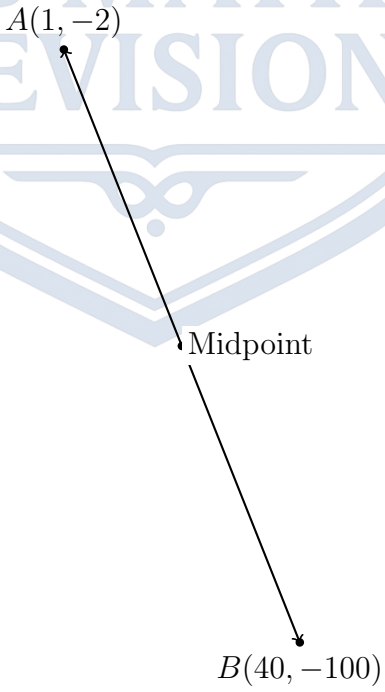
Topic: Coordinate Geometry & Voronoi Diagrams
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 114 marks.

1. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Consider the points $A(1, -2)$ and $B(40, -100)$.

   (a) Calculate the exact distance between points A and B.

   (b) Find the exact coordinates of the midpoint of the line segment [AB].



2. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
The points P and Q have coordinates $P(-2, 5)$ and $Q(4, 1)$.

   (a) Find the coordinates of the midpoint, M, of the line segment [PQ].

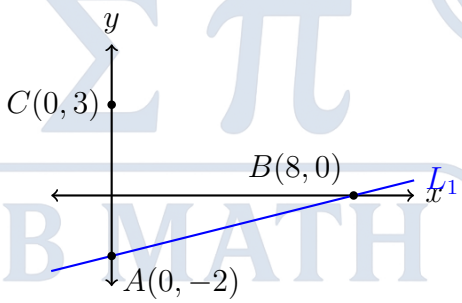
   (b) Find the gradient of the line segment [PQ].

   (c) Find the gradient of a line that is perpendicular to [PQ].

3. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
A straight line L_1 passes through the points $A(0, -2)$ and $B(8, 0)$.

   (a) Find the gradient of L_1.

   (b) A second line, L_2, is parallel to L_1 and passes through the point $C(0, 3)$. Find the equation of L_2, giving your answer in the form $y = mx + c$.



4. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
The equation of a line L_1 is $y = 0.5x + 3$. A second line, L_2, is perpendicular to L_1 and passes through the point $A(2, -1)$.

   (a) Find the gradient of L_2.

   (b) Find the equation of L_2. Give your answer in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$.

5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The line L_1 has equation $3x - 4y = 8$. The point P has coordinates $(9, 1)$.

   (a) Justify why point P is not on the line L_1.

   (b) Find the gradient of L_1.

   (c) A second line, L_2, is perpendicular to L_1 and passes through P. Find the equation of L_2, giving your answer in the form $y = mx + c$.

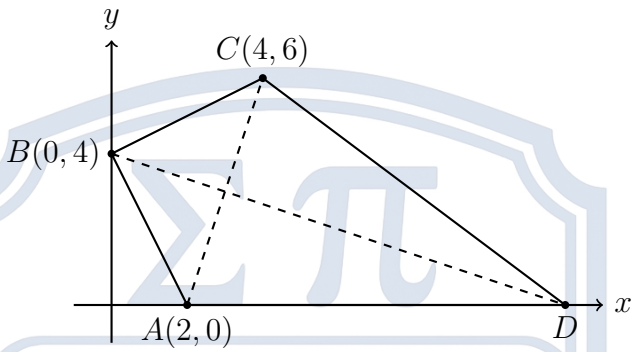
6. **[Paper 1 Style, Short Answer, Medium, 6 marks]**
Dilara is designing a kite ABCD on a coordinate grid. The coordinates of A, B, and C are $A(2, 0)$, $B(0, 4)$, and $C(4, 6)$ respectively. Point D lies on the x-axis. The diagonals [AC] and [BD] are perpendicular.

(a) Find the gradient of the line through A and C.

(b) Write down the gradient of the line through B and D.

(c) Find the equation of the line through B and D.

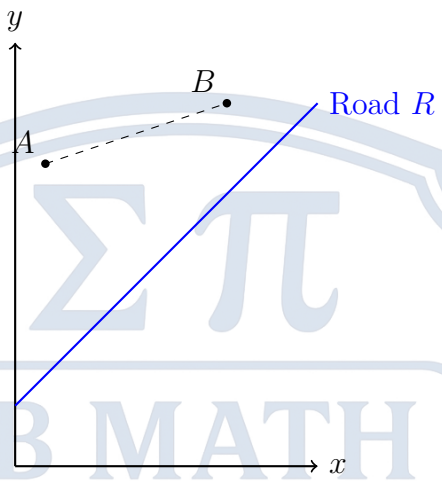
(d) Hence, write down the x-coordinate of point D.



