮

year 19 = $\frac{20000}{(1+i)^{19}}$

$$\therefore F=\frac{20000}{(1+i)^0}+\frac{20000}{(1+i)^1}\ldots\ldots\frac{20000}{(1+i)^{19}}$$

$$S=\frac{a(1-R^n)}{1-R}$$

Ammortisation

How much must you save a month for 5 years @ 0.005% p.m. (60 months) to get €100,000.

$$€100,000=P(1+0.005)^{60}\ldots\ldots P(1+0.005)^1$$

$$€100,000=P[(1.005)^{60}\ldots\ldots(1.005)^1]$$

$$€100,000=P[(1.005)^1\ldots\ldots(1.005)^{60}]$$

Geometric Series

$$€100,000=P\left[\frac{(1.005)^1(1-1.005^{60})}{1-1.005}\right]$$

Use Calculator and algebra to solve for P

How long would it take €5000 to increase to €6000 @ 2% p.A.

$$A=P(1+i)^t \Rightarrow €6000=€5000(1+0.02)^t$$

$$\frac{6000}{5000}=1.02^t$$

$$1.2=1.02^t$$

$$\ln 1.2=\ln(1.02)^t$$

$$\ln 1.2=(t)(\ln)(1.02)$$

$$\frac{\ln 1.2}{\ln 1.09}=t \qquad t=9.21\text{ years}$$

Series - Sequence - Patterns

Arithmetic

1st difference

eg 4, 7, 10, 13

Sequence $T_n = a + (n-1)d$

a = 1st term

d = difference

Series $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

a = 1st term

d = difference

n = number of terms in series

Quadratic

Second difference is the same

$T_n = an^2 + bn + c$

a = half the value of second difference

Sigma notation $\sum$

an effective way of representing a series

eg Use sigma to show 2+6+10+14.... for 45 terms.

$T_n = a+(n-1)D$

$T_n = 2+(n-1)D$

$T_n = 4n-2$ $\Rightarrow \sum_{R=1}^{R=45} (4R-2)$

Exponential Sequence

eg 2^n

Geometric Sequence

eg 2 6 18 54

$2 \underbrace{\quad}_{\times 3} 6 \underbrace{\quad}_{\times 3} 18 \underbrace{\quad}_{\times 3} 54$

$T_n = aR^{n-1}$

a is 1st term

R is common ratio ie $\frac{T_2}{T_1}$

Geometic Series

$S_n = \frac{a(1-R)^n}{1-R}$

a = 1st term

R = common ratio

Sum to Infinity
(Geometric Series)

eg Find the sum to infinity of 16+12+9+ ......

a = 16

$R = \frac{12}{16} = \frac{3}{4}$ $\frac{T_1}{T_2}$

$\lim\limits_{n \to \infty} S_n = \frac{a}{1-R} = \lim\limits_{n \to \infty} S_n \frac{16}{1-\frac{3}{4}}$

$= 64$

Linear - 1st difference is the same

Quodratic - 2nd difference is the same

Cubic - 3rd difference is the same

Exponential - Power change

Geometric - Common ratio

Recurring Decimals

eg Write $0.\dot{2}\dot{3}$ as a fraction

$0.\dot{2}\dot{3} = 0.23 + 0.0023 + 0.000023$

$= \frac{23}{100} + \frac{23}{10000} + \frac{23}{1000000} + ...$

$\lim\limits_{n \to \infty} S_n = \frac{a}{1-R} = \frac{\frac{23}{100}}{1 - \frac{1}{100}} = \frac{23}{99}$

[R = common ratio $= \frac{1}{100}$
a = 1st term $= \frac{23}{100}$]

Trigonometry 1

Trigonometric Ratios

$\cos = \frac{A}{H}$ $S \frac{O}{H}$ $\tan = \frac{O}{A}$

Pythagorus' theorem

$H^2 = A^2 + O^2$

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B}$ ← opposite side ← angle

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle

$\frac{1}{2} ab \sin C$

General Solutions:

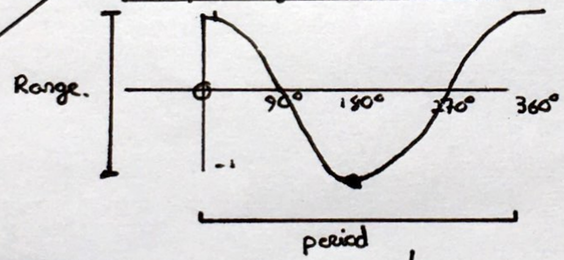
Solve for two answers in CAST and then add $n360°$ or $2n\pi$ to each answer.

All solutions: After finding the general solution we slot in 1,2,3 .... in for n.

