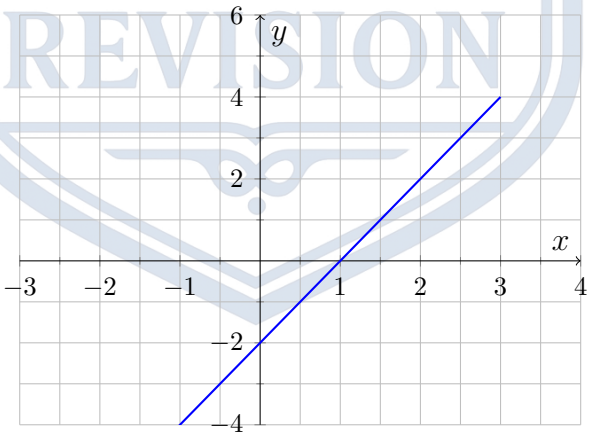
**Topic 2: Graphing Linear, Quadratic, Exponential & Logarithmic Functions**
**IB Math AI SL**

*Answer all questions. Show all working where appropriate. Use your Graphic Display Calculator (GDC) to find roots, intersections, and extrema where necessary. Total: 114 marks.*

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
   The graph of a linear function $L_1$ is shown below.

   (a) Write down the $y$-intercept and the $x$-intercept of $L_1$.

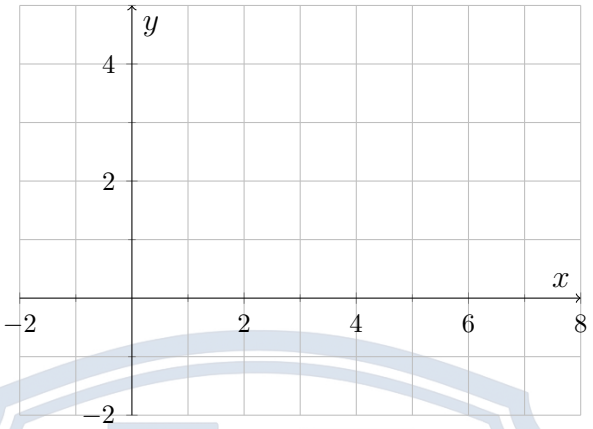
   (b) Find the gradient of $L_1$ and hence write down its equation in the form $y = mx + c$.



2. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
Consider the linear function $f(x) = -0.5x + 3$.

(a) Find the coordinates of the axes intercepts.

(b) Sketch the graph of $f(x)$ on the grid provided below.

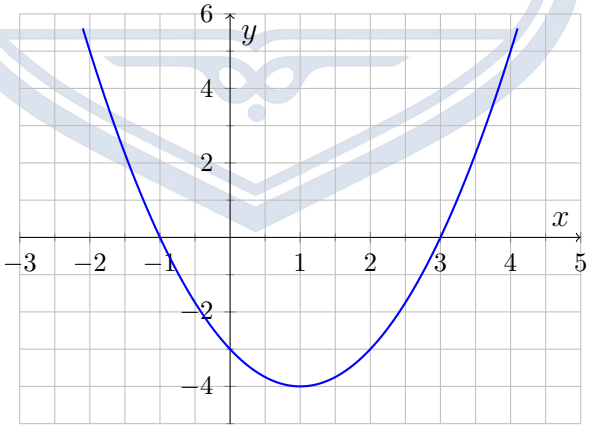



3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
The graph of a quadratic function $y = x^2 - 2x - 3$ is shown below.

(a) Write down the coordinates of the $x$-intercepts.

(b) Write down the equation of the axis of symmetry.

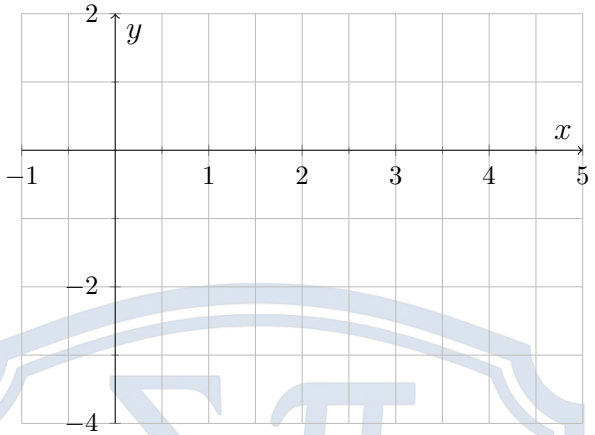
(c) Find the exact coordinates of the vertex.



4. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
Consider the quadratic function $g(x) = -x^2 + 4x - 3$.

