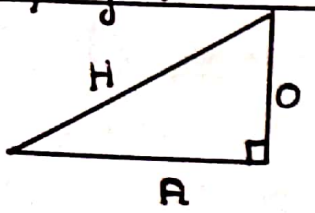
# Trigonometry

## Pythagoras Theorem




$H^2 = A^2 + O^2$

When we have two sides of a right angle triangle and we are looking for the lenght of the thied side

## Sin Rule

We can use this if we know the lenght of a side and the size of its opposite angle

Sin Rule = $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

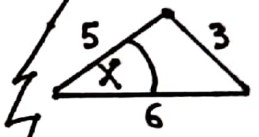
## Cosine Rule

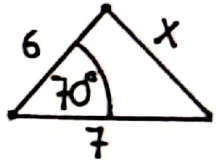
We use it when we dont have the lenght of a side and the size of its opposite angle

$a^2 = b^2 + c^2 - 2bc\cos A$

work this part out together

we use it to work out these



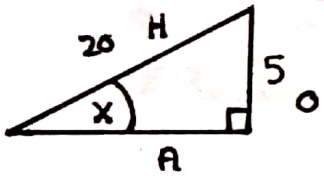

Right angle triangles

None right angle triangles

## C$\frac{A}{H}$ S$\frac{O}{H}$ T$\frac{O}{A}$

Could a hungry Stupid Ostrich have taken over America

### Finding the angle

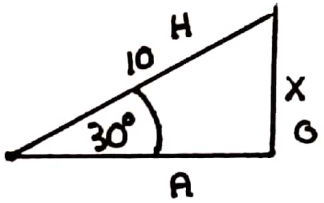



We use Sin because we have O and H

press Shift Sin $\frac{5}{20}$ =

Answer = 14°

### Finding the side




We use Sin because we have H and want to find O.

$\sin 30° = \frac{x}{10}$

$0.5 = \frac{x}{10}$

$0.5 \times 10 = x$

$5 = x$

## Remember:

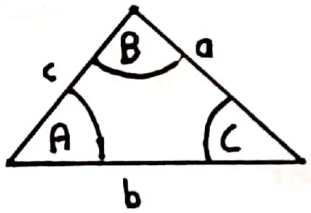
Capitel Letters represent angles

Small Letters repaesents sides

Side "a" will be opposite angle "A"

Side "b" will be opposite angle "B"

Side "c" will be opposite angle "C"




## Area of triangle

$\frac{1}{2}$ ab Sin C

OR

$\sqrt{S(S-a)(S-b)(S-c)}$

where $S = \frac{a+b+c}{2}$

OR

$\frac{1}{2}$ base x height

OR

$\frac{1}{2}|x_1 y_2 - x_2 y_1|$

# The Circle

## Equation of a Circle

With centre (0,0): $x^2+y^2=r^2$

So if $x^2+y^2=100$

the circle has radius of 10 and a centre of (0,0)

---

centre not (0,0)

$(x-h)^2+(y-k)^2=r^2$

eg: If $(x-4)^2+(y+6)^2=100$

Then radius is 10

centre is (4,-6)

Remember to change Sign

## Is a point inside, outside or on a circle?

eg: is (4,2) inside circle $x^2+y^2=25$

Step 1: Slot in x and y number

$(4)^2+(2)^2=25$

$16+4=25$

$20=25$

If $x^2+y^2=r$ .... point is on the circle

If $x^2+y^2>r$ .....point is outside the circle

If $x^2+y^2<r$.... point is inside the circle

* To find where the circle crosses the x axis..... Let y=0 and solve
* To find where the circle crosses the y axis..... let x=0 and solve

## Finding where a line and a circle cross each other

Question. Find where the line $x-3y-10=0$ and the circle $x^2+y^2=10$ cross each other.

**Step 1** Write the equation in terms of x [or y].

$x=3y-10$

**Step 2** put this into circle equation in place of x

$(3y-10)^2+y^2=10$

**Step 3** Solve for y

$(3y-10)^2+y^2=10$

$(3y-10)(3y-10)+y^2=10$

$9y^2+30y+30y+100+y^2=10$

$9y^2+60y+100+y^2-10=0$

$\div 10$ $10y^2+60y+90=0$

$y^2+6y+9=0$

$(y+3)(y+3)=0$

$y+3=0$ | $y+3=0$

$y=-3$ | $y=-3$

**Step 4** Slot value for y back into line equation and solve for x

$x-3(-3)-10=0$

$x+9-10=0$

$x-1=0$

$x=1$

The line and circle cross at (1,-3)

They only cross at 1 point.