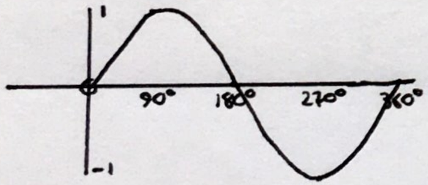
Radians: 2π Radians $= 360°$ Arc lenght $= R\emptyset$

π Radians $= 180°$ Area of sector $= \frac{1}{2}R^2\emptyset$

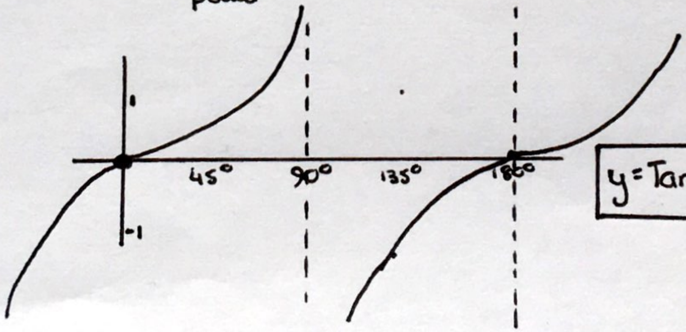
Graphs of Sin, Cos, Tan



$y = \cos X$



$y = \sin X$

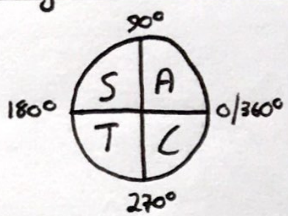


$y = \tan X$

* If $\sin X$ has a period of 360° then $\sin 2x$ has a period of 180°

* $2\sin X$ has a range of 2
$3\sin X$ has a range of 3 etc

Trigonometric Functions



eg1 Find in Surd form $\sin 120°$

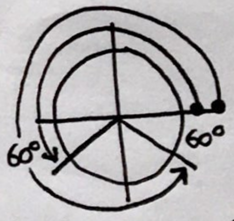
1. Get reference angle 60°
2. Check $\sin 60°$ in Log book $[\frac{\sqrt{3}}{2}]$
3. Check in Sin + or - in this quadrant

* It is + in this Quadrant

$\therefore$ answer is $\sin 120° = \frac{\sqrt{3}}{2}$

eg2 If $\sin X = \frac{-\sqrt{3}}{2}$ find X in degrees

1. $\sin X = -\frac{\sqrt{3}}{2}$
2. $\frac{\sqrt{3}}{2} = 60°$ [from Logbook]
3. Sin is negative in 3rd and 4th Quadrant.
4. Sketch



$X = 240°$ or $X = 300°$

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$\left[\begin{array}{l} R = \text{common ratio} = \frac{1}{100} \\ a = 1^{st} \text{ term} = \frac{23}{100} \end{array}\right]$

$\lim S_n = \frac{a}{1-R} = \frac{\frac{23}{100}}{} = \frac{23}{99}$

Complex Numbers

$i^2 = -1$

$\sqrt{-16} = 4i$

Conjugate $[\bar{z}] \Rightarrow 3+2i$ becomes $3-2i$

Modulas $|z|$ $\sqrt{a^2+b^2}$
Tells use the distance to (0,0)

Adding: $(3+4i)-(5+2i) = -2+2i$

Multiplying: $(3+4i)(5+2i)$

Dividing: Multiply top and bottom by conjugate of bottom

$$\frac{3+4i}{5+2i} \Rightarrow \frac{(3+4i)(5-2i)}{(5+2i)(5+2i)}$$

Remember $i^2 = -1$

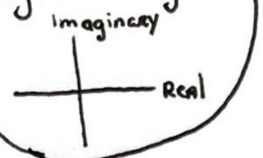
Equality

$2x + 3yi = 10 + 9i$
R I R I

Ⓡ
$2x = 10$
$x = 5$

Ⓘ
$3yi = 9i$
$3y = 9$
$y = 3$

Argand Diagram



high powers of i

$i^{4n} = 1$
$i^{4n+1} = i$
$i^{4n+2} = -1$
$i^{4n+3} = -1$

$Z \times i$: Rotates 90°
$Z \times i^2$: Rotates 180°
$Z \times i^3$: Rotates 270°
$Z \times i^4$: Rotates 360°
$Z \times (-i)$: Rotates -90°

anti Clockwise

Transformations

- Multiplying by a Real number extends the modulus

eg

- adding Complex numbers creates a translation
- Multiplying by Complex number
  1) multiplying by i Rotates anti clockwise.
  2) multiply $(2+i)$ by $(3+i)$ will first rotate $2+i$ by 90° and then stretch by 3

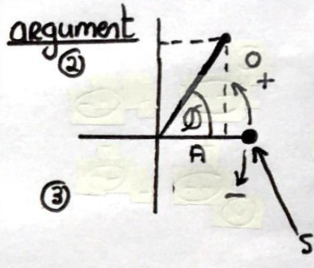
* if Z is a Root of an equation the so to is $\bar{Z}$

polar form

$R(\cos\phi + i\sin\phi)$

* $\phi = \tan^{-1}\frac{O}{A}$ $R = \sqrt{a^2+b^2}$

argument



product in polar form

If $Z_1 = R_1(\cos\phi_1 + i\sin\phi_1)$
and $Z_2 = R_2(\cos\phi_2 + i\sin\phi_2)$
then $Z_1 \cdot Z_2 = R_1 \cdot R_2[\cos(\phi_1+\phi_2) + i\sin($ [illegible] $\phi_1 +$ [illegible] $\phi)]$
and $\frac{Z_1}{Z_2} = \frac{R_1}{R_2}[\cos(\phi_1-\phi_2) + i\sin(\phi_1-\phi_2)]$

also if $Z = R(\cos\phi + i\sin\phi)$
then $\frac{1}{z} = \frac{1}{R}[\cos(-\phi) + i\sin(-\phi)]$

