Unit 5: Rules of Differentiation
IB Math AA SL

Answer all 15 questions. Show all working. For Paper 1 questions, use analytical algebraic methods. For Paper 2 questions, use your graphic display calculator (GDC) efficiently.

1. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   The equation of a curve is given by $y = \frac{3}{2}x^2 - 15x + 2$.
   (a) Find an expression for $\frac{dy}{dx}$.
   (b) Find the exact coordinates of the point on the curve where the gradient of the tangent is equal to 0.

2. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   Consider the function with rational exponents $f(x) = x^{\frac{2}{3}} + \frac{5}{x}$ for $x > 0$.
   (a) Rewrite $f(x)$ so that all terms are in the form ax^n. Hence, find $f'(x)$.
   (b) Use your graphic display calculator to evaluate the exact gradient of the curve at $x = 8$.

3. **[Paper 1 Style, Non-Calculator, Easy, 4 marks]**
   A function is defined by $g(x) = 4\cos x - 3\sin x$ for $0 \leq x \leq 2\pi$.
   (a) Find $g'(x)$.
   (b) Evaluate $g'\left(\frac{\pi}{2}\right)$.

4. **[Paper 2 Style, Calculator Required, Easy, 4 marks]**
   Let $h(x) = \ln(2x^3)$ for $x > 0$.
   (a) Use the chain rule, or the laws of logarithms, to show that $h'(x) = \frac{3}{x}$.
   (b) Use your GDC to find the equation of the tangent to the curve at $x = 4$, giving your answer in the form $y = mx + c$.

5. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Consider the function $f(x) = e^x \sin x$, for $0 \leq x \leq \pi$.

   (a) Use the product rule to find $f'(x)$.

   (b) The curve has a local maximum point in the given domain. By setting $f'(x) = 0$, find the exact x-coordinate of this maximum.

6. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
A curve has the equation $y = \frac{-7x}{x^3-1}$, for $x \neq 1$.

   (a) Use the quotient rule to find $\frac{dy}{dx}$.

   (b) Hence, use your GDC to find the x-coordinate of the local minimum point on the curve, giving your answer correct to 3 significant figures.

7. **[Paper 1 Style, Non-Calculator, Medium, 5 marks]**
Use the chain rule to differentiate the function $y = \sin(3x^2 + 5)$.

8. **[Paper 2 Style, Calculator Required, Medium, 5 marks]**
Consider the function $f(x) = (3x - 1)e^{\sin x}$.

   (a) Use the product rule and chain rule to find $f'(x)$.

   (b) Calculate the instantaneous rate of change of $f(x)$ at $x = \pi$, giving your answer correct to 3 decimal places.

9. **[Paper 1 Style, Non-Calculator, Hard, 5 marks]**
A curve has the equation $y = e^{-3x} + \ln x$, for $x > 0$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find the exact gradient of the normal line to the curve at the point where $x = 1$.

10. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
Consider the function $g(x) = \frac{\cos 3x}{4-5x^3}$.

   (a) Find $g'(x)$ using the quotient rule and chain rule.

   (b) Use the numerical derivative function on your GDC to evaluate the gradient of the curve at $x = 1.5$.

11. **[Paper 1 Style, Non-Calculator, Hard, 6 marks]**
Let $f(x) = (12x^2 - 7)e^{-2x}$.

   (a) Find an expression for $f'(x)$.

   (b) Show that $f'(x)$ can be written in the fully factorised form $f'(x) = -2e^{-2x}(ax^2 + bx + c)$, where a, b, and c are integers to be found.

12. **[Paper 2 Style, Calculator Required, Hard, 6 marks]**
A curve is given by $y = \cos(x^2 - 3x + 7) + \sin(e^x)$.

   (a) Find an expression for $\frac{dy}{dx}$.

   (b) Calculate the y-coordinate and the gradient of the tangent to the curve at $x = 0$.

   (c) Hence, find the equation of the tangent at $x = 0$, giving your answer in the form $y = mx + c$.

13. **[Paper 1 Style, Non-Calculator, Very Hard, 6 marks]**
Use the chain rule repeatedly to find the derivative of the composite logarithmic function $h(x) = \left(\ln(2x^2 - x - 2)\right)^5$. Leave your answer as a single algebraic fraction.

14. **[Paper 2 Style, Calculator Required, Very Hard, 6 marks]**
Consider the function f defined by $f(x) = xe^{3\cos x}$ for $-\pi \leq x \leq \pi$.

   (a) Show algebraically that $f'(x) = e^{3\cos x}(1 - 3x\sin x)$.

   (b) Find the number of points in the given interval at which the graph of f has a horizontal tangent. Justify your answer using your GDC.

15. **[Paper 1 Style, Non-Calculator, Very Hard, 7 marks]**
Let $f(x) = \frac{\ln(5x)}{kx}$ for $x > 0$, where k is a positive constant.

   (a) Use the quotient rule to show that $f'(x) = \frac{1-\ln(5x)}{kx^2}$.

   (b) By differentiating $f'(x)$, show that the second derivative is $f''(x) = \frac{2\ln(5x)-3}{kx^3}$.

   (c) The graph of f has exactly one point of inflection at point Q. Find the exact x-coordinate of Q.