∴ the line is a tangent to the circle

# The Line

## Formula in Log book
- Distance
- Mid Point
- Equation
- Slope

## Finding the equation of a line
put the slope and one point into $y-y_1=m(x-x_1)$ and simplify

## Finding the slope when given the equation.
Write equation in the form $y=mx+c$
$y$ is the slope

## Remember:
- Parallel lines have the same slope.
- perpendicular lines slopes multiply to $-1$

To get a perpendicular slope, turn upside down and change one sign.
eg $\frac{3}{5}$ is perpendicular to $-\frac{5}{3}$

To find where two lines cross each other we use Simultaneous Eq.

## Drawing a line from an equation
* Let $y=0$ and solve for $X$
  This will get you the $X$ intercept
* Let $X=0$ and solve for $y$.
  This will get you the $y$ intercept.
* Join both points

## To check if a point is on a line.
eg check if $(4,2)$ is on the line $2x+3y=13$

Slot 4 and 2 in for $X$ and $Y$

$2(4)+3(2)=13$

$8+6=13$

$14=13$

Not true. $\therefore$ $(4,2)$ is <u>not</u> on the line

## Area of triangle
$\frac{1}{2}|x_1y_2-x_2y_1|$

Remember to translate one point to $(0,0)$

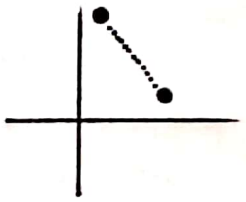
$(2,3) \xrightarrow{-2\ -3} (0,0)$

$(4,1) \xrightarrow{-2-3} (2,-2)$

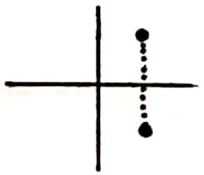
$(-1,2) \xrightarrow{-2-3} (-3,-1)$

now use $(2-2)$ and $(-3,-1)$ as $x_1y_1x_2y_2$ in the formula

Translation

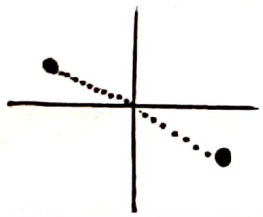


axial Symetry [through X axis]



axial Symetry [through y axis]

central Symetry (through the origin)



**Mode:** Most common number

**Median:** List from smallest to biggest and choose middle number

**Mean:** Add the numbers and divide answer by the number of numbers

**Range:** Biggest number - smallest number

**Inter Quartile Range:**

2 3 (5) 6 7 (9) 11 12 (14) 16 17

Q1 = 5
Q2 = 9
Q3 = 14

IQR = Q3 - Q1

Data Types
- Numerical
  - Discrete
  - Continuous
- Categorical
  - Nominal
  - Ordinal

- Desk Research [secondary]
- Field Research [Primary]

## Empirical Rule

68% of data lies within 1 standard Deviation of its mean.
95% of data lies within 2 standard Deviations of its mean
99.7% of data lies within 3 standard Deviations of its mean

# Statistics

## Correlation [between -1 and 1]

0 = no correlation
negative number = negative correlation.
positive number = positive correlation.


## Frequency table

| Value | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Freq. | 2 | 4 | 3 | 1 |

Mode: Value above most Common Frequency

Median: Add all frequencys Divide answer by 2. Find which frequency gets you up to this number.