Know

If $Z^n = a+bi$
then $Z = (a+bi)^{\frac{1}{n}}$

eg $Z^3 = (1+i)$
$Z = (1+i)^{\frac{1}{3}}$

De Moivre's theorem

$(\cos\phi + i\sin\phi)^n = \cos n\phi + i\sin\phi$

$\therefore$ if $Z = R(\cos\phi + i\sin\phi)$
$Z^n = [R(\cos_n\phi + i\sin_n\phi]^n$
$Z^n = R^n(\cos_n\phi + i\sin_n\phi)$

know how to prove De Moivre's theorem with Induction

General Polar form

If $Z = a+bi$
the $Z = R[\cos(\phi+2n\pi)$
$+ i\sin(\phi+2n\pi)$

Formula in Logbook

Distance $= \sqrt{(x_2-x_1)^2+(y^2-y_1)^2}$

Midpoint $= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

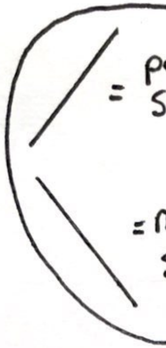
Slope $= \frac{y_2-y_1}{x_2-x_1}$ or $\frac{Rise}{Run}$

Equation $= y-y_1=m(x-x_1)$

$y = mx + c$

↑

Slope



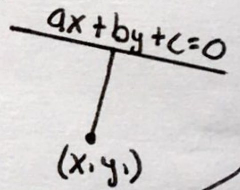
Parallel lines have the same slopes.
perpendicular lines slopes multiply to give -1. eg $\frac{2}{3} \times \frac{-3}{2} = -1$

Area of triangle $= \frac{1}{2}|x_1y_2 - x_2y_1|$ * must translate triangle so that one of its vertices lies on (0,0)

Remember: Equation of a line parallel to $ax+by+c=0$ is $ax+by+k=0$
Equation of a line perpendicular to $ax+by+c=0$ is $bx-ay+k=0$

perpendicular distance from a point to a line:

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

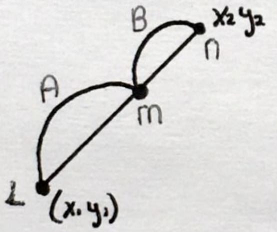


The Line

Find X intercept: Let $y=0$ and solve
Find y intercept: Let $x=0$ and solve.

Dividing a line Into a given ratio

① Internal: $m = \left(\frac{bx_1+ax_2}{b+a}, \frac{by_1+ay^2}{b+a}\right)$



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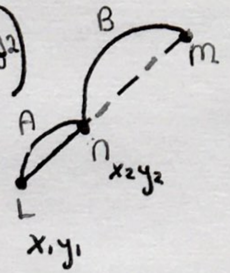
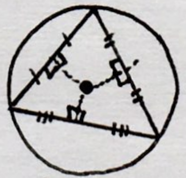


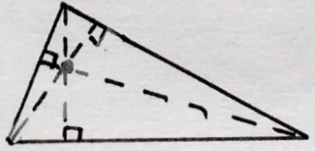
Centroid

① $\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$

② Circumcentre



③



Orthocentre

Scale Factor $= \frac{\text{Lenght of Image Side}}{\text{lenght of corresponding Image Side}}$

Angle between two lines

$\tan \phi = \pm \frac{m_1-m_2}{1+m_1m_2}$

Know Constructions

The Line

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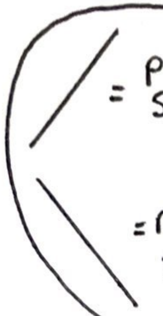
Slope = $\frac{y_2-y_1}{x_2-x_1}$ or $\frac{\text{Rise}}{\text{Run}}$

Equation = $y-y_1=m(x-x_1)$

$y=mx+c$

↑

Slope



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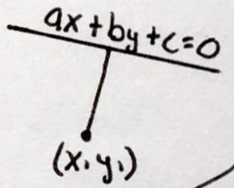

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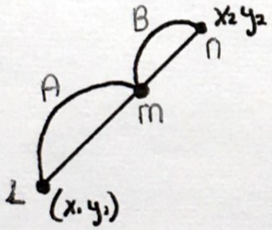

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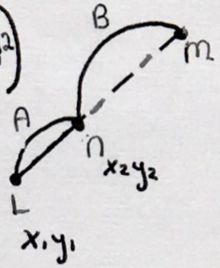

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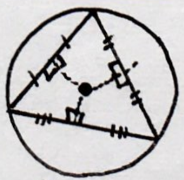

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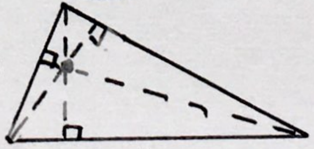

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Algebra 1

perfect Squares

$(x+a)^2 = x^2 + 2ax + a^2$

$(x-a)^2 = x^2 - 2ax + a^2$

Division

$$\begin{array}{r|l} & 2x^2 - 3x + 1 \\ \hline x-3 & 2x^3 - 9x^2 + 10x - 3 \\ & \underline{2x^3 - 6x^2} \\ & \quad -3x^2 + 10x \\ & \quad \underline{-3x^2 + 9x} \\ & \qquad x - 3 \\ & \qquad \underline{x - 3} \\ & \qquad 0 \end{array}$$

Sim. Equations

$A = x + y + z = 6$

$B = 2x + y - z = 1$

$C = 4x - 3y + 2z = 4$

- *Take A and B and eliminate Z to make equation D
- *Take B and C to eliminate Z to make equation E.
- * Use D and E to find X and y

Algebraic Identities

eg $(2x+a)^2 = 4x^2 + 12x + b$

$4x^2 + 4ax + a^2 = 4x^2 + 12x + b$

Compare like powers of X

$4ax = 12x$

$4a = 12$

$\therefore a = 3$

Remember:

$\dfrac{3x+4}{\frac{8}{10}} = (3x+4) \times \dfrac{10}{8}$

Remember

$\dfrac{2}{x+1} \div \dfrac{2x}{2x+3}$

is the same as...

