Teaching Math in the Primary Grades

The Learning Trajectories Approach

Julie Sarama and Douglas H. Clements

Two kindergarten teachers sit down for lunch during a professional development workshop. One says, "I think it's ridiculous. The children are still babies. They're trying to teach them too much." Her friend nods. Soon they are joined by a colleague from another school, who bubbles, "Isn't this great? The children are going to know so much more!"

Most of us can sympathize with both perspectives. What *should* we be teaching in the early grades? Three research findings provide some guidance in mathematics instruction.

1. Learning substantial math is critical for primary grade children.

The early years are especially important for math development. Children's knowledge of math in these years predicts their math achievement for later years—and throughout their school career. Furthermore, what they know in math predicts their later reading achievement as well (Duncan et al. in press). Given that early math learning predicts later math and reading achievement, math appears to be a core component of learning and thinking.

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2. All children have the potential to learn challenging and interesting math.

Primary grade children have an often surprising ability to do abstract math—that is, math that is done by reasoning mentally, without the need for concrete objects. Listen to the worries of this first-grader.

> "I find it easier not to do it [simple addition] with my fingers because sometimes I get into a big muddle with them [and] I find it much harder to add up because I am not concentrating on the sum. I am concentrating on getting my fingers right . . . It can take longer to work out the sum [with fingers] than it does to work out the sum in my head." [In her head, Emily imagined dot arrays. Why didn't she just use those?] "If we don't use our fingers, the teacher is going to think, 'Why aren't they using their fingers . . . they are just sitting there thinking' . . . We are meant to be using our fingers because it is easier . . . which it is not." (Gray & Pitta 1997, 35)

Should the teacher encourage Emily to use concrete objects to solve math problems? Or should she encourage children like Emily to use arithmetic reasoning?

Primary grade children often know, and can definitely learn, far more challenging and interesting math than they are taught in most U.S. classrooms. That does not necessarily mean math pushed down from higher grades. It means letting children invent their own strategies for solving a variety of types of problems. How can teachers best support creative thinking in mathematics?

3. Understanding children's mathematical development helps teachers be knowledgeable and effective in teaching math.

Children's thinking follows natural developmental paths in learning math. When teachers understand these paths and offer activities based on children's progress along them, they build math learning environments that are developmentally appropriate and particularly effective. A useful tool in understanding and supporting the development of children's mathematical reasoning is a *math learning trajectory.* There are learning trajectories for mathematics at all age levels, from birth throughout the school years, and for learning all kinds of content—from specific math concepts such as number and operations to specific science concepts like understanding electricity.

Learning trajectories

Math learning trajectories have three parts: a mathematical **goal**, a **developmental path** along which children's math knowledge grows to reach that goal, and a set of **instructional tasks**, or activities, for each level of children's understanding along that path to help them become proficient in that level before moving on to the next level. Let's examine each of these three parts.

Goal. The first part of a learning trajectory is the goal. Goals should include the big ideas of math, such as "numbers can be used to tell us how many, describe order, and measure" and "geometry can be used to understand and to represent the objects, directions, and locations in our world, and the relationship between them" (Clements, Sarama, & DiBiase 2004). In this article, we look at the goal of knowing how to solve a variety of addition and subtraction problems.

Developmental path. The second part of a learning trajectory consists of levels of thinking, each more sophisticated than the last, leading to achieving the mathematical goal. That is, the developmental path describes a typical learning route children follow in developing understanding of and skill in a particular mathematics topic.

Learning trajectories are important because young children's ideas and their interpretations of situations are different from those of adults. Teachers must interpret what the child is doing and thinking and attempt to see the situation from the child's viewpoint. Knowledge of developmental paths enhances teachers' understanding of children's thinking, helping teachers assess children's level of understanding and offer instructional activities at that level. Similarly, effective teachers consider the instructional tasks from the child's perspective.

Instructional tasks. The third part of a learning trajectory consists of sets of instructional tasks or activities matched to each level of thinking in a developmental progression. The tasks are designed to help children learn the ideas and practice the skills needed to master that level. Teachers use instructional tasks to promote children's growth from one level to the next

The National Association of Early Childhood Specialists in State Departments of Education (NAECS/SDE) works to improve instruction, curriculum, and administration in education programs for young children and their families. Of Primary Interest is written by members of NAECS/SDE for kindergarten and primary teachers. The column appears in March, July, and November issues of *Young Children* and Beyond the Journal (online at **www.journal.naeyc.org/btj**).