Mean: $\frac{2(1)+4(2)+3(3)+1(4)}{2+4+3+1} = 2.3$

## Standard Deviation

$$\sqrt{\frac{\sum f\,[\text{number} - \text{mean}]^2}{\sum f}}$$

| V | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| F | 1 | 4 | 9 | 6 |

Mean = 2

Standard Deviation: $\sqrt{\frac{1(2-1)^2+4(2-2)^2+9(3-2)^2+6(4-2)^2}{1+4+9+6}}$

**Margin of Error** = $\frac{1}{\sqrt{n}}$ [n is sample size]

**Sample proportion** = $\hat{p} = \frac{\text{amount}}{\text{Sample}}$

**Confidence Interval** = $\hat{p} - \frac{1}{\sqrt{n}} \leq p \leq \frac{1}{\sqrt{n}} + \hat{p}$

**Hypothesis Test:** Ho [null hypothesis]
H1 [alternate hypothesis]

* Accept or Reject null hypothesis *

# Probability

And = X
OR = +

Probability of Rolling a 6 and a 2
$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

probability of rolling a 6 or a 2
$\frac{1}{6} + \frac{1}{6} = \frac{1}{12}$

Remember:
$4! = 4 \times 3 \times 2 \times 1$

Probabilities lie between 0 and 1
0 being Impossible
1 being certain

## Bernaulli trials

Combining two Events

eg Find the probability of tossing a tails and throwing a 6

## Permulations

① How many ways can we arrange the Letters A, B, C, D.
Answer $4 \times 3 \times 2 \times 1 = 24$.

② If the word must begin with A
answer $1 \times 3 \times 2 \times 1 = 6$

③ If the word must begin with A and the next Letter B
answer $1 \times 1 \times 2 \times 1 = 2$

④ If we can repeat the Letters
answer $4 \times 4 \times 4 \times 4 = 256$

## Theoretical Probability

$$P[\epsilon] = \frac{\text{number of successful outcomes}}{\text{Total number of possible Outcome}}$$

eg Probability of picking an Ace from a deck
$= \frac{4}{52}$

probability of an event not happening is 1 − probability of the event happening

Expected Frequency = Probability × number of trials

## Expected Value


| Outcome(x) | Probability(P) | (x)×(P) |
|---|---|---|
| €10 | $\frac{1}{2}$ | €5 |
| €4 | $\frac{1}{4}$ | €1 |
| €5 | $\frac{1}{4}$ | €1.25 |
| | | €7.25 |

Expected Value

* If it cost more than the expected value to play the game the in the long Run we will Loose money.
* If cost to play = E.V. then it is a "Fair Game".

## Sample Space Diagram

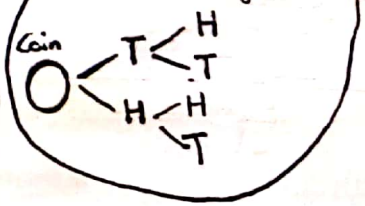
Dice 1

| Dice 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

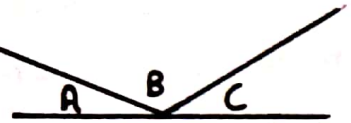
## Relative frequency

$$R.F. = \frac{\text{number of successful trials}}{\text{Total number of trials}}$$

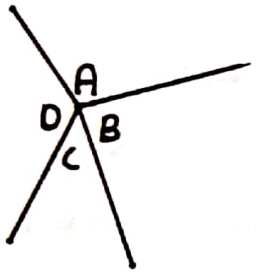
## Tree Diagrams



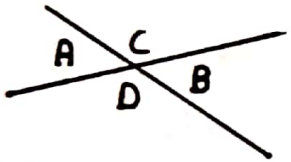
## Properties of Angles

① 

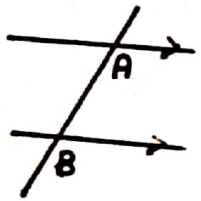

$A+B+C = 180°$

② 

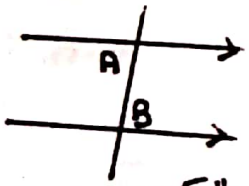

$A+B+C+D = 360°$

③ 


$A=B \quad C=D$

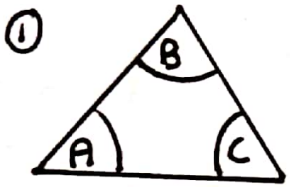
④ 


$A=B$ [corresponding angles]

⑤ 


$A+B=180°$ [alternate angles]

