Topic 1: Worked Solutions & Mark Scheme

Marks are awarded for Method (M), Accuracy (A), and Reasoning (R). (M1) or (A1) indicates an implied mark.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**

   (a) 0.0451. **A1**

   (b) 0.045. **A1**

   (c) 4.51×10^{-2}. **A1A1**

2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

   (a) $PE = \left|\frac{8.2-8.5}{8.5}\right| \times 100\%$. **(M1)(A1)**
   $= \left|\frac{-0.3}{8.5}\right| \times 100\% = 3.53\%$ $(3.52941\ldots\%)$. **A1**

   (b) It is an underestimate. **A1**
   Because $8.2 < 8.5$ (the measured value is lower than the exact value). **R1**

3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**

   (a) Half of 0.1 is 0.05.
   Lower bound: 6.35 cm. Upper bound: 6.45 cm. **A1A1**

   (b) Max perimeter uses the upper bound. **(M1)**
   Max Perimeter $= 4 \times 6.45$. **(A1)**
   $= 25.8$ cm. **A1**

4. **[Paper 1 Style, Short Answer, Medium, 6 marks]**

   (a) $T = 4(3.32 \times 10^{-5})(5.67 \times 10^{10})$. **(M1)**
   $T = 7529760$. **A1**

   (b) 7.52976×10^{6}. **A1A1**

   (c) $PE = \left|\frac{7500000-7529760}{7529760}\right| \times 100\%$. **(M1)**
   $= 0.395\%$ $(0.395231\ldots\%)$. **A1**

5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**

(a) $6.89 \to 7$, $21.3 \to 20$, $0.485 \to 0.5$. **(A1)**
$P \approx \frac{7 \times 20}{0.5} = \frac{140}{0.5} = 280$. **A1**

(b) $P_{exact} = 302.591\ldots$ **A1**

(c) $PE = \left|\frac{280 - 302.591\ldots}{302.591\ldots}\right| \times 100\%$. **(M1)**
$= 7.47\%$ $(7.4661\ldots\%)$. **A1**

6. **[Paper 1 Style, Short Answer, Medium, 6 marks]**

(a) Half of 5 is 2.5.
LB of $L = 120 - 2.5 = 117.5$ m. **A1**
LB of $W = 85 - 2.5 = 82.5$ m. **A1**

(b) Min Area $= 117.5 \times 82.5$. **(M1)**
$= 9693.75$ m^2. **A1**

(c) 9690 (to 3 s.f.) $\implies 9.69 \times 10^3$ m^2. **A1A1**

7. **[Paper 1 Style, Short Answer, Medium, 6 marks]**

(a) Half of 10 is 5. Upper bound $d = 205$ m. **A1**

(b) Half of 1 is 0.5. Lower bound $t = 23.5$ s. **A1**

(c) Max speed relies on max distance and min time. **(M1)**
$v_{max} = \frac{205}{23.5}$. **(A1)**
$v_{max} = 8.72$ m/s $(8.7234\ldots)$. **A1**

8. **[Paper 1 Style, Short Answer, Hard, 6 marks]**

(a) $y = \frac{3(4.2)^2}{1.5} = \frac{3(17.64)}{1.5}$. **(M1)**
$y = 35.28$. **A1**

(b) $y \approx 35$. **A1**

(c) $PE = \left|\frac{35 - 35.28}{35.28}\right| \times 100\%$. **(M1)(A1)**
$= 0.794\%$ $(0.79365\ldots\%)$. **A1**

9. **[Paper 1 Style, Short Answer, Hard, 6 marks]**

(a) Lower bound $r = 6.45$ cm. **A1**

(b) $V_{min} = \frac{4}{3}\pi(6.45)^3$. **(M1)**
$V_{min} = 1124.004\ldots$ **(A1)**
Correct to 3 s.f.: 1120 cm^3. **A1**

(c) No, it is not possible. **A1**
The absolute minimum possible volume is $1124.004\ldots$ cm^3, so the exact volume cannot be as low as 1123 cm^3. **R1**

10. **[Paper 2 Style, Longer Question, Hard, 12 marks]**

(a) Half of 100 is 50.
Lower Bound = 4450 kg. Upper Bound = 4550 kg. **A1A1**

(b) Max 1 crate $= \frac{4550}{50} = 91$ kg. **(M1)A1**
Min 1 crate $= \frac{4450}{50} = 89$ kg. **A1**

(c) Worst case scenario means the crates weigh their maximum: 91 kg. **(M1)**
Number of crates $= \frac{2000}{91} = 21.978\ldots$ **(M1)(A1)**
Must round down to ensure safety: 21 crates. **A1**

