

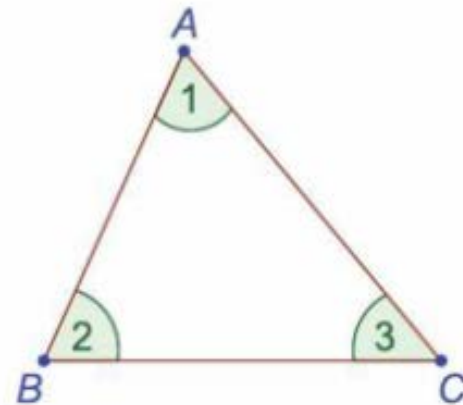
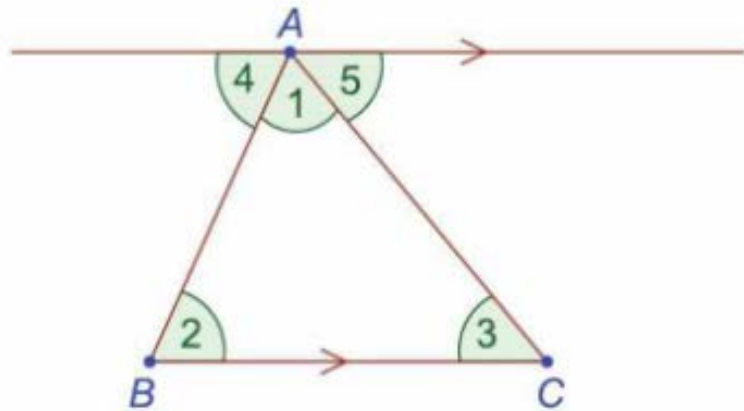
### Theorem 4

The angles in any triangle add up to  $180^\circ$ .

**Given:** A triangle with angles  $\angle 1$ ,  $\angle 2$  and  $\angle 3$ .

**To prove:**  $|\angle 1| + |\angle 2| + |\angle 3| = 180^\circ$ .

**Construction:** Draw a line through  $A$ , parallel to  $BC$ . Label angles 4 and 5.



**Proof:**

Statement	Reason
$ \angle 4  +  \angle 1  +  \angle 5  = 180^\circ$	Straight angle
$ \angle 2  =  \angle 4 $	Alternate
$ \angle 3  =  \angle 5 $	Alternate
$\Rightarrow  \angle 4  +  \angle 1  +  \angle 5  =  \angle 2  +  \angle 1  +  \angle 3 $	
$\Rightarrow  \angle 1  +  \angle 2  +  \angle 3  = 180^\circ$	
Q.E.D.	

## Theorem 6

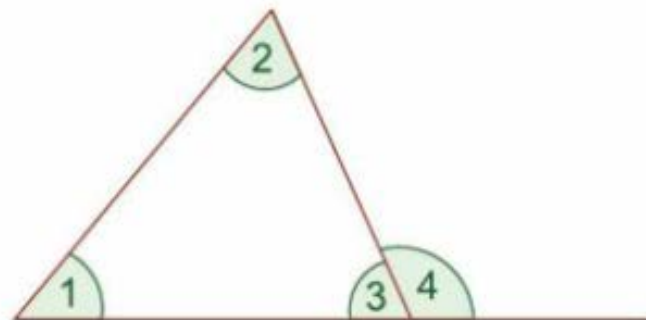
Each exterior angle of a triangle is equal to the sum of the interior remote angles.

**Given:** A triangle with interior angles  $\angle 1$ ,  $\angle 2$  and  $\angle 3$ , and an exterior angle  $\angle 4$ .

**To prove:**  $|\angle 1| + |\angle 2| = |\angle 4|$ .

**Proof:**

Statement	Reason
$ \angle 3  +  \angle 4  = 180^\circ$	Straight angle
$ \angle 1  +  \angle 2  +  \angle 3  = 180^\circ$	Angles in a triangle
$\Rightarrow  \angle 1  +  \angle 2  +  \angle 3  =  \angle 3  +  \angle 4 $	Both = $180^\circ$
$\Rightarrow  \angle 1  +  \angle 2  =  \angle 4 $	Subtracting $ \angle 3 $
Q.E.D.	