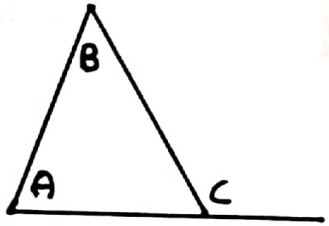
## Triangle Properties

① 

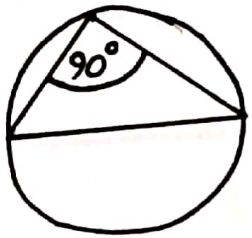

$A+B+C = 180°$

② 

In an Isosceles triangle the base angles are equal

③ 


$C = A+B$

④ 

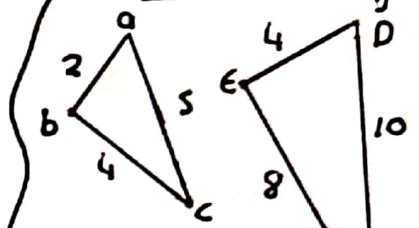

angle in a semi circle is always 90°

## Congruent triangles

[the same]

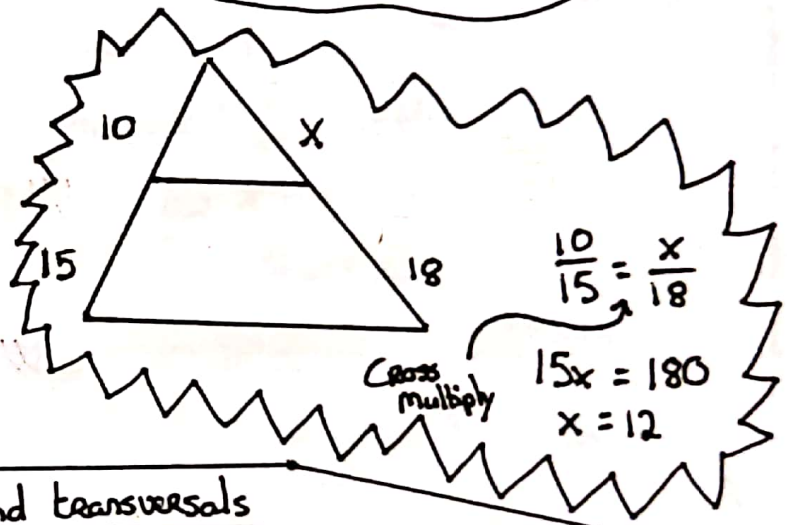
Side Side Side
Side Angle Side
Angle Side Angle

## Similar triangles

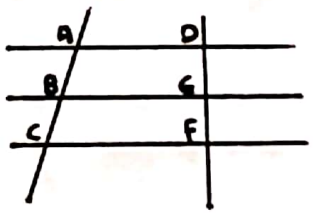



$\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|} = \frac{|BC|}{|EF|}$

# Geometry




## Parallel lines and transversals




if $|AB| = |BC|$ then $|DE| = |EF|$

Area of triangle = $\frac{1}{2}$ base × Perpendicular Height

Area of parallelogram = Base × Perpendicular Height

* Know: Theorems, Constructions, Enlargements

# Calculus

## Rates of Change

Distance – Don't Differentiate
Velocity or Speed – Differentiate once
Acceleration – Differentiate twice

eg A body moves S metres in t seconds such that $S = 2t^2 - 4t + 4$

1) Find the distance travelled after 6 seconds

* Just put 6 in for t and Solve

$2(6)^2 - 4(6) + 4$

52 metres

2) Find the Speed after 4 seconds

* Differentiate and put 4 in for t.

$\frac{ds}{dt} = 8t - 4$

$= 8(4) - 4$

$= 28$ mps

## Differentiation

Multiply the Coefficient by the power and reduce the power by 1

eg $y = 3x^2 - 5x + 4$

$\frac{dy}{dx} = 6x^1 - 5$

## To find the slope of a curve at a certain point:

1) Differentiate
2) put in X coordinate.
3) Solve

eg Find the slope of $y = 3x^2 + 4x - 5$ where $x = 1$

Differentiate : $\frac{dy}{dx} = 6x + 4$

Slot in 1 for x: $6(1) + 4$

Slope = 10

## Finding Max or Min turning points.

Step 1: Differentiate
Step 2: Let differentiated equation = 0
Step 3: Solve for X
Step 4: put X back into original equation and solve for y.

