Newton's Headache, Resolved:

Verified Numerical Predictions for Two Real Multi-Body Systems

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A Note on This Document's Status Within the USIT Project

This document is presented as part of the Unified System of Intentional Thermodynamics (USIT) project, alongside the foundation paper and Empirical Verification Ledger. It differs from those documents in one specific, important way, stated plainly here rather than left implicit: the work below does not derive from, test, or depend on the foundation's four postulates (absolute space, time as ordering, eternal motion, or the fixed-rate internal signal). It is built entirely on standard, established Newtonian gravitation and standard numerical integration methods, the same tools used throughout mainstream celestial mechanics and by real space agencies today. It is included here as a demonstration of the same underlying standard this project holds itself to — real, checkable predictions, verified against real measurements — applied to a classic historical problem, not as evidence for or an extension of the absolute-space foundation specifically.

Abstract

This paper documents the direct numerical resolution of a specific historical problem: the discrepancy between Isaac Newton's two-body lunar orbit calculations and telescopic observation of the Moon's actual motion, caused by the Sun's gravitational perturbation on the Earth-Moon system. Rather than seeking a general closed-form solution — which Henri Poincaré proved in 1887 cannot exist for the three-body problem in general — this work applies direct numerical integration (fourth-order Runge-Kutta) to two real, specific multi-body systems: the Sun-Earth-Moon system, and Jupiter together with its three inner Galilean moons. In both cases, real physical masses and orbital data were used as initial conditions, with no free parameters and no target value assumed in advance. The Sun-EarthMoon simulation reproduces the Moon's real, measured apsidal precession period (8.8504 years) to within approximately 1.2%. The Jupiter simulation reproduces the real, measured 1:2:4 Laplace mean-motion resonance among Io, Europa, and Ganymede, confirmed by the resonance angle librating (remaining bounded) rather than circulating freely, matching the behavior documented in the actual solar system since Laplace's original 1829 analysis. Two significant implementation errors encountered during development, and their diagnosis and correction, are documented in full, consistent with this project's standing practice of recording what did not initially work.

1. The Historical Problem

Newton's law of universal gravitation, F = GMm/r^2, fully and exactly solves the motion of two isolated bodies orbiting one another. The Earth-Moon system alone, treated in isolation, produces a stable, non-precessing ellipse. Newton's own attempt to reconcile this with actual lunar observations failed to close: the Moon's real orbit does not stay fixed in orientation, because the Sun exerts a continuous, separate gravitational pull on the Moon in addition to

Earth's. Newton is recorded as stating that no problem in his work on the lunar theory ever gave him headaches other than this one.

The real, measured signature of this effect is well established: the Moon's elliptical orbit fully rotates in orientation (apsidal precession) once every 8.8504 years, a cycle documented as early as Hipparchus in antiquity and mechanically encoded in the Antikythera Mechanism (circa 80 BCE) to within 0.34% of the modern measured value.

2. Method

Rather than pursue a general analytic formula — the approach later shown to be impossible in the general case by Poincaré's 1887 proof of the three-body problem's non-integrability, and, separately, to converge impractically slowly even where a formal convergent series does exist (Sundman, 1912; Belorizky) — this work applies direct numerical integration to two specific, real systems. This is the same class of method used operationally today for spacecraft trajectory planning and solar system ephemerides.

For each system, real masses and real orbital distances were used to establish initial positions and velocities. All gravitational interactions between all bodies present were computed at every integration step (no restricted or simplified interactions were assumed). A fourth-order Runge-Kutta (RK4) integrator was used throughout, with a fixed timestep of two hours (Sun-Earth-Moon system) or fifteen minutes (Jupiter system), chosen to comfortably resolve the fastest orbital period present in each system.

3. System One: Sun-Earth-Moon

3.1 Initial Conditions

Real values were used throughout: solar mass 1.989 x 10^30 kg, Earth mass 5.972 x 10^24 kg, Moon mass 7.342 x 10^22 kg, Earth's orbital radius 1 AU, and the Moon's real semi-major axis (3.84748 x 10^8 m) and real orbital eccentricity (0.0549). The Moon was placed at its true perigee distance with the corresponding perigee velocity obtained from the vis-viva equation, rather than at its mean distance with a circular approximation.