   (a) Using your GDC or algebraic factoring, find the roots of $g(x) = 0$.

   (b) Sketch the graph of $g(x)$ on the grid below, clearly marking the vertex and intercepts.

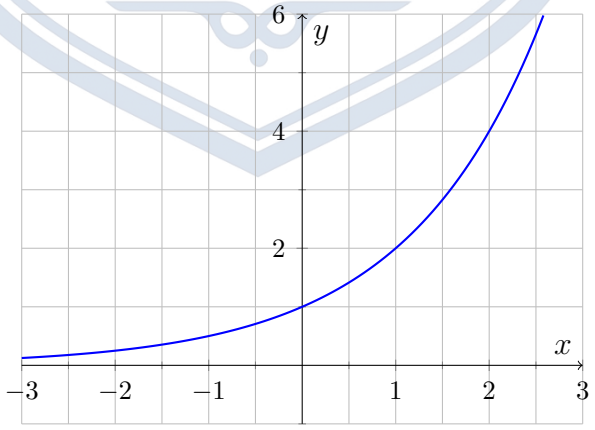



5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The graph of the exponential function $y = 2^x$ is shown below.

   (a) Write down the $y$-intercept.

   (b) Write down the equation of the horizontal asymptote.

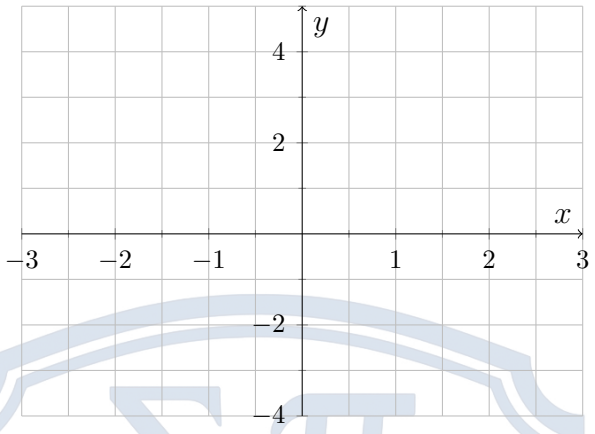
   (c) State the domain and range of the function.



6. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Consider the exponential function $h(x) = e^x - 3$.

   (a) Write down the equation of the horizontal asymptote.

   (b) Sketch the graph of $h(x)$ on the grid below, showing the asymptote and axes intercepts.

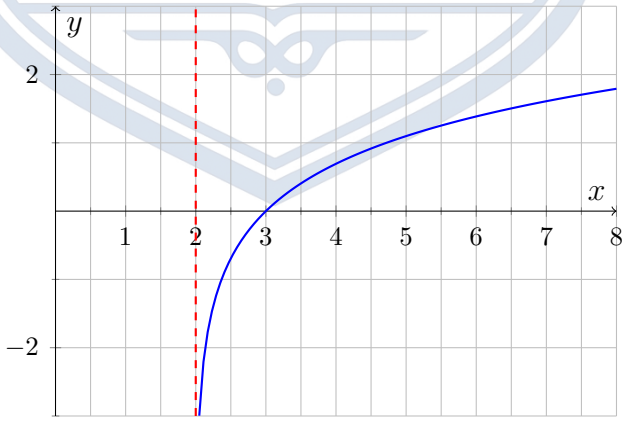



7. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The graph of a natural logarithmic function $y = \ln(x - 2)$ is shown below.

   (a) Write down the equation of the vertical asymptote.

   (b) Write down the exact coordinate of the $x$-intercept.

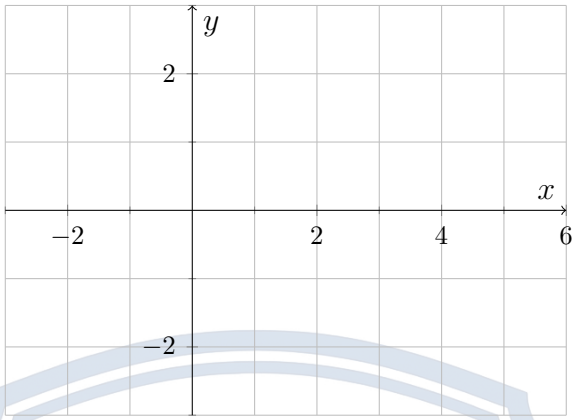
   (c) State the domain of the function.



8. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
Consider the logarithmic function $f(x) = \log_2(x+2)$.

(a) Find the exact values of the $x$-intercept and $y$-intercept.