7. **[Paper 2 Style, Longer Question, Medium, 7 marks]**
Two schools are represented by points $A(2, 20)$ and $B(14, 24)$ on a map. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. A town planner is asked to determine the location of a new bus stop on the road such that it is exactly the same distance from the two schools.

   (a) Find the equation of the perpendicular bisector of [AB]. Give your equation in the form $y = mx + c$.

   (b) Determine the exact coordinates of the point on road R where the bus stop should be located.



8. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
Three towns, A, B and C are represented as coordinates on a map. Town A is located at $(-6, -1)$ and town B is located at $(8, 6)$. A straight highway runs exactly along the perpendicular bisector of the line segment [AB].

   (a) Find the coordinates of the midpoint of [AB].

   (b) Find the equation of the line that the highway follows.

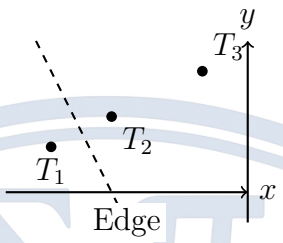
   (c) Town C is located on the highway and has an x-coordinate of -6. Find the y-coordinate of Town C.

9. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
The Voronoi diagram below shows three identical cellular phone towers, T_1, T_2, and T_3. The dashed lines represent the edges of the Voronoi cells. Tower T_1 has coordinates $(-13, 3)$ and Tower T_2 has coordinates $(-9, 5)$.

   (a) Find the coordinates of the midpoint between T_1 and T_2.

   (b) Find the gradient of the line segment connecting T_1 and T_2.

   (c) Find the equation of the Voronoi edge between T_1 and T_2, giving your answer in the form $y = mx + c$.



10. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
A Voronoi diagram is being constructed for two coffee shops, A and B. The location of coffee shop B is $(4, 6)$. The equation of the perpendicular bisector between sites A and B, which forms the Voronoi edge, is $y = -2x + 9$.

   (a) By considering the gradient of the edge, find the gradient of the line segment [AB].

   (b) Let the coordinates of A be (x, y). Form an equation in x and y using the gradient of [AB].

   (c) Show that the coordinates of coffee shop A are $(0, 4)$.

11. **[Paper 1 Style, Short Answer, Hard, 6 marks]**
Another Voronoi diagram contains sites C and D. The location of site C is $(2, 3)$. The Voronoi edge separating C and D has the equation $y = -x + 6$.

   (a) Find the equation of the line passing through C and D.

   (b) Hence, find the coordinates of site D.

12. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
The equation of a straight coastline is modelled by the line $L_1 : 2y - x - 10 = 0$. A boat is anchored at point $M(8, 18)$. The coastguard needs to find the shortest distance from the boat to the coastline.

(a) Find the gradient of the coastline L_1.

(b) Find the equation of the line L_2, which passes through M and is perpendicular to L_1.

(c) Find the coordinates of point D, the intersection of L_1 and L_2.

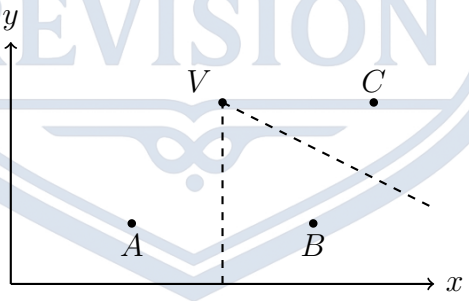
(d) Calculate the shortest distance from the boat to the coastline.

13. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
There are three stations used by fire wardens in a national forest, located at $A(4, 2)$, $B(10, 2)$, and $C(12, 6)$ on a coordinate plane. The region is partitioned into a Voronoi diagram.

(a) Write down the equation of the perpendicular bisector of [AB].

(b) Find the equation of the perpendicular bisector of [BC].

(c) A central command post is to be built at the Voronoi vertex V where the boundaries meet. Determine the exact coordinates of V.