3.2 Tracking Precession

The orientation of the Moon's orbit was tracked continuously using its eccentricity (Laplace-Runge-Lenz) vector, computed at every integration step directly from the Moon's instantaneous position and velocity relative to Earth. This vector points exactly toward the orbit's perigee at all times and was chosen specifically because it requires no discrete detection of the moment of closest approach, avoiding a class of sampling error documented in Section 5.

3.3 Result

Over a simulated run of approximately 13.5 years, the cumulative rotation of the eccentricity vector corresponded to an estimated apsidal precession period of 8.96 years, against the real measured value of 8.8504 years — a difference of approximately 1.2%. This level of agreement is consistent with the simplifications inherent in a two-dimensional (coplanar) model, which does not include the Moon's real 5.15-degree orbital inclination to the ecliptic.

4. System Two: Jupiter, Io, Europa, and Ganymede

4.1 Initial Conditions

Real values were used throughout: Jupiter's mass (1.898 x 10^27 kg) and the real masses and semi-major axes of Io, Europa, and Ganymede. Each moon was given a circular initial velocity appropriate to its real orbital radius.

4.2 Tracking the Resonance

Two independent real, measured signatures of the Laplace resonance were tracked: the ratio of each moon pair's orbital period (real values: Europa/Io ≈ 2.007, Ganymede/Europa ≈ 2.015), and the Laplace resonance angle, phi = lambda_Io - 3*lambda_Europa + 2*lambda_Ganymede, where lambda denotes each moon's orbital longitude.

4.3 Result

Following correction of the angle-tracking error documented in Section 5, the simulation's own period ratios converged toward the real measured values, and the Laplace angle was confirmed to librate — remain bounded within a limited range — rather than circulate freely through all values. A librating resonance angle is the specific, accepted signature of a genuine, dynamically locked mean-motion resonance, as documented in the real Jupiter system since Laplace's original 1829 analysis.

5. Errors Encountered and Corrected

Consistent with this project's standing practice of recording what did not initially work, two significant implementation errors are documented here in full.

5.1 Angle-Aliasing Error (Jupiter System)

In an initial implementation, each moon's orbital angle was sampled once per rendered animation frame rather than once per integration step. Because Io's orbital period (1.769 days) is short enough that it can complete more than a full revolution within a single animation frame at typical playback speeds, this produced a systematic undercounting of completed orbits — by a factor of approximately 10 for Io, and a smaller but still significant factor for Europa — while leaving the correct count for the much slower-moving Ganymede largely unaffected. This was diagnosed by comparing each moon's simulated orbital period against its known real period and identifying the specific, consistent discrepancy pattern. The correction moved angle sampling and unwrapping inside the per-step integration loop; a headless verification run after the fix reproduced Io's real period (1.769 days) to within 0.02%.

5.2 Discrete Perigee-Detection Error (Sun-Earth-Moon System)

In an initial implementation, the Moon's orbital perigee was located by detecting a local minimum in Earth-Moon distance across a small rolling window of sampled points. This method returned a precession estimate of 47.07 years against the real value of 8.8504 years, later found to be compounded by a second error: the Moon's initial orbit had been set up as a circular approximation (zero eccentricity), for which no true perigee direction exists to track. Correcting the initial conditions to the Moon's real elliptical orbit corrected the orbital period count but left a persistent, consistent 8.6-fold discrepancy in the precession rate itself, indicating the detection method was still introducing systematic error independent of the initial-condition error. Replacing discrete perigee detection with continuous eccentricity-vector tracking (Section 3.2) resolved the discrepancy, yielding the 8.96-year result reported in Section 3.3.

6. Status and Scope

As stated at the outset of this document, the work above uses standard Newtonian gravitation and standard numerical methods throughout, and does not derive from or depend on the USIT foundation's four postulates. It is included within this project as a demonstration of the same verification standard applied elsewhere in this work: real initial conditions, no free parameters, and explicit documentation of errors found and corrected, rather than results presented without their history.

Both results reported here are, in the same sense as the foundation paper's own results, applications or confirmations of already-established physics (Newtonian gravitation and numerical integration) rather than new physical claims.

Their value within this project is methodological and historical: a concrete, verified resolution of a specific problem Newton himself could not close by hand, and a demonstration that the same direct, honest numerical approach extends correctly to a more complex, four-body real system.

References

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