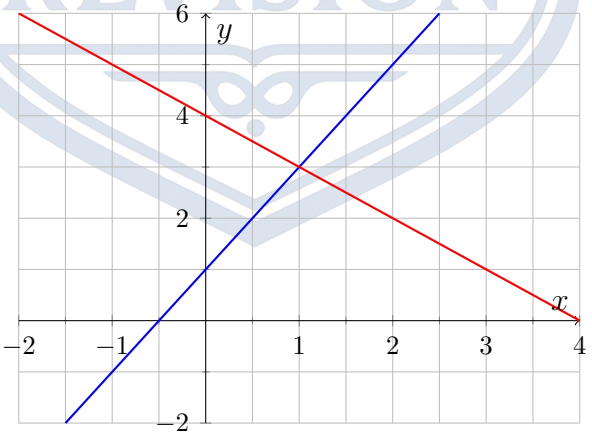
(b) Sketch the graph of $f(x)$ on the grid below, including the vertical asymptote.




9. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The graphs of two linear functions, $L_1 : y = 2x + 1$ and $L_2 : y = -x + 4$, are shown below.

(a) Using the graph, write down the coordinates of their point of intersection.

(b) Verify your answer algebraically or by using the intersection tool on your GDC.

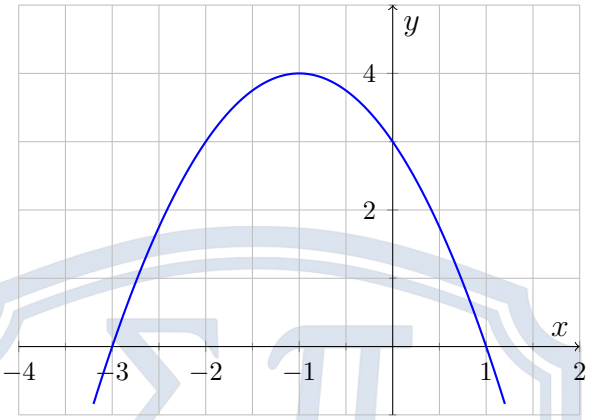


10. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
The graph of a downward-opening parabola is shown below. Its vertex is at $(-1, 4)$ and it passes through the $y$-axis at $(0, 3)$.

(a) Write the equation of the parabola in the form $y = a(x - h)^2 + k$.

(b) Find the value of $a$.

(c) Find the exact $x$-intercepts.

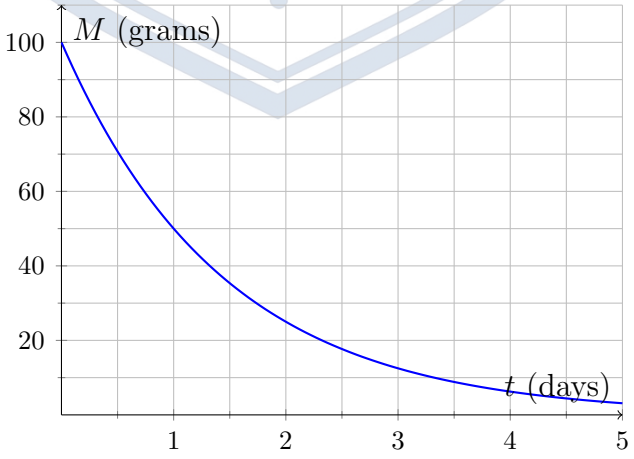



11. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
The decay of a radioactive substance is shown in the graph below. The mass $M$ in grams remaining after $t$ days is modelled by $M(t) = A(0.5)^t$.

(a) Use the $y$-intercept to find the value of $A$.

(b) Using the graph or otherwise, find the mass remaining after 2 days.

(c) Using your GDC, find the exact time it takes for the mass to drop to 10 grams.

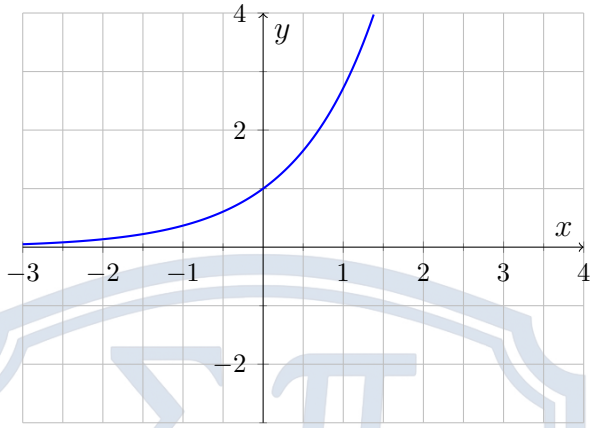


12. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
Consider the exponential function $f(x) = e^x$. The graph of $f(x)$ is drawn below.

