Math Formulas: Complex numbers

Definitions:
A complex number is written as $a + bi$ where $a$ and $b$ are real numbers and $i$, called the imaginary unit, has the property that $i^2 = -1$.

The complex numbers $z = a + bi$ and $\overline{z} = a - bi$ are called complex conjugate of each other.

Formulas:

Equality of complex numbers
1. $a + bi = c + di \iff a = c$ and $b = d$

Addition of complex numbers
2. $(a + bi) + (c + di) = (a + c) + (b + d)i$

Subtraction of complex numbers
3. $(a + bi) - (c + di) = (a - c) + (b - d)i$

Multiplication of complex numbers
4. $(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$

Division of complex numbers
5. \[
\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i
\]

Polar form of complex numbers
6. $a + bi = r \cdot (\cos \theta + i \sin \theta)$

Multiplication and division of complex numbers in polar form
7. $[r_1 (\cos \theta_1 + i \sin \theta_1)] \cdot [r_2 (\cos \theta_2 + i \sin \theta_2)] = r_1 \cdot r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)]$
8. \[
\frac{r_1 (\cos \theta_1 + i \sin \theta_1)}{r_2 (\cos \theta_2 + i \sin \theta_2)} = \frac{r_1}{r_2} [\cos (\theta_1 - \theta_2) + i \sin (\theta_1 - \theta_2)]
\]

De Moivre’s theorem
9. $[r (\cos \theta + i \sin \theta)]^n = r^n (\cos (n\theta) + i \sin (n\theta))$

Roots of complex numbers
10. \[
[r (\cos \theta + i \sin \theta)]^{1/n} = r^{1/n} \left( \cos \left( \frac{\theta + 2k\pi}{n} \right) + i \sin \left( \frac{\theta + 2k\pi}{n} \right) \right) \quad k = 0, 1, \ldots, n - 1
\]