(d) Estimated mass $= 21 \times 90 = 1890$ kg. **(A1)**
Exact mass $= 21 \times 89.5 = 1879.5$ kg. **(A1)**
$PE = \left|\frac{1890-1879.5}{1879.5}\right| \times 100\%$. **(M1)**
$= 0.559\%$ $(0.55865\ldots\%)$. **A1**

11. **[Paper 2 Style, Longer Question, Hard, 12 marks]**

(a) $t = \frac{d}{c} = \frac{1.496\times10^8}{3.00\times10^5}$. **(M1)**
$t = 498.666\ldots$ s. **(A1)**
$t = 4.99 \times 10^2$ s (or $4.98666\cdots \times 10^2$). **A1**

(b) $t_{mins} = \frac{498.666\ldots}{60}$. **(M1)**
$t_{mins} = 8.31$ minutes. **A1**

(c) Student $t = \frac{1.5\times10^8}{3\times10^5} = 500$ s. **(M1)(A1)**
$t_{mins} = \frac{500}{60} = 8.33$ minutes $(8.3333\ldots)$. **A1**

(d) Exact value is $8.3111\ldots$ and approximate is $8.3333\ldots$ **(M1)**
$PE = \left|\frac{8.3333\cdots-8.3111\ldots}{8.3111\ldots}\right| \times 100\%$. **(M1)(A1)**
$= 0.267\%$ $(0.26737\ldots\%)$. **A1**

12. **[Paper 1 Style, Short Answer, Very Hard, 6 marks]**

(a) Half of 0.1 is 0.05.
Lower Bound = 15.45 m. Upper Bound = 15.55 m. **A1A1**

(b) Min Length $= \frac{15.45}{4}$. **(M1)**
$= 3.8625$ m. **A1**

(c) The student incorrectly divided the measured value 15.5 by 4 instead of using the upper bound. **R1**
True max length $= \frac{15.55}{4} = 3.8875$ m. **A1**

13. [**Paper 2 Style, Longer Question, Very Hard, 12 marks**]

(a) $8.97 \times 10^{18} = 8970 \times 10^{15}$.
Value = 8970 quadrillion EUR. **(M1)A1**

(b) Nearest integer $\implies$ half is 0.5.
$LB = 112.5$ km, $UB = 113.5$ km. **A1A1**

(c) $V_{max} = \frac{4}{3}\pi(113.5)^3$. **(M1)**
$= 6128913\ldots$ km^3. **(A1)**
$= 6.13 \times 10^6$ km^3. **A1A1**

(d) Max value per km^3 occurs when volume is minimized (Lower Bound). **(M1)**
$V_{min} = \frac{4}{3}\pi(112.5)^3 = 5964115\ldots$ km^3. **(A1)**
Ratio $= \frac{8.97\times10^{18}}{5964115\ldots}$. **(M1)**
$= 1.50 \times 10^{12}$ EUR/km^3 $(1.50399\cdots \times 10^{12})$. **A1**

14. [**Paper 2 Style, Longer Question, Very Hard, 10 marks**]

(a) $\pi \approx 3 + \frac{1}{6.8125} = 3 + \frac{16}{109}$. **(M1)**
$= 3.146788\cdots \implies 3.1468$. **A1A1**

(b) $PE = \left|\frac{3.1468-\pi}{\pi}\right| \times 100\%$. **(M1)**
$= 0.166\%$ $(0.16575\ldots\%)$. **A1**

(c) Sophia $\approx 3.1666\ldots$ **(A1)**
$PE = \left|\frac{(19/6)-\pi}{\pi}\right| \times 100\%$. **(M1)**
$= 0.800\%$ $(0.7995\ldots\%)$. **A1**

(d) Katya has the more accurate approximation. **A1**
Her percentage error (0.166%) is lower than Sophia's (0.800%). **R1**

15. [**Paper 2 Style, Longer Question, Very Hard, 13 marks**]

(a) Mass: $42.45 \leq M < 42.55$. **A1A1**

(b) Volume: $4.75 \leq V < 4.85$. **A1A1**

(c) Max density $= \frac{\text{Upper Bound Mass}}{\text{Lower Bound Volume}}$. **(M1)**
$= \frac{42.55}{4.75}$. **(A1)**
$= 8.96$ kg/L $(8.95789\ldots)$. **A1**

(d) Min density $= \frac{\text{Lower Bound Mass}}{\text{Upper Bound Volume}}$. **(M1)**
$= \frac{42.45}{4.85}$. **(A1)**
$= 8.75$ kg/L $(8.75257\ldots)$. **A1**

(e) Engineer's estimate $= \frac{8.95789\cdots+8.75257\ldots}{2} = 8.85523\ldots$ **(M1)**
$PE = \left|\frac{8.85523\cdots-8.80}{8.80}\right| \times 100\%$. **(M1)**
$= 0.628\%$ $(0.62765\ldots\%)$. **A1**