### Theorem 9

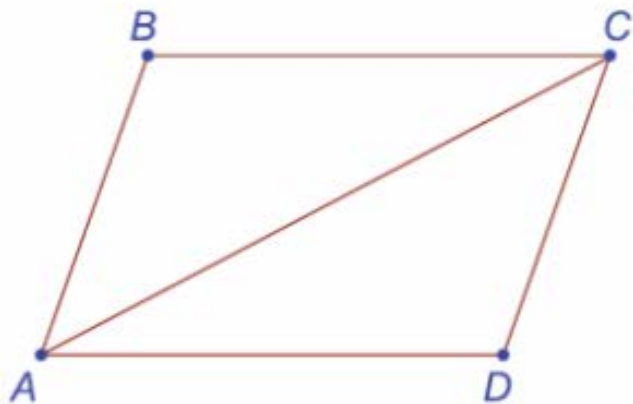
In a parallelogram, opposite sides are equal and opposite angles are equal.

**Given:** A parallelogram  $ABCD$ .

**To prove:**

- (i)  $|AB| = |CD|$  and  $|BC| = |AD|$  (opposite sides are equal)
- (ii)  $|\angle ABC| = |\angle ADC|$ ,  $|\angle BAD| = |\angle BCD|$  (opposite angles are equal)

**Construction:** Draw the diagonal  $[AC]$ .



**Proof:**

Statement	Reason
$ \angle BCA  =  \angle CAD $	Alternate
$ AC  =  AC $	Common (shared)
$ \angle BAC  =  \angle ACD $	Alternate
$\Rightarrow \triangle BAC \cong \triangle ADC$	ASA
$\Rightarrow  AB  =  CD $ and $ BC  =  AD $	Corresponding sides
Also, $ \angle ABC  =  \angle ADC $	Corresponding angle
Similarly, $ \angle BAD  =  \angle BCD $	
Q.E.D.	

### Theorem 14: Theorem of Pythagoras

In a right-angled triangle, the square of the hypotenuse is the sum of the squares of the other two sides.

**Given:** A right-angled triangle  $ABC$  with  $|\angle ABC| = 90^\circ$ .

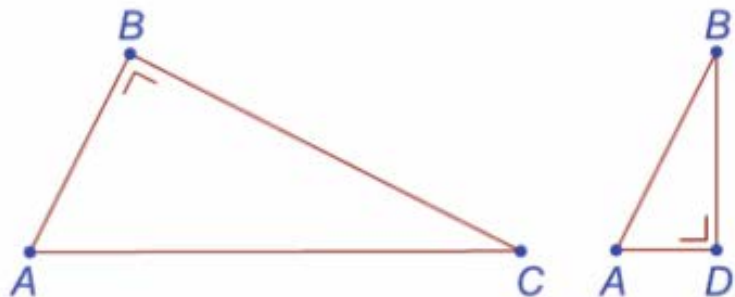
**To prove:**  $|AC|^2 = |AB|^2 + |BC|^2$ .

**Construction:** Draw  $BD \perp AC$ .

**Proof:**

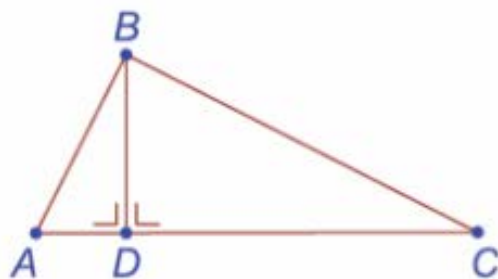
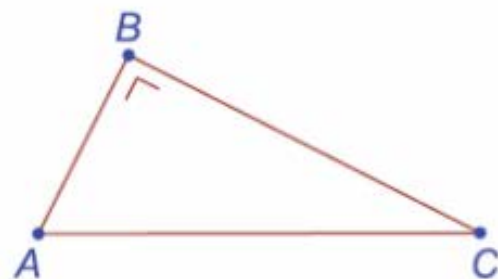
**Step 1**

Consider the triangles  $ABC$  and  $ADB$ .



