IB MATHEMATICS AI HL
UNIT 5: CALCULUS

Advanced Integration & Kinematics

Instructions to Candidates

- This question booklet contains **15 questions**.
- The paper targets **AHL** syllabus components 5.11, 5.12, and Kinematics.
- Answer all questions, showing all step-by-step working clearly.

Difficulty Progression

- **Questions 1 - 5 (Easy):** Basic indefinite integrals, integration of e^x and $1/x$, simple kinematics ($v \to s$), and the trapezoidal rule.
- **Questions 6 - 10 (Medium):** Area bounded by the x-axis, integration by substitution, volumes of revolution, and applying boundary conditions.
- **Questions 11 - 15 (Hard):** Area between two intersecting curves, complex total distance travelled vs displacement, and kinematics from acceleration.

SECTION A: EASY (Fundamentals)

Question 1 (4 Marks)

Find the indefinite integral:

$$\int (4x^3 - 6x^2 + 5)\,dx$$

Question 2 (4 Marks)

Find the exact indefinite integral:

$$\int \left(e^{3x+1} + \frac{2}{x}\right) dx$$

Question 3 (4 Marks)

A particle moves in a straight line such that its velocity at time t seconds is given by $v(t) = 6t^2 - 4t$ ms^{-1}.
Given that the particle's initial displacement from the origin is 5 metres ($s(0) = 5$), find an expression for its displacement $s(t)$ at any time t.

Question 4 (4 Marks)

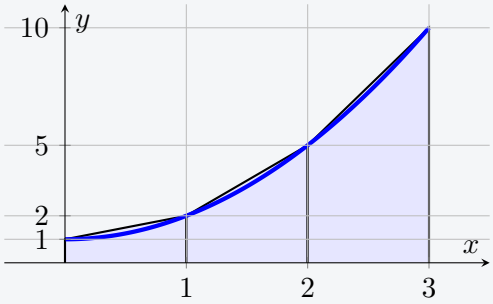
Evaluate the exact value of the definite integral:

$$\int_0^2 (3x^2)\,dx$$

CG50 Tip: The Trapezoidal Rule

When finding the area under a curve manually with the trapezoidal rule, use the `MENU 7 (Table)` function to instantly generate the sequence of y-values ($y_0, y_1, y_2 \dots$) instead of calculating each one one-by-one!

Question 5 (5 Marks)



Use the trapezoidal rule with exactly $n = 3$ intervals to approximate the area under the curve $y = x^2 + 1$ from $x = 0$ to $x = 3$.

SECTION B: MEDIUM (Application & Modelling)

Question 6 (5 Marks)

Find the exact area of the region totally bounded by the curve $y = 4x - x^2$ and the x-axis.

Question 7 (6 Marks)

Use the method of integration by substitution to find the indefinite integral:

$$\int 2x(x^2 + 1)^3 \, dx$$

CG50 Tip: Total Distance Travelled vs Displacement

Displacement is $\int v(t)dt$. However, **Total Distance** is $\int |v(t)|dt$. To evaluate this on your CG50 without manually splitting the integral at the roots, press `MATH` $\rightarrow$ `Int`, then inside the integral press `OPTN` $\rightarrow$ `NUM` $\rightarrow$ `Abs` to place the absolute value bars!

Question 8 (6 Marks)

A particle moves in a straight line with velocity $v(t) = t^2 - 3t + 2$ ms^{-1} for $t \geq 0$.
Calculate the exact total distance travelled by the particle in the first 4 seconds of motion.

Question 9 (6 Marks)

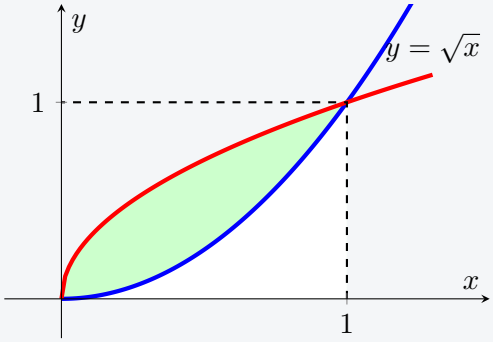
The region bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 4$ is rotated a full 360° (2π radians) about the x-axis.
Calculate the exact volume of the solid of revolution formed.

Question 10 (6 Marks)

The gradient function of a curve is given by $f'(x) = 2e^{2x} + 1$.
Given that the curve passes through the point $(0, 4)$, find the exact equation of the curve $f(x)$.

SECTION C: HARD (Synthesis & Proof)

Question 11 (7 Marks)



The diagram shows the curves $y = x^2$ and $y = \sqrt{x}$.
Determine the exact x-coordinates of their points of intersection, and hence calculate the exact area of the shaded region enclosed between the two curves.

Question 12 (8 Marks)

A particle moves in a straight line with an acceleration given by $a(t) = 3t^2 - 14t + 8$ ms^{-2} for $t \geq 0$.
Given that the initial velocity of the particle is 3 ms^{-1}, find an expression for its velocity $v(t)$, and determine the exact interval of time during which the velocity of the particle is strictly decreasing.

Question 13 (8 Marks)

Water flows into a large tank such that the rate of flow is given by $R(t) = 50e^{-0.05t}$ litres per minute, where t is the time in minutes.
Find the exact total volume of water that flows into the tank during the first 20 minutes. Give your final answer in the form $A - \frac{B}{e}$, where A and B are integers.

Question 14 (7 Marks)

The region bounded by the curve $y = e^x$, the x-axis, the y-axis, and the line $x = 2$ is rotated completely about the x-axis to form a solid.
Show algebraically that the exact volume of this solid is $\frac{\pi}{2}(e^4 - 1)$.

Question 15 (9 Marks)

A particle travels in a straight line such that its velocity is given by $v(t) = 3 - \frac{1}{2}\sqrt{t+1}$ ms^{-1} for $t \geq 0$.
(a) Find an exact expression for the acceleration of the particle, $a(t)$. **[3 marks]**

(b) Given that the particle's initial displacement is $s(0) = 16$ m, use integration to find an exact expression for its displacement $s(t)$ at any time t. **[6 marks]**