IB MATHEMATICS AA HL
AHL TOPIC 2 PRACTICE

Transformations & Modulus Functions

Instructions to Candidates

- This practice paper contains **20** questions progressing from Easy to Very Hard.
- Each question indicates whether it is styled for **Paper 1 (No Calculator)** or **Paper 2 (Calculator Allowed)**.
- The paper tests syllabus topics SL 2.11 and AHL 2.16: Translations, stretches, reflections, composite transformations, solving modulus equations/inequalities, and graphing $y = |f(x)|$, $y = f(|x|)$, $y = [f(x)]^2$, and $y = 1/f(x)$.
- Answer all questions, showing all your working clearly.
- Total marks available: **95**.

Difficulty Progression

- **SECTION A (Easy):** Extracting translation vectors, solving basic absolute value equations, determining transformed coordinates, and simple inequality regions.
- **SECTION B (Medium):** Sequencing composite transformations, solving modulus equations yielding extraneous roots, sketching $y = 1/f(x)$ asymptotes, and algebraic inequalities by squaring.
- **SECTION C (Hard):** Graphing $y = [f(x)]^2$ turning points, area bounded by modulus graphs, complex piecewise absolute value combinations, and mapping inverse transformation sequences.

SECTION A: EASY (Fundamentals)

Question 1 (2 Marks) — Paper 1 (No Calculator Allowed)

The graph of $y = f(x)$ is translated by the vector $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$. Write down the equation of the transformed graph in terms of $f(x)$.

Question 2 (3 Marks) — Paper 1 (No Calculator Allowed)

Solve the absolute value equation $|4x - 1| = 11$ algebraically.

Question 3 (3 Marks) — Paper 1 (No Calculator Allowed)

The point $A(6, -8)$ lies on the curve $y = f(x)$. Find the exact coordinates of the image of point A under the transformation $y = \frac{1}{2}f(3x)$.

Question 4 (3 Marks) — Paper 1 (No Calculator Allowed)

Describe the full sequence of three geometric transformations that maps the graph of $y = f(x)$ to the graph of $y = -f(x - 5) + 2$.

Question 5 (3 Marks) — Paper 1 (No Calculator Allowed)

Solve the modulus inequality $|2x + 3| \leq 9$ algebraically.

Question 6 (3 Marks) — Paper 1 (No Calculator Allowed)

A function $f(x)$ has x-intercepts at $x = -2$ and $x = 7$. State the equations of the vertical asymptotes of the reciprocal graph $y = \frac{1}{f(x)}$.

Question 7 (4 Marks) — Paper 1 (No Calculator Allowed)

Let $f(x) = x^3 - 4x$. Find the explicit equation of $g(x)$ if the graph of $f(x)$ is reflected in the y-axis and then vertically stretched by a scale factor of 3. Give your answer in the form $g(x) = ax^3 + bx$.

SECTION B: MEDIUM (Application & Algebraic Methods)

Question 8 (5 Marks) — Paper 2 (Calculator Allowed)

Solve the quadratic modulus equation $|x^2 - 3x| = 4$.

Question 9 (4 Marks) — Paper 1 (No Calculator Allowed)

Explain the geometric difference between the transformations $y = |f(x)|$ and $y = f(|x|)$.

Question 10 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the inequality $|3x - 1| \geq |x + 5|$ algebraically by squaring both sides.

Question 11 (6 Marks) — Paper 1 (No Calculator Allowed)

The function $f(x) = e^x$ is mapped to $g(x) = e^{2x-6} + 1$. Describe a sequence of two transformations that maps $f(x)$ onto $g(x)$.

Question 12 (5 Marks) — Paper 1 (No Calculator Allowed)

Solve the modulus equation $\left|\frac{x+4}{x-2}\right| = 3$ algebraically.

Question 13 (5 Marks) — Paper 1 (No Calculator Allowed)

The graph of $y = f(x)$ has a horizontal asymptote of $y = 3$ and a vertical asymptote of $x = -4$. Find the equations of the horizontal and vertical asymptotes of the transformed function $g(x) = -2f(x - 5) + 1$.

Question 14 (6 Marks) — Paper 2 (Calculator Allowed)

Solve the modulus inequality $|x^2 - 16| < 9$ algebraically or using your graphic display calculator.

SECTION C: HARD / VERY HARD (Synthesis & Proof)

Question 15 (6 Marks) — Paper 1 (No Calculator Allowed)

Solve the equation $|x^2 - 6x| = 4x$ algebraically. You must justify the validity of all potential roots.

Question 16 (6 Marks) — Paper 1 (No Calculator Allowed)

A continuous function $f(x)$ passes through the origin, has a local minimum at $(3, -4)$, and a local maximum at $(6, 2)$. Consider the squared transformation $g(x) = [f(x)]^2$. Find the coordinates and nature of the new turning points on the graph of $g(x)$ corresponding to $x = 3$ and $x = 6$.

Question 17 (7 Marks) — Paper 2 (Calculator Allowed)

Find the exact area of the triangle formed by the intersection of the modulus graph $y = 8 - |2x - 6|$ and the x-axis.

Question 18 (6 Marks) — Paper 1 (No Calculator Allowed)

Solve the piecewise modulus equation $|x - 1| + |x + 4| = 9$ algebraically.

Question 19 (7 Marks) — Paper 1 (No Calculator Allowed)

Let $f(x) = \frac{ax+b}{cx+d}$. The graph of f has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 3$. It passes through the point $(3, 11)$. If the graph of $f(x)$ is translated by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ to form $g(x)$, find the explicit expression for $g(x)$ in the form $\frac{px+q}{rx+s}$.

Question 20 (6 Marks) — Paper 2 (Calculator Allowed)

Use your graphic display calculator to determine the exact number of real solutions to the equation:

$$e^{|x|} - 2 = |x^2 - 5|$$

Briefly explain your method.