(a) On the same grid, sketch the line $y = x$.

(b) Sketch the inverse function, $f^{-1}(x)$, on the same grid.

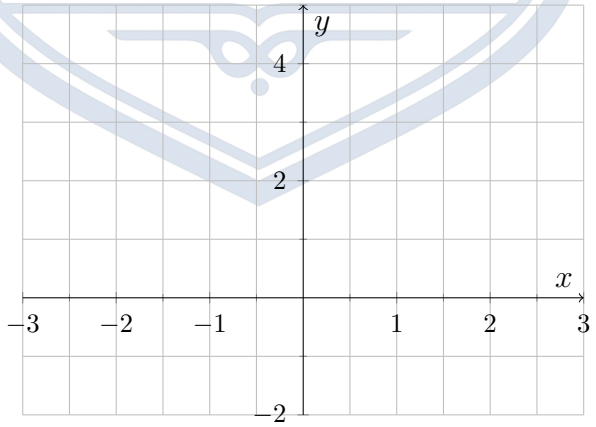
(c) Write down the mathematical equation of the inverse function.




13. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
Consider the quadratic function $y = x^2 - 1$ and the linear function $y = x + 1$.

(a) Sketch both graphs on the grid provided below.

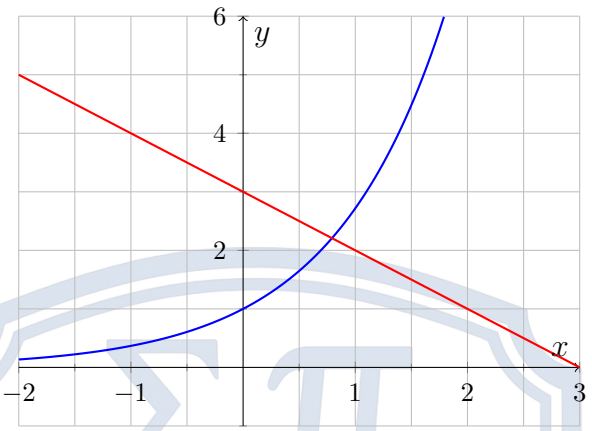
(b) Using your GDC's intersection tool, find the exact coordinates of the points where the two graphs intersect.



14. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
The graphs of $y = e^x$ and $y = 3 - x$ are plotted on the same axes below.

(a) Using the intersection feature on your GDC, find the coordinates of the point of intersection, correct to 3 significant figures.

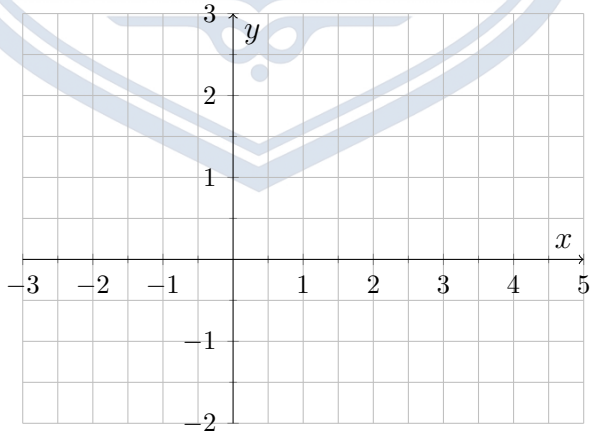
(b) Briefly explain why an algebraic method (without a GDC) cannot easily solve the equation $e^x = 3 - x$.




15. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
Consider the logarithmic function $f(x) = \ln(x + 2)$ and the horizontal line $y = 1$.

(a) Sketch both graphs on the grid provided below.

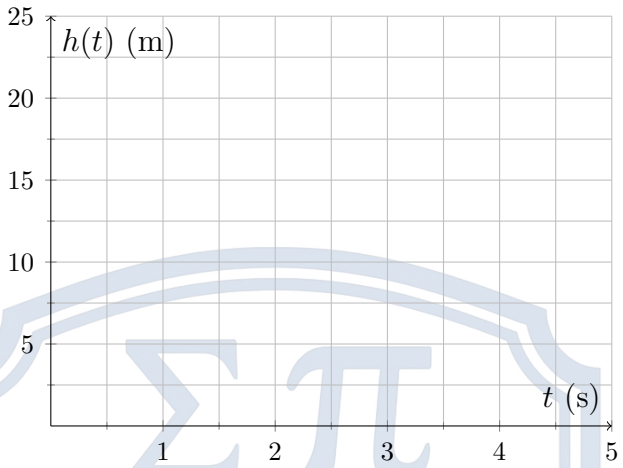
(b) Find the exact $x$-coordinate where the two graphs intersect. Give your answer in terms of $e$.



16. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
A projectile is fired upwards. Its height, $h$, in metres after $t$ seconds is modelled by the quadratic function $h(t) = -5t^2 + 20t + 2$.

(a) Sketch the path of the projectile on the grid provided.

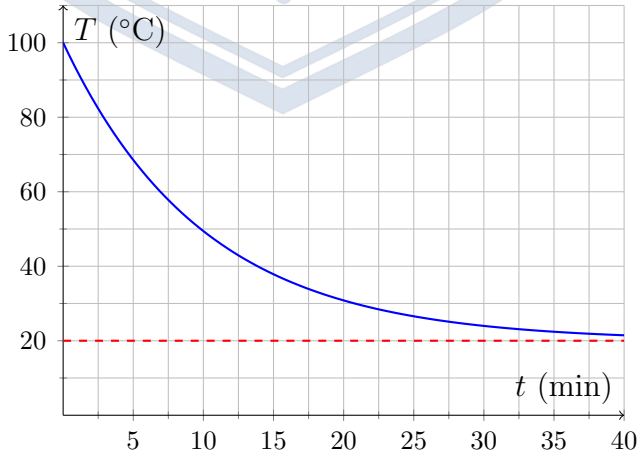
(b) Use your GDC's maximum finding tool to find the maximum height reached by the projectile and the time it takes to reach it.




17. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
A hot cup of tea cools down over time. Its temperature $T$ (in °C) after $t$ minutes is modelled by $T(t) = 20 + 80e^{-0.1t}$. The graph is shown below.

(a) Write down the initial temperature of the tea.

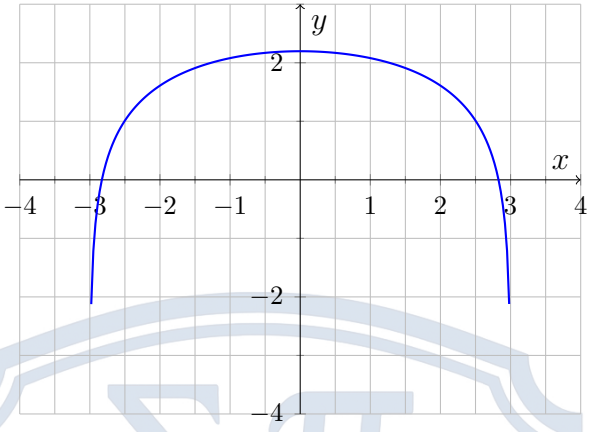
(b) Write down the equation of the horizontal asymptote and explain its physical meaning in the context of the room temperature.



18. **[Paper 1 Style, Short Answer, Very Hard, 5 marks]**
The graph of a composite logarithmic function $y = \ln(9 - x^2)$ is shown below.

(a) Find the exact $x$-intercepts.

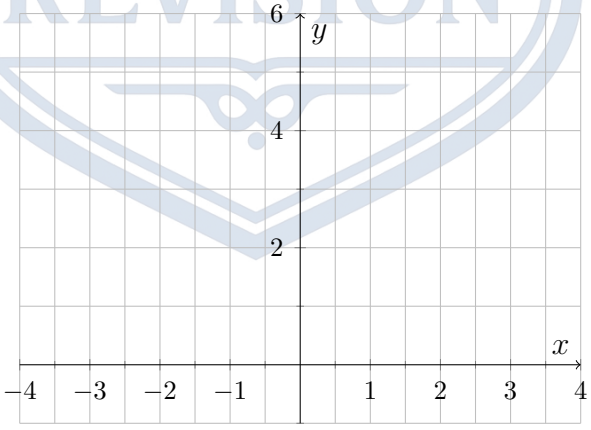
(b) State the domain of this function, explaining how the vertical asymptotes define this boundary.




19. **[Paper 1 Style, Short Answer, Very Hard, 6 marks]**
Consider the absolute value quadratic function $f(x) = |x^2 - 4|$.

(a) Sketch the graph of $f(x)$ on the grid below.

(b) Find the coordinates of the three turning points (extrema) of this graph.



20. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
The value of Car A (in thousands of dollars) is modelled by an exponential decay function $V_A(t) = 20(0.8)^t$. The value of an Antique Car B (in thousands of dollars) is modelled by an exponential growth function $V_B(t) = 10(1.05)^t$. The graphs are shown below.

(a) Determine which graph (the solid blue line or the dashed red line) represents Car A.

(b) Using your GDC, find the exact time $t$ when both cars are worth the exact same amount, and state this value.