If $\frac{dy}{dx} > 0$ the slope is increasing

If $\frac{dy}{dx} < 0$ the slope is decreasing

## acceleration

eg $S = 3t^3 - 4t + 6$

Find acceleration after 3 seconds.

Differentiate twice, Slot in 3 for X and solve

$\frac{ds}{dt} = 9t^2 - 4$ $\quad \frac{d^2s}{dt^2} = 18t$ $\quad$ acceleration $= 18(3) = 54$

To find the X intercept let $y = 0$ and solve.
To find the y intercept let $x = 0$ and solve.

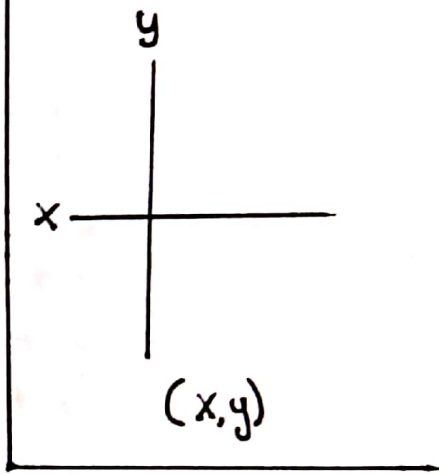
If we are asked to draw a line when given its equation .... we find the X and the y intercept and joint them.

## Drawing Graphs

eg: Draw $f(x) = x^2 - 2x - 3$ in the domain $-2 \leq x \leq 2$

| X | $x^2 - 2x - 3$ | y | point |
|---|---|---|---|
| -2 | $(-2)^2 - 2(-2) - 3$ | 5 | (-2, 5) |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

*when we have all the points we can draw the graph



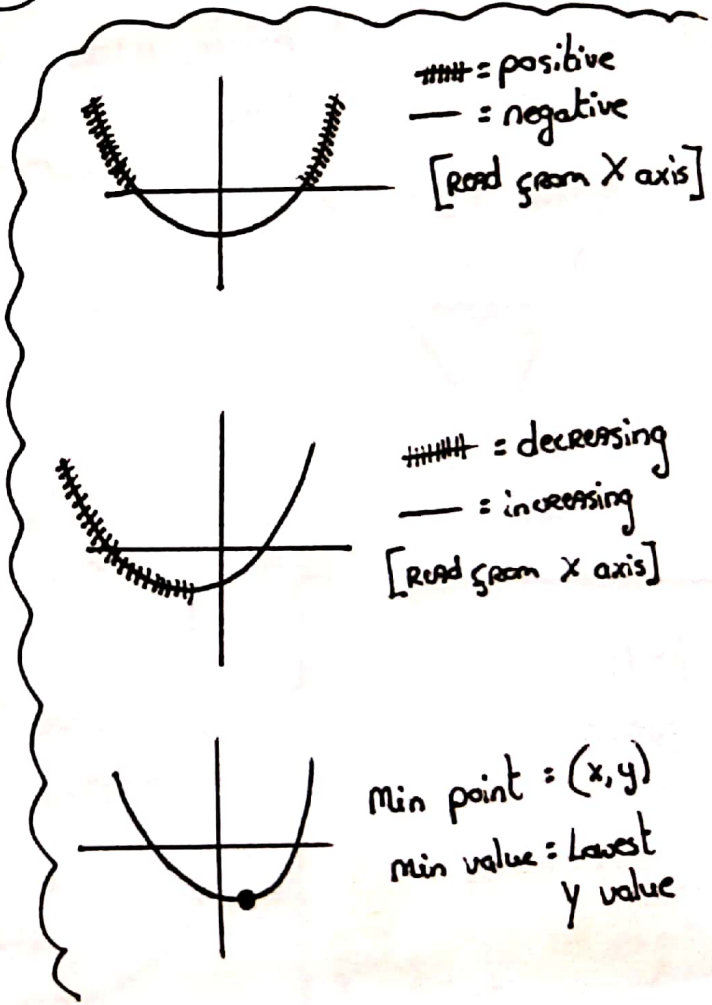

## Functions

Remember: $f(x)$ is just another name for y

* Finding the roots of an equation.
  [where the line crosses the X axis]
  Let equation = 0 and solve.