Teaching challenging and interesting math

The three research findings—the importance of math learning in the primary grades, all children's potential to learn math, and teachers' need to understand children's learning development—have implications for teaching primary grade math well. We suggest the following approach:

- Know and use learning trajectories.
- Include a wide variety of instructional activities. The learning trajectories provide a guide as to which activities are likely to challenge children to invent new strategies and build new knowledge.
- Use a combination of teaching strategies. One effective approach is to (a) discuss a problem with a group, (b) follow up by having children work in pairs, and then (c) have the children share solution strategies back with the group. Discuss strategies with children in pairs and individually. Differentiate instruction by giving groups or individual children different problem types.

Alexander and Entwisle state that "the early grades may be precisely the time that schools have their strongest effects" (1988, 114). Math is so important to children's success in school, in the primary grades and in future learning, that it is critical to give children motivating, substantive educational experiences. Learning trajectories are a powerful tool to engage all children in creating and understanding math.

References

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Learning Trajectory for Addition and Subtraction: Sample Levels of the Developmental Path and Examples of Instructional Tasks

This chart gives simple labels and a sampling of levels in the developmental learning progressions for ages 5 through 7 years. The ages in the first column are not exact indications—children in challenging educational environments often create strategies that are surprisingly sophisticated. The second column describes four main levels of thinking in the addition and subtraction learning trajectory. These levels are samples—there are many levels in between them (for full learning trajectories, see Clements & Sarama 2009 and Sarama & Clements 2009). The third column briefly describes examples of instructional tasks.

Age	Developmental path—Sample levels	Instructional tasks
5	**Find Change.** Children find the missing addend (5 + _ = 7) by adding on objects.	**Word Problems.** For example, say to the children, "You have 5 balls and then get some more. Now you have 7 balls in all. How many more balls did you get?" Children use balls in 2 colors to solve such problems.
5½	**Counting Strategies.** Children find sums for joining problems ("You have 8 apples and get 3 more . . .") and part-part-whole problems ("6 girls and 5 boys . . .") with finger patterns [*counting using fingers and quickly recognizing the quantity*] and/or by counting on. **Counting On.** The teacher asks, "How much is 4 and 3 more?" A child replies, "4 . . . 5, 6, 7 [*uses a rhythmic or finger pattern to keep track*]. 7!" **Counting Up.** A child may solve a missing addend (3 + _ = 7) or compare problems by counting up; for example, the child counts "4, 5, 6, 7" while putting up fingers, and then counts or recognizes the 4 fingers raised. Or the teacher asks, "You have 6 balls. How many more do you need to have 8 balls?" The child says, "6, 7 [*puts up a finger*], 8 [*puts up a second finger*]. 2!"	**How Many Now? Problems.** For example, have the children count objects as you place them in a box. Ask, "How many are in the box now?" Add 1, repeating the question, then check the children's responses by counting all the objects. Repeat, checking occasionally. When children are ready, sometimes add 2, and eventually more, objects.  **Double Compare.** Children compare sums of 2 cards to determine which sum is greater. Encourage the children to use more sophisticated strategies, such as counting on.  **Bright Idea.** Using a numeral and a frame with dots, children count on from the numeral to identify the total amount. They then move forward a corresponding number of spaces on a game board.
6	**Part-Whole.** The child has an initial part-whole understanding and can solve all the preceding problem types using flexible strategies (may use some known combinations, such as "5 + 5 is 10").	**Hidden Objects.** Hide 4 counters under a dark cloth and show children 7 counters. Tell them that 4 counters are hidden and challenge them to tell you how many counters there are in all. Or tell the children there are 11 counters in all, and ask how many are hidden. Have the children discuss their solution strategies. Repeat with different sums.  **Barkley's Bones.** Children determine the missing addend in problems such as 4 + _ = 7.
7	**Deriver.** The child uses flexible strategies and derived combinations ("7 + 7 is 14, so 7 + 8 is 15") to solve all types of problems.	**Twenty-one.** Play this card game, whose object is to have the sum of one's cards be 21 or as close as possible without exceeding 21. An ace is worth either 1 or 11, and cards for 2 through 10 are worth their face value. A child deals everyone 2 cards, including herself. • In each round, if a player's sum is less than 21, the player can request another card or stand pat, saying, "Hold." • If the new card makes the player's sum greater than 21, the player is out. • Play continues until everyone holds. The player whose sum is closest to 21 wins. **Multidigit Addition and Subtraction.** "What's 28 + 35?"

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