$\dfrac{2}{x+1} \times \dfrac{2x+3}{2x}$

Simplifying Algebraic Fractions

Use Common Denominator

adding $\dfrac{2}{x+2} + \dfrac{3}{x+4} = \dfrac{2(x+4)+3(x+2)}{(x+2)(x+4)}$ ← now Simplify

Multiplying $\dfrac{2}{x+1} \times \dfrac{2x}{2x+3} = \dfrac{4x}{(x+1)(2x+3)}$

Remember: $\dfrac{\overset{2}{\cancel{6}} \times 12}{\cancel{3}^{1}}$ when multiplying

$\dfrac{\overset{2}{\cancel{6}} + \cancel{12}^{4}}{\cancel{3}^{1}}$ when adding

Factorising

1. Grouping: $2c^2 - 4cd + c - 2d$
2. HCF: $5x^2 - 10x$
   $= 5x(x-2)$
3. Difference of two squares:
   $9a^2 - 4 = (3a-2)(3a+2)$
4. Quadratics: trial and error.
   Guide number
   -B formula.
5. Sum and Difference of two cubes
   Know: $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
   $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

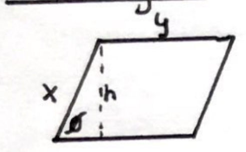
Know how to Manipulate formula

← Tip: Get $\sqrt[3]{x^3}$ and $\sqrt[3]{y^3}$ first

Remembee: Polygons are made up of isoceles triangles

Area and Volume

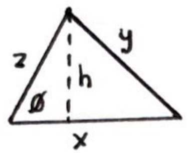
Parallelogramm



$h = x \sin \emptyset$

$\therefore$ area $= y \times \sin \emptyset$

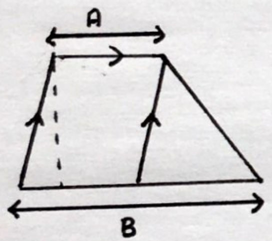
Triangle



$h = z \sin \emptyset$

$\therefore$ area $= \frac{1}{2} \times z \sin \emptyset$

Trapezium [one pair of parallel lines]



Area $= \left(\frac{a+b}{2}\right) h$

Trapezoidal Rule

Log book: Area $= \frac{h}{2}\left[y_1 + y_n + 2(y_2 + y_3 + y_4 + \dots y_{n-1})\right]$

ie $\frac{h}{2}\left[1^{st} + \text{Last} + 2(\text{everything else})\right]$

Sector of circle

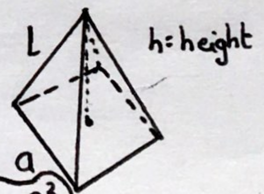
Arc $= \left[\frac{\text{degrees}}{360°} \text{ or } \frac{\text{radians}}{2\pi}\right] 2\pi R$

Area $= \left[\frac{\text{degrees}}{360°} \text{ or } \frac{\text{radians}}{2\pi}\right] \pi R^2$

Pyramid

Volume $= \frac{1}{3} a^2 \times h$

S.A $= a^2 + 4\left(\frac{1}{2}al\right) = a^2 + 2al$



$* \; L = h^2 + \frac{a^2}{4}$

Algebra 3

Inequalities

$\frac{3x-2}{x+1} \geq 2$

multiply both sides by $(x+1)^2$ because we dont know if $x+1$ is + or −

Modulus

Solve $|3x+5| = 2$

Square both sides

$(3x+5)^2 = 2$

and solve.

Proof of Abstract Inequalities

Remember: If a and b are real numbers

Then
$$\left.\begin{array}{l} a^2 \geq 0 \\ b^2 \geq 0 \\ (a+b)^2 \geq 0 \\ (a-b)^2 \geq 0 \\ -(a+b)^2 \leq 0 \\ -(a-b)^2 \leq 0 \end{array}\right\}$$

eg prove $x^2+4x+6 \geq 0$

$\underbrace{x^2+4x+4}-4+6 \geq 0$

$(x+2)^2 + 2 \geq 0$

This must $b \geq 0$ ∴ True

Know Rules of Indices

page 21 of log book

Know rules of Logs

page 21 of Log book.

prove $\sqrt{2}$ is irrational

Assume True ∴ can be written as $\frac{a}{b}$

$\sqrt{2} = \frac{a}{b}$

$\sqrt{2}^2 = \frac{a^2}{b^2}$

$2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$ [a^2 is even so a is Even.

∴ a can be written as 2c

$a^2 = 4c^2 = 2b^2$

$2c^2 = b^2$

∴ b is also even

a and b are even and therefore have a common factor of 2.

∴ $\sqrt{2}$ is irrational

Induction: Seris

* Prove $1+2+3+4 \ldots n = \frac{n}{2}(n+1)$ for all n.