$ \angle ABC  =  \angle ADB $	$90^\circ$
$ \angle BAC  =  \angle BAD $	Common

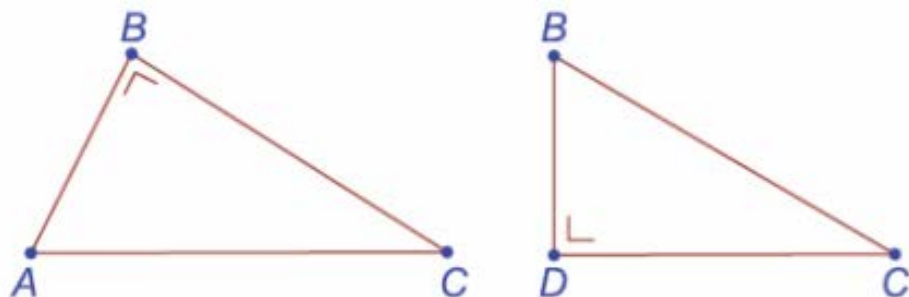
$\therefore \triangle ABC$  and  $\triangle ADB$  are similar.



Statement	Reason
$\triangle ABC$ and $\triangle ADB$ are similar.	Construction
$\Rightarrow \frac{ AC }{ AB } = \frac{ AB }{ AD }$	Theorem
$\Rightarrow  AB  \cdot  AB  =  AC  \cdot  AD $	
$\Rightarrow  AB ^2 =  AC  \cdot  AD $	

## Step 2

Consider the triangles  $ABC$  and  $BDC$ .



$ \angle ABC  =  \angle BDC $	$90^\circ$
$ \angle ACB  =  \angle DCB $	Common

$\therefore \Delta ABC$  and  $\Delta BDC$  are similar.

## Step 3

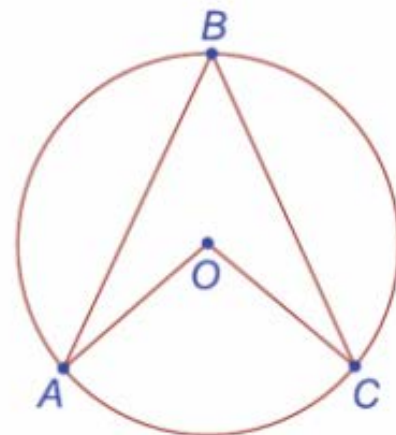
$ AB ^2 +  BC ^2 =  AC  \cdot  AD  +  AC  \cdot  DC $
$=  AC  \cdot ( AD  +  DC )$
$\Rightarrow  AB ^2 +  BC ^2 =  AC  \cdot  AC $ (Since $ AD  +  DC  =  AC $ )
$ AB ^2 +  BC ^2 =  AC ^2$
Q.E.D.

Statement	Reason
$\Delta ABC$ and $\Delta BDC$ are similar.	Construction
$\Rightarrow \frac{ AC }{ BC } = \frac{ BC }{ DC }$	Theorem
$\Rightarrow  BC  \cdot  BC  =  AC  \cdot  DC $	
$\Rightarrow  BC ^2 =  AC  \cdot  DC $	



### Theorem 19

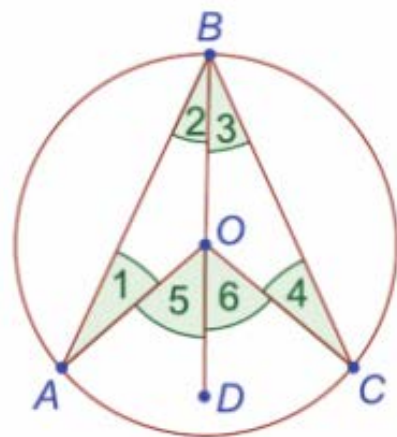
The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.



**Given:** A circle with centre  $O$  and an arc  $AC$ .  
A point  $B$  on the circle.

**To prove:**  $|\angle AOC| = 2|\angle ABC|$ .

**Construction:** Join  $B$  to  $O$  and continue to a point  $D$ . Label angles 1, 2, 3, 4, 5 and 6.



$$|\angle AOC| = |\angle 5| + |\angle 6|$$

$$|\angle ABC| = |\angle 2| + |\angle 3|$$

**Proof:**

Statement	Reason
$ OA  =  OB $	Radii
$ \angle 1  =  \angle 2 $	Isosceles triangle
$ \angle 5  =  \angle 1  +  \angle 2 $	Exterior angle
$\Rightarrow  \angle 5  = 2 \angle 2 $	Since $ \angle 1  =  \angle 2 $
Similarly, $ \angle 6  = 2 \angle 3 $	
$ \angle 5  +  \angle 6  = 2 \angle 2  + 2 \angle 3 $	
$\Rightarrow  \angle 5  +  \angle 6  = 2( \angle 2  +  \angle 3 )$	
$ \angle AOC  = 2 \angle ABC $	
Q.E.D.	