* Find $f(x) = 2$
  Go to 2 on y axis, draw horizontal line. to where it hits the curve.
  Now draw a vertical line to X axis. from here
  Our X value is the answer.

* Find $f(2)$
  Go to 2 on X axis.
  Draw vertical line to where it hits curve
  Now Draw a horizontal line from here to y axis
  Our y value is our answer



# Arithmetic

## Compound Interest

$F = P(1+i)^t$

## Depreciation

$F = P(1-i)^t$

## Foreign Exchange

€1 = $1.20

Find €250

| ×250 | €1 = €1.20 | ×250 |
|---|---|---|
| | €250 = ? | |

Commission is taken away at the end.

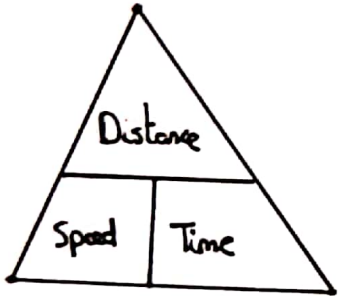
## Income Tax

Cut off point: you are taxed at the lower rate up to this amount. Any amount you earn more than this is taxed at the higher rate.

Tax credit: money taken away from tax owed

Gross: Wages before deductions

Net: Wages after deductions




## Ratios

Divide €200 into the ratio 8:2

+ add 8+2 = 10
* divide €200 by 10 = 20
* multiply 20 by 8 = 160
  multiply 20 by 2 = 40
∴ €40 : €160

## percentages

Remember: Cost price and price excluding vat is 100%

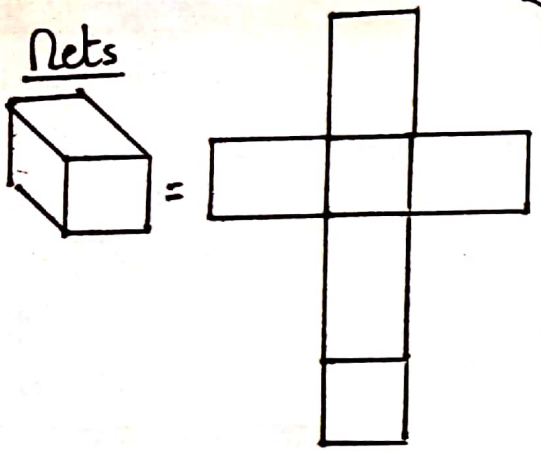
Sell price and price excluding vat is 100% + Vat%

So if a sofa cost €242 inclusive of Vat [21%] find the cost exclusive of Vat.

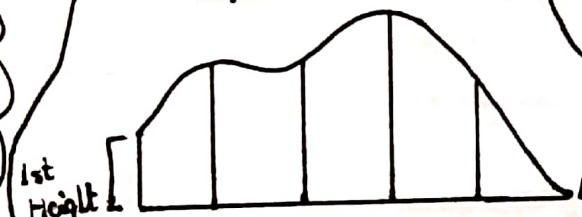
Divide €242 by 121 to get 1% and multiply answer by 100 to get 100 [which is the price excluding Vat]