1) Prove for n=1
2) Assume of n=k : $1+2+3\ldots k = \frac{k}{2}(k+1) = \frac{k(k+1)}{2}$
3) Prove for n=k+1

$\underbrace{1+2+3\ldots\ldots k}+(k+1) = \frac{(k+1)(k+1+1)}{2}$?

$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$

$\frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$

$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}.$

Induction: Divisibility

prove 7^n-1 is always divisible by 6

1. Prove for n=1
2. Assume for n=k
3. Prove for n=k+1

$f(k+1) - f(k)$ divisible by 6 ?

$(7^{k-1}-1) - (7^k-1)$

$(7^k.7-1) - (7^k-1)$

$7^k.7-1-7^k+1$

$7^k.6$

∴ is divisible by 6

Inequality Proof: $2^n \geq n^2$ where $n \geq 4$

1. prove for n=4
2. Assume for n=k
3. Prove for n=k+1

$2^k \geq k^2$ assumed

$2(2^k) \geq 2(k^2)$

$2^{k+1} \geq 2k^2$

$\underline{a}$ $\underline{b}$

$2k^2 \geq (k+1)^2$

a c

$2k^2 \geq k^2+2k+1$

$k^2 \geq 2k+1$

$k^2-2k-1 \geq 0$

$(k-1)^2 \geq 2$ [true for $k \geq 4$]

$a \geq b$

$b \geq c$

∴ $a \geq c$

∴ $2^n \geq n^2$ for $n \geq 4$

Algebra 2

Surds

- * $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$
- * $\sqrt{\frac{50}{64}} = \frac{\sqrt{50}}{\sqrt{64}} = \frac{\sqrt{25} \times \sqrt{2}}{8} = \frac{5\sqrt{2}}{8}$
- * $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$
- * $\sqrt{27} - \sqrt{12} = \sqrt{9 \times 3} - \sqrt{4 \times 3} = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$
- * $\sqrt{4} \times \sqrt{4} = 4$

  $\sqrt{5} \times \sqrt{6} = \sqrt{30}$
- * $\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

Forming Quadratics from Roots

$x^2 - x(\text{sum of the roots}) + \text{Product of the roots} = 0$

Completing the square

x^2+4x+1

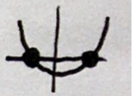
* Take the x term, half it, square it, add it and subtract it

eg $\underbrace{x^2+4x+4}-4+1$

$(x+2)^2-3=0$ ∴ $(-2,-3)$ is the min point.

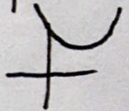
Discriminant (b^2-4ac)

- If $b^2-4ac>0$ there are two real roots



- If $b^2-4ac=0$ there is one real root

- If $b^2-4ac<0$ there are imaginary roots



- If b^2-4ac is a perfect square there are Rational Roots

The Factor Theorem

Factorise $2x^3+x^2-13x+6=0$

* Trial and error to find 1st factor

eg put in 0 for x and see if it $=0$

put in 1 for x and see if it$=0$

etc.

In this example when I put in 2 for x the equation $=0$ ∴ $(x-2)$ is a factor

now $x-2 \overset{2x^2+5x-3}{\overline{\left) 2x^3+x^2-13x+6 \right.}}$

now factorise $2x^2+5x-3$

$(x-2)(x+3)(2x-1)$ are the factors

Sued Equations

$\frac{1}{2\sqrt{x+2}} = 2$

$1 = 4\sqrt{x+2}$

$1 = 16(x+2)$

$1 = 16x+32$

finish

Solve $x^2+y^2=5$

$x-y+1=0$

* write x in terms of y: $x=y-1$
* now slot this in for x

$(y-1)^2+y^2=5$

* Solve for y
* put y back into $x-y+1=0$ to solve for x

Sometimes we need to subsitute when solving Quadratics.

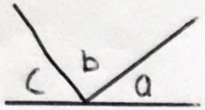
eg $\left(t-\frac{6}{t}\right)^2 - 6\left(t-\frac{6}{t}\right)+5=0$

let $u=\left(t-\frac{6}{t}\right)$ ∴ $u^2-6u+5=0$

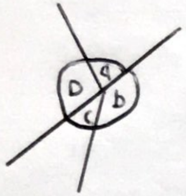
and solve

Geometry

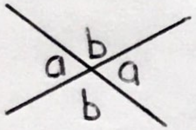
Rules

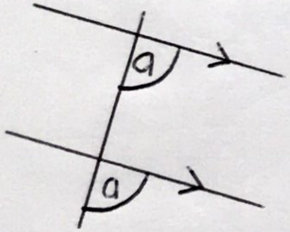


$a+b+c=180°$

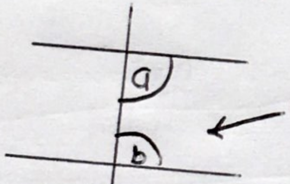


$A+B+C+D=360°$



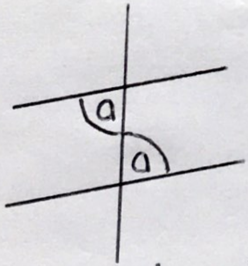


↑ Corresponding angles



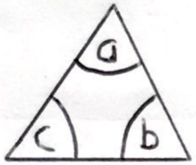
← $a+b=180°$

Interior angles

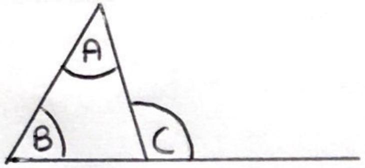


Alternate angles

Triangles



$A+b+C=180°$



$C=A+B$

Congruent triangles

Tests: Side Side Side
Side Angle Side
Angle Side Angle

Theorems to Learn: 11,12 and 13
Learn Constructions:

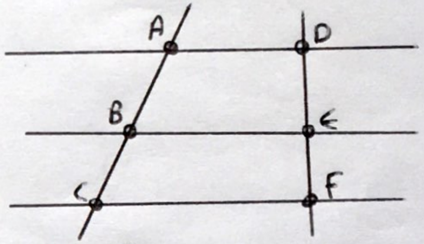
Area of triangle

$\frac{1}{2}$ base × Perpendicular height

Area of Parallelogram

Base × Perpendicular Height

Ratios



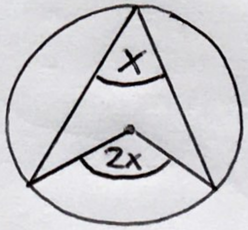
$\frac{AB}{BC} = \frac{DE}{EF}$

Scale factor = $\frac{\text{lenght of Image Side}}{\text{lenght of Corresponding object Side}}$

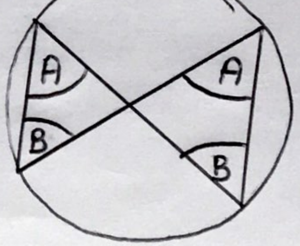
Circle theorems

① perpendicular from centre to Cord, bisects the Cord

②



③



A=A
B=B

probability

permutations [order matters]

How many different 4 letter words can be made from THURSDAY

$\boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5}$

Combinations [order does not matter]

How many ways can I select 3 letters from 7.

$\frac{7 \times 6 \times 5}{3 \times 2 \times 1} = {}^{7}C_{3} = 35$

* P [event not happening] = 1 - P [event happening]

* P [event happening at least one] = 1 - P [event not happening at all]

Remember: If letters have to be together, treat them a 1. Then multiply your answer by how many ways these letters can be rearranged

Theoretical Probability = $\frac{\text{\# of successful outcomes}}{\text{Total \# of outcomes}}$

Experimental Probability [Relative frequency] = $\frac{\text{\# of successful trials}}{\text{total \# of trials}}$

Expected Frequency = Probability X number of trials

and = X

OR = −

Imperical Rule

68% 1 SD

95% 2SD

99.7% 3SD

either side of the mean

Z Scores = $\frac{x - \mu}{\sigma}$

x = given score

μ = given mean

σ = standard deviation

Bernoulli Trials

$^{n}C_{R}\, p^{R}\, q^{n-R}$

eg Dice is thrown 5 times
Find the probability of getting 3 6's

$p = \frac{1}{6}$ ↑ success

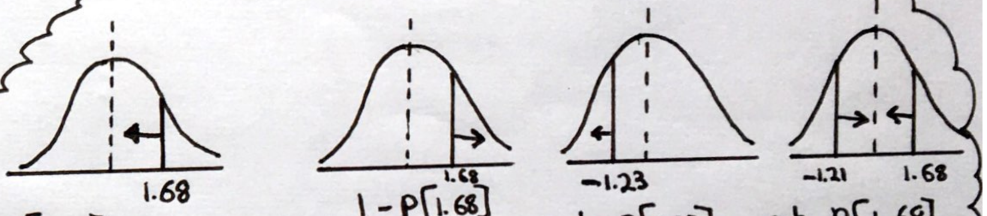
$q = \frac{5}{6}$ ↑ Fail

$n = 5$ ↑ all throws

$R = 3$ ↑ success

$\binom{5}{C_3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{5-3}$

Normal Distribution



Read P[1.68] from tables.

1 - P[1.68]

1 - P[1.23]

get P[1.68]
get 1-P[1.21]
Minus from each other

Addition Rule

Mutually Exclusive Events: P(A or B) = P(A) + P(B)

non mutually Exclusive Events: P(A or B) = P(A) + P(B) - P(A and B)

Multiplication Rule P(A and B) = P(A) x P(B) [Independent Events]

Conditional probability: P(A and B) = P(A) x P(B|A)

[not Independent]

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

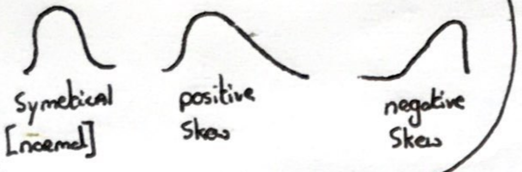
To show that events are Independent.

If $P(A \cap B) = P(A) \times P(B)$
Then the events are Independent.

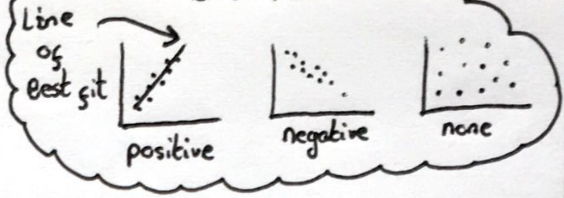
Statistics

Know: Types of Data
Mean, Mode, Median
Range, IQR
Standard Deviation
Histograms
Stem and Leaf plots
Empirical Rule

Shape of Distribution



Correlation



Finding Correlation Coefficient

1) Put calculator into stat
Linear mode
$\boxed{\text{Mode}}$ $\boxed{2}$ stat $\boxed{2}$ A+Bx

Data = (150, 65) (148, 68) (172, 75)

2) Input data
150 = 148 = 172 = $\boxed{\downarrow}$ $\boxed{\rightarrow}$ 65 = 68 = 7

3) Calculate
$\boxed{\text{ac}}$ $\boxed{\text{Shift}}$ $\boxed{1}$ stat $\boxed{5}$ $\boxed{3}$ =

Confidence Interval for population Proportion

$$\hat{p} - 1.96\sqrt{\frac{p(1-p)}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{p(1-p)}{n}}$$

Hypothesis test for pop. proportion

Ho
H₁

Use confidence interval: If Ho is within confidence interval we accept null hypothesis.