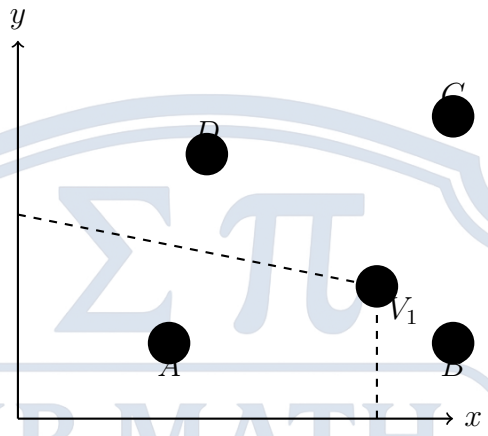


14. **[Paper 2 Style, Longer Question, Hard, 6 marks]**
Each year the popular music festival "Exit" is held in a large field. The Voronoi diagram below shows the positions of four toilets: $A(200, 100)$, $B(750, 100)$, $C(800, 400)$, and $D(250, 350)$. The coordinates are measured in metres.

(a) Write down the equation of the perpendicular bisector of [AB].

(b) The equation of the perpendicular bisector of [AD] is $x + 5y = 1350$. Find the coordinates of the Voronoi vertex V_1 that is equidistant from A, B, and D.

(c) Determine the distance from V_1 to toilet A.



15. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
Consider the Voronoi diagram for a city that contains four towns A, B, C, and D. The equation of the perpendicular bisector between sites A and D, denoted PB(A,D), is found to be $y = x - 25$. The coordinates of towns C and D are $C(39, 14)$ and $D(29, -26)$.

(a) Determine the equation of the perpendicular bisector between C and D, PB(C,D).

(b) Hence, determine the coordinates of the intersection Voronoi vertex V.

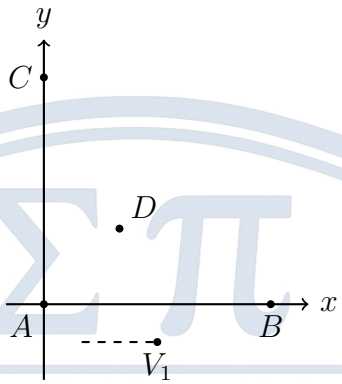
(c) Determine the optimal position for a new toxic waste dump in this district, such that it is as far away from the towns as possible, and calculate its distance from town D.

16. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0,0)$, $B(6,0)$, $C(0,6)$, and $D(2,2)$. The vertex V_1 is equidistant from A, B, and D.

(a) Find the midpoint of [BD].

(b) Find the equation of the perpendicular bisector of [BD].

(c) Given that the perpendicular bisector of [AD] is $y = -x + 2$, find the coordinates of V_1.



17. **[Paper 2 Style, Longer Question, Very Hard, 7 marks]**
The Voronoi diagram below shows four veterinary clinics represented by the points $A(-3,-1)$, $B(5,-1)$, $C(-1,5)$, and $D(1,1)$. All distances are in kilometres.

(a) Find the midpoint of [AD].

(b) Hence, find the equation of the perpendicular bisector of [AD].

(c) Find the equation of the perpendicular bisector of [CD].

(d) Calculate the coordinates of the Voronoi vertex V equidistant from A, C, and D.

18. **[Paper 2 Style, Longer Question, Very Hard, 5 marks]**
A king rules a small kingdom which is in the form of a square of length 4 kilometres. The square is described by the coordinate system $0 \leq x \leq 4$, $0 \leq y \leq 4$. He has four children, and places four castles at $(1, 1)$, $(3, 1)$, $(1, 3)$, and $(3, 3)$. The land is divided into a Voronoi diagram such that each child controls the land closest to their castle.

(a) Write down the equations of the two perpendicular bisectors separating the cells.

(b) State the coordinates of the central Voronoi vertex.

(c) Determine the exact area of the land controlled by the child whose castle is at $(1, 1)$.

19. **[Paper 2 Style, Longer Question, Very Hard, 4 marks]**
Three data centres are located at $P_1(1, 2)$, $P_2(5, 6)$, and $P_3(7, 2)$. The region is divided into a Voronoi diagram.

(a) Find the equation of the perpendicular bisector of P_1 and P_2.

(b) Find the equation of the perpendicular bisector of P_1 and P_3.

(c) Determine the exact coordinates of the Voronoi vertex V separating the three data centres.

20. **[Paper 2 Style, Longer Question, Very Hard, 5 marks]**
A town has three pizza restaurants: $A(2, 5)$, $B(6, 5)$, and $C(4, 1)$. A famous chef wishes to open a new pizza restaurant in the town but wants it to be located at the Voronoi vertex, as far away as possible from the three current restaurants.

(a) Write down the equation of the perpendicular bisector of [AB].

(b) Show that the equation of the perpendicular bisector of [AC] is $y = 0.5x + 1.5$.

(c) Determine the exact coordinates where the famous chef should position the new restaurant.

(d) Calculate the distance of the new restaurant from restaurant C.