# Area and Volume

## Nets

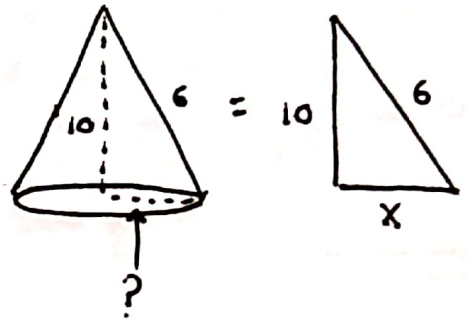


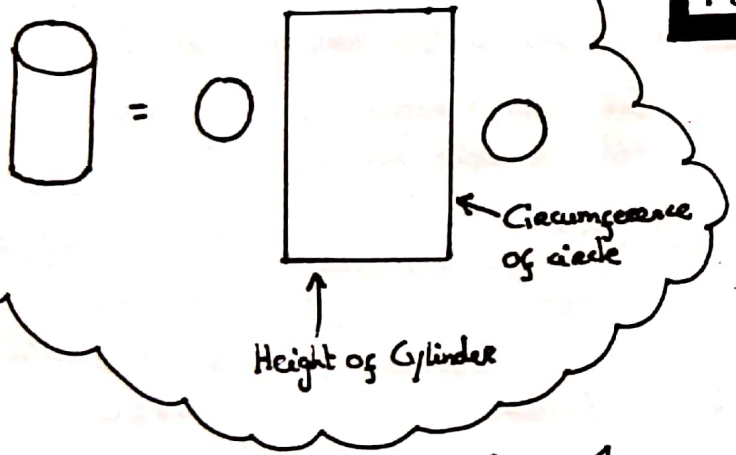
## Trapezoidal Rule




$$\text{Area} = \frac{w}{2}\left[1^{st} + \text{Last} + 2[\text{the rest}]\right]$$

Sometimes we need to use pythagoras theorem

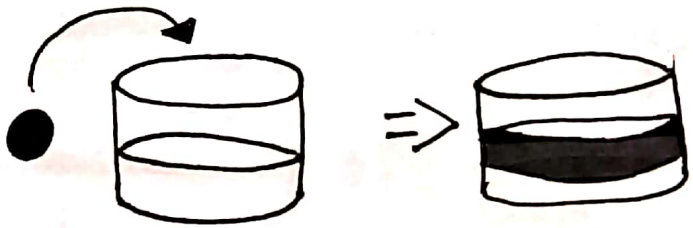



Formula in Log Book

Remember: perimeter and Circumference measured in cm, m etc
- Area measured in $cm^2$
- Volume measured in $cm^3$

## Equal Volumes



Water rises by the same volume as what was put in.

∴ Vol of sphere = Raise in vol of water which is now in the shape of a cylinder.

So we let ● = ⬭

$\frac{4}{3}\pi R^3 = \pi R^2 H$ ← Sub in what we know and solve.

# Algebra

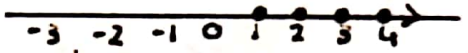
## Rules of Multiplication

Like signs give + answer
Unlike signs give − answer

eg (3)(4) = 12
(−3)(4) = −12
(−3)(−4) = 12
(3)(−4) = −12

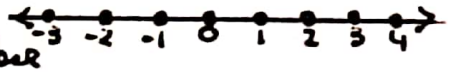
## Natural numbers N:

positive whole numbers



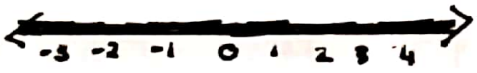

## Integers Z:

any whole number




## Real numbers R:

any number




## Sim. Equations: One linear and one quadratic

$x+y=3$ ①
$x^2+y^2=17$ ②

* write ① in terms of y: $x=3-y$
* put this in for x in ②
* now solve for y.
* when you find y put the answer back into ① and solve for x

## Simultaneous Equations

$2x-5y=9$
$3x+2y=4$

* Remember: get a positive and negative pair

## Rules of Addition

* when the signs are the same, we add the numbers and use the common sign
* when the signs are different, we subtract the numbers and use the sign of the bigger number

eg 4+3 = 7
−4−3 = −7
−4+3 = −1
4−3 = 1

## Multiplying brackets

$(4x+3)(2x-4)$
$8x^2-16x+6x-12$
$8x^2-10x-12$

## Solving Algebraic Fractions

$\frac{x+4}{3}-\frac{x+2}{4}=\frac{7}{6}$ LCM = 12

$\frac{4(x+4)-3(x+2)=2(7)}{12}$

now ignore denominator and solve for x

## Laws of Indices

$4^2\times4^3=4^5$

$\frac{4^5}{4^2}=4^3$

$(4^2)^3=4^6$

$4^0=1$

## Simplifying

$3x^2+4x+3y+2+x^2+3y+2y+4$
$4x^2+4x+8y+6$

## Adding Algebraic Fractions

$\frac{5}{x+3}+\frac{2}{x-4}$

$\frac{5(x-4)+2(x+3)}{(x+3)(x-4)}$

$\frac{5x-20+2x+3}{(x+3)(x-4)}$

## 4 ways to factorise

1) HCF eg $4x-8$
$4(x-2)$

2) Grouping
$2d+4cd+3a+6ac$
$2d(1+2c)+3a(1+2c)$
$(2d+3a)(1+2c)$

3) Difference of two squares
eg $9x^2-16$
$(3x-4)(3x+4)$

4) Factorising Quadratics eg $x^2+6x+9$
$(x+3)(x+3)$ solving ↓
$x+3=0$ $x+3=0$
$x=-3$ $x=-3$

