Topic: Probability (Venn, Trees & Sample Spaces)
IB Math AI SL

Answer all questions. Show all working where appropriate. Total: 83 marks.

1. **[Paper 1 Style, Short Answer, Easy, 3 marks]**
A fair coin is tossed and a fair 4-sided die (numbered 1 to 4) is rolled.

   (a) List the complete sample space for this experiment.

   (b) Find the theoretical probability of obtaining a Head and a prime number.

2. **[Paper 1 Style, Short Answer, Easy, 4 marks]**
Events A and B are mutually exclusive. It is given that $P(A) = 0.25$ and $P(B) = 0.35$.

   (a) Find $P(A \cup B)$.

   (b) Find $P(A' \cap B')$, the probability that neither event A nor event B occurs.

3. **[Paper 1 Style, Short Answer, Easy, 5 marks]**
A survey was conducted among 50 students regarding their dominant hand. The results are shown in the two-way table below.

	Left-handed	**Right-handed**	**Total**
Boys	4	21	25
Girls	3	22	25
Total	7	43	50

   A student is chosen at random from the surveyed group.

   (a) Find the probability that the student is left-handed.

   (b) Given that a randomly chosen student is left-handed, find the probability that the student is a boy.

4. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
The probability that it rains on any given day is 0.4. The weather on any day is independent of the weather on the previous day.

   (a) Draw a fully labelled tree diagram for the weather over two consecutive days.

   (b) Find the probability that it rains on exactly one of the two days.

5. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Two fair 5-sided dice, each numbered 1 to 5, are rolled. The sum of the two numbers shown is recorded.

+	1	2	3	4	5
1					
2					
3					
4					
5					

   (a) Find the probability that the sum of the two dice is strictly greater than 7.

   (b) Given that the sum of the two dice is strictly greater than 7, find the probability that the sum is exactly 9.

6. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
In a group of 60 people, 35 drink tea (T), 40 drink coffee (C), and 10 drink neither tea nor coffee.

   (a) Find the number of people who drink both tea and coffee.

   (b) Draw a fully labelled Venn diagram to represent this information.

   (c) A person is chosen at random from the group. Find the probability that they drink tea, given that they drink coffee.

7. **[Paper 2 Style, Longer Question, Medium, 6 marks]**
A bag contains 4 red marbles and 5 green marbles. Two marbles are drawn from the bag in succession **without replacement**.

   (a) Draw a fully labelled tree diagram to represent this experiment.

   (b) Calculate the probability that both marbles drawn are green.

   (c) Calculate the probability of drawing exactly one red marble and one green marble, in any order.

8. **[Paper 1 Style, Short Answer, Medium, 5 marks]**
Two events E and F are such that $P(F) = 0.4$, $P(E) = 0.6$, and $P(E \cup F) = 0.76$.

   (a) Find $P(E \cap F)$.

   (b) Determine, with clear mathematical reasoning, whether events E and F are independent.

9. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
A factory has two machines, A and B. Machine A produces 60% of the factory's items, and Machine B produces the remaining 40%. It is known that 2% of the items produced by Machine A are defective, and 5% of the items produced by Machine B are defective.

   (a) Draw a tree diagram to represent this situation.

   (b) Calculate the probability that a randomly chosen item from the factory is defective.

   (c) A randomly chosen item is inspected and found to be defective. Find the probability that it was produced by Machine B.

10. **[Paper 1 Style, Short Answer, Hard, 4 marks]**
Events A and B are mutually exclusive. It is known that $P(A) = 2x$ and $P(B) = x$. Given that $P(A \cup B) = 0.9$, find the value of x.

11. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
Sara regularly flies from Geneva to London. She takes either a direct flight or a non-direct flight. The probability that she takes a non-direct flight is 0.2.
If she takes a direct flight, the probability that her baggage arrives in London is 0.99. If she takes a non-direct flight, the probability that her baggage arrives in London is 0.95.

   (a) Complete a tree diagram for this scenario.

   (b) Find the overall probability that Sara's baggage arrives in London.

   (c) Given that Sara's baggage arrived safely in London, find the probability that she took a direct flight.

12. **[Paper 2 Style, Longer Question, Hard, 7 marks]**
Two fair six-sided dice are simultaneously rolled in a game. Let the random variable X be the **absolute difference** between the numbers shown on the two dice.

(a) Using a 2D sample space grid, determine the probabilities for all possible values of X. Construct a probability distribution table for X.

(b) Calculate $E(X)$, the expected value of the difference when rolling the two dice.

13. **[Paper 2 Style, Longer Question, Very Hard, 8 marks]**
A survey of 100 university students was conducted to see which subjects they study out of Physics (P), Chemistry (C), and Biology (B).

- 5 students study all three subjects.
- 12 students study Physics and Chemistry.
- 15 students study Biology and Chemistry.
- 10 students study Physics and Biology.
- 40 students study Chemistry in total.
- 30 students study Physics in total.
- 18 students study none of these three subjects.

(a) Draw a Venn diagram to represent this information, finding the number of students in each region.

(b) Find the probability that a randomly chosen student studies Biology but **not** Chemistry.

14. **[Paper 2 Style, Longer Question, Very Hard, 6 marks]**
Andre is playing in a tennis tournament. The probability that he wins his semi-final match is p. If he wins the semi-final, he advances to the final. The probability that he wins the final, given that he won the semi-final, is 0.7. If he loses the semi-final, he is eliminated and cannot win the final.
The overall probability that Andre will **not** be the champion (meaning he does not win the final) is 0.58.

(a) Find the value of p.

(b) Given that Andre did not become the champion, find the probability that he lost in the semi-final.

15. **[Paper 1 Style, Short Answer, Very Hard, 5 marks]**
For two events A and B, you are given that $P(A) = 0.4$, $P(B|A) = 0.25$, and $P(A \cup B) = 0.6$.

(a) Calculate $P(A \cap B)$.

(b) Find the exact value of $P(B)$.