OR

Use Z score: $Z = \frac{x - \mu}{\sigma}$

If Z score is within + 1.96 and -1.96 accept null hypothesis

Confidence Interval for Mean

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \leq X \leq \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

Hypothesis test for mean

Ho
H₁

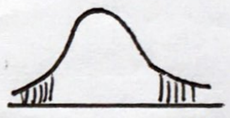
Use confidence interval: If Ho is within confidence Interval accept null hypothesis.

OR

Use Z score $Z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$

If Z score is within + 1.96 and -1.96 accept null hypothesis

p Values



$$p\text{value} = 2\left[\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right]$$

If P Value > 0.05 accept
If P Value ≤ 0.05 reject

Gives us a more precise level of significance

Central Limit theorem

- * If sample size is greater than 30 it will be distributed normally
- * The mean of the sample will be the same as the population mean.
- * The sample distribution of mean [mean of means] will also be normal.

Functions

Notation

$f(x) = x^2+6$

$f: x \to x^2+6$

$y = x^2+6$

Domain: numbers IN

Range: numbers OUT

Continuity

eg Show that $f(x) = \frac{3}{x-3}$ is not continuous at $x=3$

put 3 in for x

$f(3) = \frac{3}{3-3} = \frac{3}{0}$ ← Undefined and cant be factorised

$\therefore f(x) = \frac{3}{x-3}$ is discontinuous at $x=3$

$f(x) = x^2$

$f(x)+1$ ..... moves graph up 1

$f(x)-1$ ..... moves graph down 1

$y = f(x+1)$ .... moves graph left 1

$f(x+1)$ .... moves graph right 1

$-f(x)$ give symetry in x axis.

$2f(x)$ increases range by a factor of 2

Drawing functions

- * Know the shape of the function in question
- * Use roots of the equation to find where function crosses x axis
- * Draw function

Find:

$f(2)$ ..... put 2 in for X and solve

$f(x)=2$ ..... let the function equal 2 and solve for X

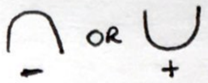
Types of functions

Linear

$y=2x-4$

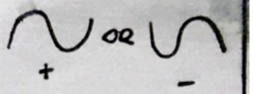
Quadratic

$y=x^2+2x+4$



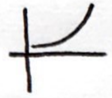
cubic

$2x^3+4x^2-3x+2$



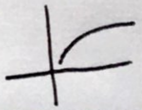
exponential

$y=3^x$



Log functions

$y=\log_{10} x$



$g \circ f$ OR gf

$g(x) = x^2-1$

$f(x) = x+3$

find $gf(-1)$

① Do $f(-1)$ first

$f(-1) = (-1)+3 = 2$

② now put answer into $g(x)$

$g(2) = (2)^2-1$

$4-1$

$=3$

$\therefore gf(-1) = 3$

Inverse functions

If $f(x) = 5x-3$, find $f^{-1}(x)$

1. Let $y = 5x-3$
2. Write function in terms of x

   $x = \frac{y+3}{5}$
3. Change x to $f^{-1}(x)$ and y to x

   $f^{-1}(x) = \frac{x+3}{5}$

Limit to Infinity

* Divide all terms by highest power of x and then let $x=0$

eg $\lim_{x\to\infty} \frac{4x+1}{2x+3} \Rightarrow \lim_{x\to\infty} \frac{4+\frac{1}{x}}{2+\frac{3}{x}} = \frac{4+0}{2+0} = \frac{4}{2}$

Limits

eg 1. $\lim_{x\to 2} \frac{3x+2}{x+4}$

to find the limit as x approachs 2 Simply let $x=2$ and solve.

$\frac{3(2)+2}{2+4} = \frac{4}{3}$

eg 2. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$ [when we put 2 in for x we get 0 as a denominator. ∴ wont work]

* now we factorise *

$\lim_{x\to 2} \frac{x^2+x-6}{x-2} \Rightarrow \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} (x+3)$

* now put 2 in for x ..... $\lim_{x\to 2} (2+3) = 5$

Drawing function in domain $-2 \leq x \leq 2$

- * Put in -2, -1, 0, 1 and 2 into function. get range [y] numbers.
- * Use these points to plot function

Differential Calculus

1st Principles

$$\lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$$

Rule: $y = x^3 + 3x^2 + 5x + 4$

$\frac{dy}{dx} = 3x^2 + 6x + 5$

Second Derivatives

$\frac{d^2y}{dx^2}$ OR $f''(x)$

Derivative → $\frac{dy}{dx}$ OR $f'(x)$

- Increasing is $\frac{dy}{dx} > 0$
- decreasing is $\frac{dy}{dx} < 0$
- To find equation of a tangent to a curve at a point:
  1) differentiate
  2) Slot in x value
- Stationary / Turning points
  1) Differentiate
  2) Let $= 0$ and solve.
- Max or min.
  1) After finding max/min point:
  2) differentiate again and slot in X values
     if $f''(x) > 0$ .... minimum turning point
     if $f''(x) < 0$ ..... maximum turning point.