OR $-b$ formula

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

# Complex Number

$\sqrt{-1} = i$

$i^2 = -1$

$\therefore \sqrt{-16} = 4i$

**Complex Number**

$3 + 2i$

↑ Real part ↑ Imaginary part

**Addition**

$z_1 = 2+3i$

$z_2 = 4-2i$

$z_1 + z_2 = (2+3i) + (4-2i)$

$= 6+1i$

**Multiplication**

$z_1 = 2+3i \quad z_2 = 4+2i$

$(2+3i)(4+2i)$

$8+4i+12i+6i^2$

$8+16i+6(-1)$ ← Remember $i^2 = -1$

$8+16i-6$

$2+16i$

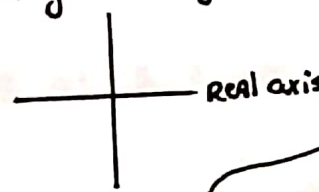
**Conjugate $\bar{z}$**

Just change the sign of the imaginary part

eg $z = 4-3i$

$\bar{z} = 4+3i$

**Argand Diagram**




**Modulus = $\sqrt{a^2+b^2}$**

The modulus is the distance from the origin (0,0) to the complex number on the argand diagram.

**Equality in Complex numbers**

$\overset{R}{2}x + \overset{I}{3}y i = \overset{R}{10} + \overset{I}{9}i$

Separate real numbers with real number and imaginary numbers with imaginary

| R | I |
|---|---|
| $2x = 10$ | $3y = 9$ |
| $x = 5$ | $y = 3$ |

**Division**

Rule: Multiply top and bottom by conjugate of bottom

eg $z_1 = 1+7i \quad z_2 = 4+3i$

Find $\frac{z_1}{z_2}$

$\frac{1+7i}{4+3i}$ (conjugate is $4-3i$)

$\frac{1+7i}{4+3i} \times \frac{4-3i}{4-3i}$

Top $(4-3i)(1+7i) = 4+25i-21i^2$

$= 4+25i+21$

$= 25+25i$

Bottom $(4+3i)(4-3i) = 16-9i^2$

$= 16+9$

$= 25$

$\frac{25+25i}{25} = 1+1i$

Each number in a sequence is called a term.

$T_1$ stands for the first term

$T_4$ stands for the fourth term.

$T_n$ stands for a general term/formula to help you calculate any term in the sequence

Finding $T_n$ [the $n^{th}$ term] of an arithmetic Sequence

$T_n = a + (n-1)d$

$a$ = 1st term

$d$ = difference

If $T_n = 4n - 3$

Find $T_2$

$T_2 = 4(2) - 3$

$= 5$

## Types of Sequence

① 5 7 9 11 13 — Arithmetic [1st difference is equal]

(2 2 2 2)

② 3 6 11 18 27 — Quadratic [2nd difference is equal]

(3 5 7 9; 2 2 2)

④ 2 4 8 16 32 — Exponential.

## Number Patterns

$T_n = 4n - 3$

What term will give 11

- we could use trial and ERROR
- Better to Let $4n - 3 = 11$ and work back to find n

## Arithmetic Series [Sum of]

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

($a$: 1st term; $d$: Difference)

*Remember: $T_n = S_n - S_{n-1}$

## Quadratic Sequence

$T_n = an^2 + bn + c$

3 10 21 36

7 11 15

4 4 ← a = half 2nd difference.

$T_n = 2n^2 + bn + c$

we find b and c using sim. equations.

eg $T_1 = 3$

$\therefore 2(1) + b(1) + c = 3$

$b + c = 1$

$T_2 = 10$

and $\therefore 2(2) + b(2) + c = 10$

$4 + 2b + c = 10$

$2b + c = 6$

Now use sim eq:

$2b + c = 6$

$b + c = 1$