

IB MATHEMATICS AA HL AHL TOPIC 1 PRACTICE

Complex Numbers & De Moivre's Theorem

Instructions to Candidates

0.2cm

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests the full AHL syllabus: Cartesian, Polar, and Euler forms, roots of complex numbers, geometric interpretations, and De Moivre's Theorem.
- Answer all questions, showing all your working clearly.
- Total marks available: **77**.

Difficulty Progression

0.2cm

- **SECTION A (Easy):** Basic Cartesian operations, finding the conjugate, calculating modulus and argument, and simple Argand diagram applications.
- **SECTION B (Medium):** Multiplication and division in polar/Euler forms, fundamental applications of De Moivre's Theorem, solving basic equations, and complex polynomial roots.
- **SECTION C (Hard):** Generating complex n -th roots, geometric transformations, algebraic proof using double/triple angle identities, and synthesizing complex numbers with infinite geometric series.

SECTION A: EASY (Fundamentals)**Question 1 (2 Marks) — Paper 1 (No Calculator Allowed)**

Consider the complex number $z = 3 - 4i$. Find its complex conjugate z^* , and calculate its exact modulus $|z|$.

Question 2 (2 Marks) — Paper 1 (No Calculator Allowed)

Expand and simplify the product $(2 + i)(3 - 2i)$, giving your answer in the Cartesian form $a + bi$.

Question 3 (4 Marks) — Paper 2 (Calculator Allowed)

By multiplying the numerator and denominator by the complex conjugate of the denominator, express $z = \frac{5+5i}{1-2i}$ in the form $a + bi$.

Question 4 (3 Marks) — Paper 1 (No Calculator Allowed)

Two complex numbers are given as $z_1 = 2 + i$ and $z_2 = -1 + 3i$. Calculate the exact geometric distance between these two points when plotted on an Argand diagram.

Question 5 (3 Marks) — Paper 1 (No Calculator Allowed)

Express the complex number $z = 1 + \sqrt{3}i$ in the polar (modulus-argument) form $r \operatorname{cis}(\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.

Question 6 (2 Marks) — Paper 1 (No Calculator Allowed)

Express the complex number $w = 4e^{i\frac{\pi}{4}}$ in the Cartesian form $a + bi$.

Question 7 (3 Marks) — Paper 1 (No Calculator Allowed)

Solve the quadratic equation $x^2 - 4x + 13 = 0$ for $x \in \mathbb{C}$.

SECTION B: MEDIUM (Application & Transformations)**Question 8 (3 Marks) — Paper 1 (No Calculator Allowed)**

Given $z_1 = 4 \operatorname{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = 2 \operatorname{cis}\left(\frac{\pi}{6}\right)$, find the product $z_1 z_2$, leaving your answer in the simplest Cartesian form $a + bi$.

Question 9 (4 Marks) — Paper 1 (No Calculator Allowed)

Let $z = a + bi$ where a and b are real numbers. Solve the equation:

$$3z + z^* = 8 - 4i$$

Question 10 (4 Marks) — Paper 2 (Calculator Allowed)

Use De Moivre's Theorem to evaluate $(1 - i)^6$. Give your final answer in the form $a + bi$.

Question 11 (3 Marks) — Paper 1 (No Calculator Allowed)

Find the exact modulus and the principal argument of the complex number $w = \frac{4 \operatorname{cis}\left(\frac{5\pi}{6}\right)}{2 \operatorname{cis}\left(\frac{\pi}{3}\right)}$.

Question 12 (4 Marks) — Paper 2 (Calculator Allowed)

A cubic polynomial $P(z) = z^3 + az^2 + bz + 10$ has entirely real coefficients $a, b \in \mathbb{R}$. Given that $z_1 = 1 + 2i$ is a root of the equation $P(z) = 0$, find the other two roots.

Question 13 (2 Marks) — Paper 1 (No Calculator Allowed)

A geometric transformation in the complex plane maps the point z to a new point w such that $w = iz$. Describe fully the geometric nature of this transformation.

Question 14 (5 Marks) — Paper 1 (No Calculator Allowed)

By setting $z = a + bi$, solve the equation $z^2 = 3 + 4i$ algebraically to find the exact square roots of $3 + 4i$.

SECTION C: HARD / VERY HARD (Synthesis & Proof)**Question 15 (6 Marks) — Paper 2 (Calculator Allowed)**

Find the three distinct complex cube roots of $w = 27 \operatorname{cis} \left(\frac{3\pi}{4} \right)$. Give your answers in the Euler form $re^{i\theta}$, where $-\pi < \theta \leq \pi$.

Question 16 (6 Marks) — Paper 1 (No Calculator Allowed)

Find the four complex roots of the equation $z^4 + 16 = 0$. Plot them roughly on an Argand diagram, and state the geometric shape formed by connecting the roots.

Question 17 (6 Marks) — Paper 1 (No Calculator Allowed)

By applying De Moivre's Theorem to $(\cos \theta + i \sin \theta)^3$ and expanding using the binomial theorem, prove the trigonometric identity:

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

Question 18 (4 Marks) — Paper 1 (No Calculator Allowed)

Let $z = \operatorname{cis} \theta$. Show algebraically that $z^n + z^{-n} = 2 \cos(n\theta)$ for any integer n .

Question 19 (4 Marks) — Paper 1 (No Calculator Allowed)

Let $\omega = e^{\frac{2\pi i}{3}}$ be one of the complex cube roots of unity. Show algebraically that $1 + \omega + \omega^2 = 0$.

Question 20 (7 Marks) — Paper 2 (Calculator Allowed)

Consider the infinite geometric series:

$$S_c = 1 + \frac{1}{2} \cos \theta + \frac{1}{4} \cos 2\theta + \frac{1}{8} \cos 3\theta + \dots$$

By considering the complex series $S = 1 + \frac{1}{2}e^{i\theta} + \frac{1}{4}e^{2i\theta} + \dots$, show that:

$$S_c = \frac{4 - 2 \cos \theta}{5 - 4 \cos \theta}$$