Remember $\ln e^x = X$ and $\sin^3 X$ can be written as $(\sin x)^3$

$e^{ax} \to e^{ax} . ax$ differentiated

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$x^n \to nx^{n-1}$

$\ln x \to \frac{1}{x}$

$e^x \to e^x$

$e^{ax} \to ae^{ax}$

$a^x \to a^x \ln a$

$\cos x \to -\sin x$

$\sin x \to \cos x$

$\tan x \to \sec^2 x$

$\cos^{-1}\frac{x}{a} \to -\frac{1}{\sqrt{a^2-x^2}}$

$\sin^{-1}\frac{x}{a} \to \frac{1}{\sqrt{a^2-x^2}}$

$\tan^{-1}\frac{x}{a} \to \frac{a}{a^2+x^2}$

Let $a = 1$
Let $x = \frac{x}{a}$

Product Rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = uv$

eg $\underset{u}{(6x^2+2x)}\underset{v}{(3x-2)}$

Quotient Rule: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = \frac{u}{v}$

eg $\frac{x^2+7}{3x-1}$

Chain Rule

If y is a function of u, and u is a function of x

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

eg Find $\frac{dy}{dx}$ if $y = (2x^2-1)^3$

$y = (2x^2-1)^3$

Let $u = 2x^2 - 1$

$\therefore y = u^3$

$u = 2x^2 - 1 \longrightarrow \frac{du}{dx} = 4x$

$y = u^3 \longrightarrow \frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 3u^2 \cdot 4x$

$= 3(2x^2-1)^2 \cdot 4x$

$= 12x(2x^2-1)^2$

Integration

Anti differentiation

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$

*Increase power by 1 and divide by new power

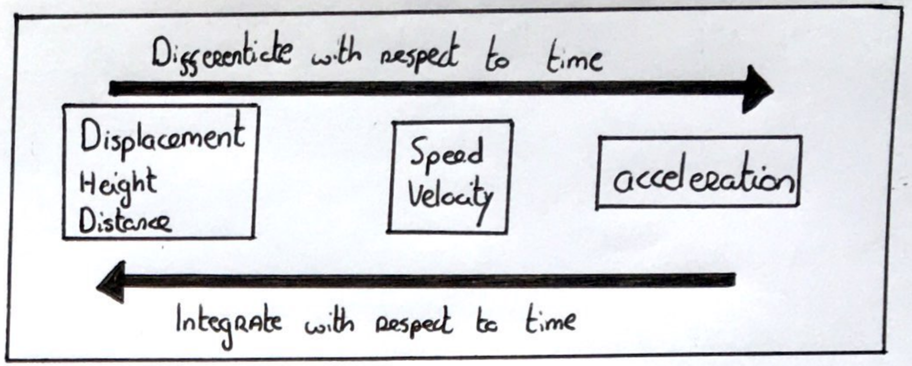
*always include "+C"

$\int (3x^2+4x+5) \, dx$

becomes . . . . .

$\frac{3x^3}{3} + \frac{4x^2}{2} + 5x + C$

$= x^3 + 2x^2 + 5x + C$



Differentiate with respect to time

Displacement
Height
Distance

Speed
Velocity

acceleration

Integrate with respect to time

Average Value of a function

$\frac{1}{b-a} \int_a^b fx \, dx$

Log Book

$\int e^x \, dx \Rightarrow e^x + C$

$\int e^{ax} \, dx \Rightarrow \frac{1}{a} e^{ax} + C$

$\int a^x \, dx \Rightarrow \frac{a^x}{\ln a} + C$

$\int \cos x \, dx \Rightarrow \sin x + C$

$\int \cos mx \, dx \Rightarrow \frac{\sin mx}{m} + C$

$\int \sin x \, dx \Rightarrow -\cos x + C$

$\int \sin mx \, dx \Rightarrow -\frac{\cos mx}{m} + C$

Finding Constant

If you are given a point on the curve put x value in for x, Let function equal y value and now you can solve for C

* If $y = \sin 3x^2$, find $\frac{dy}{dx}$

Hence find $\int 6x \cos 3x^2 \, dx$

In a question such as this we would not be expected to Integrate $\int 6x \cos 3x^2 \, dx$ we would just need to show that we understand that Integration is the inverse of differentiation

Finding area between curve and X axis

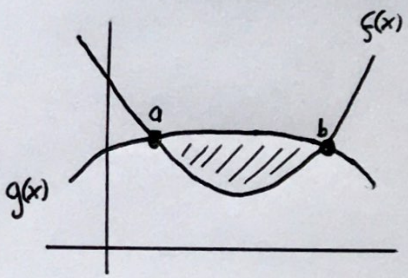
$\int_a^b f(x) \, dx$

Finding area between curve and y axis

$\int_a^b f(y) \, dy$ * must write function in terms of y.

Finding area between two curves

$\int_a^b g(x) \, dx - \int_a^b fx \, dx$



* always sketch these questions to ensure you are subtracting the correct integral.

eg: $\int_0^2 3x^2 \, dx = \left[\frac{3x^3}{3}\right]_0^2 = \left[x^3\right]_0^2$

$= [2^3] - [0^3] = 8$